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TRANSLATION

ELECTRONIC ANALOG COMPUTERS AND THEIR APPLICATION FOR INVESTIGATION OF AUTOMATIC CONTROL SYSTEMS

By

B. Ya. Kogan

FOREIGN TECHNOLOGY DIVISION

AIR FORCE SYSTEMS COMMAND

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EDITED MACHINE TRANSLATION

ELECTRONIC ANALOG COMPUTERS AND THEIR APPLICATION FOR INVESTIGATION OF AUTOMATIC CONTROL SYSTEMS

BY: B. Ya. Kogan

English Pages: 521

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TRANSLATION DIVISION
FOREIGN TECHNOLOGY DIVISION
WP-AFB, OHIO.

FTD-MT- 64-115

Date 4 February 1965

B. Ya. Kogan

ELEKTRONNYYE MODELIRUYUSHCHIYE USTROYSTVA

i ikh Primeneniye
dlya Issledovaniya Sistem

Abtomaticheskogo
Regulirovaniya

izdaniya vtoroye,
ispravlennoye
i dopolnennoye

Gosudarstvennoye Izdatel'stvo
Fisiko-Matematicheskoy Literatury

Moskva - 1963

Pages 1-510

The author dedicates this book
o the memory of his teacher and friend,

EVGENIY KONSTANTINOVICH POPOV,

Who perished in the battle on the Volga in
August, 1942.

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FOLLOWING ARE THE CORRESPONDING RUSSIAN AND ENGLISH DESIGNATIONS OF THE TRIGONOMETRIC FUNCTIONS

Russian	English
sin	sin
cos	cos
tg	tan
ctg	cot
sec	sec
cosec	csc
sh	sinh
ch	cosh
th	tanh
cth	coth
sch	sech
csch	csch
arc sin	$\sin^{-1}$
arc cos	$\cos^{-1}$
arc tg	$\tan^{-1}$
arc ctg	$\cot^{-1}$
arc sec	$\sec^{-1}$
arc cosec	$\csc^{-1}$
arc sh	$\sinh^{-1}$
arc ch	$\cosh^{-1}$
arc th	$\tanh^{-1}$
arc cth	$\coth^{-1}$
arc sch	sech^{-1}
arc csch	csch^{-1}

rot	curl
lg	log
ct	step

CYRILLIC AND OTHER DESIGNATIONS USED IN THE TEXT

БФИ - pulse-shaping unit

ВП - vibrapack

ГР - scanning generator

Д - diode

ДН - voltage divider

доп - permissible

ДП - diode generator

ЗИ - sign-inverting amplifier

$К_{\text{Д}}$ - dynamic range of voltage

$К_{\text{общ}}$ - total amplification factor

ком - kohm

$к_{\text{т}}$ - tach factor

$К_{\text{у}}$ - amplification factor

$М_{\text{в}}$ - disturbing moment

$М_{\text{дв}}$ - moment of motor

$М^{(\text{эл})}$ - electromagnetic moment

мгом - megohm

мкф - mkf

$М^{(\text{эл})}_{\text{тор}}$ - retarding moment

$М^{(\text{мех})}$ - mechanical moments

$М_{\text{п}}$ - starting moment

П - potentiometer

ПЕ - transition unit

ПД - peak detector

пф - pf

расч - calculated

САР - automatic control system

$С_{\text{ф}}$ - phase comparator

ТМ - tachomachine

уст. расч - steady-state calculated

уст. экс. - steady-state by experiment

ФП - functional generator

ФЭУ - photomultiplier

ШИ - stepping selector

ШИР - reversible stepping selector

экс - by experiment

ЭЛТ - cathode-ray tube

$С_{\text{ос}}$ - capacitor of feedback circuit

$e_{\text{вх}}$ - input voltage

$e_{\text{вх. гран}}$ - boundary input voltage

$e_{\text{вх. max}}$ - maximum input voltage

$e_{\text{вх. н}}$ - initial input voltage

$e^{\text{г}}_{\text{вх}}$ - approximate input voltage

$e_{\text{вых}}$ - output voltage

$e_{\text{вых. гран}}$ - boundary output voltage

$e_{\text{вых. и}}$ - ideal output voltage

$e_{\text{вых. max}}$ - maximum output voltage

$e_{\text{вых. н}}$ - initial output voltage

$e_{\text{вых. р}}$ - real output voltage

$e_{\text{н}}$ - initial voltage

$e_{\text{н.уст}}$ - steady-state voltage, initial

$e_{\text{оп}}$ - reference voltage

$E_{\text{пер}}$ - rectangular voltage

$e_{т}$ - tachogenerator voltage
$e_{экв}$ - equivalent voltage
$F_{в}$ - disturbing force
$I_{гран}$ - boundary current
$I_{н}$ - load current
$I_{ос}$ - feedback current
$I_{п}$ - resting current
$I_{р}$ - current in frame
$I_{ФЭУ}$ - photomultiplier current
$I_{я}$ - armature current
$t_{имп}$ - pulse length
$P_{вых.ср}$ - average output power
$P_{потр}$ - power consumed
$P_{потр.ср}$ - average power consumed
$R_{вн}$ - internal resistance
$R_{вх}$ - input resistance
$R_{вых}$ - output resistance
$R_{д}$ - diode resistance
$r_{доб}$ - additional resistance
$R_{к}$ - resistor cascade
$R_{н}$ - load resistor
$R_{ос}$ - feedback resistor
$R_{у}$ - leak resistance
$R_{я}$ - armature resistance
$t_{имп}$ - pulse time
$t_{реш}$ - computing time
$t_{уст}$ - steady-state time
$U_{доп}$ - permissible voltage
$U_{отн}$ - relative voltage
$U_{п}$ - sawtooth voltage
$U_{с}$ - signal voltage
$U_{ср}$ - mean voltage
$U_{я}$ - armature voltage
$x_{вых.уст}$ - steady-state output
$x_{уст}$ - $x_{steady-state}$
$y_{зад}$ - y_{task}
$y_{н}$ - inductance of load
$y_{экв}$ - equivalent conductance
$Z_{вх}$ - input impedance
$Z_{вых}$ - output resistance
$Z_{н}$ - load impedance

$\Delta e_{вых.случ}$ - random component of error
$\Delta e_{вых.сист}$ - systematic error
$\Delta \bar{e}_{вых\ д}$ - zero drift of output
$\Delta \bar{e}_{д}$ - zero drift
$\varepsilon_{обр}$ - inverse function error
$\varepsilon_{пр}$ - direct function error
$\varphi_{н}$ - φ_{load}

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FOREWARD TO THE FIRST EDITION

The development of systems of automatic adjustment and control confronts the theory and practice of automatic adjustment with new and increasingly complicated problems.

For the last 8-10 years in the solution of these problems d-c electronic analog computers of direct current found wide application. They are used for calculation, in the carrying out of experimental investigations and settings, and most recently as elements themselves of systems of control and adjustment. However, the value of electronic analog computers is far from exhausted by these applications alone.

The combination of electronic models with electronic digital computers, the construction of combined discrete-continuous computers opens in the technology of modeling broad new prospects.

In spite of all this, in the domestic literature to the present in essence, books containing generalised and systematised questions of theory, general principles of construction and methods of use of d-c electronic models, summarising experience accumulated in the Soviet Union and abroad, are lacking.

The purpose of the present book is to fill the existing gap in the literature on the considered question. Contents of the book are limited to the consideration of electronic models and their main decision elements, and also to questions of application of models for solution of problems of the dynamics of automatic control systems.

In this there is taken the assumption that the operational amplifier possesses ideal frequency responses, i.e., its transfer function in the open state is a constant number, equal to the amplification factor. This assumption made it possible to ignore in this book, intended for the initial familiarisation for a wide circle of readers with the technology of electronic modeling, with a number of complicated, very important, interesting, but at the same time still not completely developed questions.

The limited contents of the book also impelled the author to exclude from investigation a number of questions, which, in regard to the solution of problems of automatic control are not of prime importance or can be solved with great effectiveness with the help of other means of computing technique. The latter questions include in the first place solution with the help of electronic models of algebraic equations, partial differential equations and integral equations.

Contents of the book are calculated for the reader, who is familiar with questions of theory and practice of automatic adjustment and control, and also with the principles of electronics.

In the writing of this book there were used results of the development of a series of electronic models of type EMU*, obtained by the author and his collaborators in the Institute of Automation and Telemechanics of the Academy of Sciences of the USSR, and also material from lectures on the course "Simulation of systems of automatic adjustment," presented by the author from 1950 to 1956 for the Higher Engineering Courses at Moscow Highest Technical School, in the Institute of Automation and Telemechanics of the Academy of Sciences of USSR for scientific collaborators and in the Moscow Physicotechnical Institute.

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*EMU is Russian abbreviation for electronic analog computer. Ed. Note.

References to the literature used are given in the text with indication of the surnames of the author and ordinal number of his work (in brackets). The list of cited literature shown at the end of the book is composed in Russian alphabetical order by surnames of the authors*).

I consider it my own pleasant duty to offer my deep gratitude to A. A. Fel'dbaum, I. Z. Tsypkin, I. M. Tetel'baum and L. V. Yamshanov, who examined the manuscript and made a number of valuable remarks, which were considered in the final editing of the book. I express gratitude to V. A. Trapeznikov, whose advice I constantly used during the ten-year joint work in the fields of electronic simulation, and to the collective of collaborators with whom the author worked at the Institute of Automation and Telemechanics.

During preparation of the manuscript for publication great help was rendered to the author by F. Ye. Tranin, V. V. Gurov, A. A. Maslov and T. V. Pritullo, for which the author expresses his sincere gratitude.

Moscow, April 1957.

---

*For a fuller bibliography on questions of simulation from 1947 to 1955 see the journal Automation and Telemechanics, V. XVII, No. 3 and 4, 1956.

FOREWARD TO THE SECOND EDITION

The broad introduction of electronic analog computers (electronic modeling denies) in the practice of scientific investigations and technical developments, apparently explains the fact that the first edition of the present book was sold out very fast and there appeared the necessity for republication.

In the second edition the general structure of book is retained. The introduction was rewritten; in the first Section (Chapter IV) there is added a Sub-Section on amplifiers with parallel channels of amplification. The second Section is supplemented by two Chapters — XII and XIV. The first is devoted to solution of problems of automatic control in cases of fractional rational transfer functions, the second — to methods of investigation of the dynamics of systems, whose range of change of variables (coordinates) exceeds the dynamic range of the analog installation. Corrections are introduced in the table of installations produced in the USSR and abroad (Appendix II); photographs of obsolete samples are replaced by photographs of contemporary ones. The list of literature is completed mainly with names of books on analog computing technique, issued after the first edition of this book*).

In the second edition are also corrected noticed misprints and inaccuracies.

---

*See bibliography on questions of analog computing technique from 1955 to 1960 in the journal Automation and Telemechanics, V. XVIII, No. 9, 1957; V. XIX, No. 5, 1958; V. XX, No. 11, 12, 1959; V. XXI, No. 12, 1960; V. XXIII, No. 2, 3, 1962.

The author considers it his privilege to express deep gratitude to all comrades who indicated errors in the first edition.

Moscow, May, 1962

B. Kogan

INTRODUCTION

The development and wide propagation of systems of automatic adjustment and control in industry and military technology require the solution of a number of problems connected with questions of rational selection of structure and elements of these systems, evaluating the influence of separate parameters on the nature of transient and steady states.

Before the investigator and designer there is placed the problem of giving not only a qualitative solution of these problems, but also to bring the solution to a numerical result. Even for comparatively simple linear systems of automatic adjustment the latter presents known difficulties practically in all stages of development (composition of the mathematical description, calculations of transients, experimental investigation of systems) and requires significant expenditure of time. These difficulties increase with complication of the system of automatic adjustment (for example, in case of the multicircuit systems), with the presence in the system of elements with nonlinear characteristics and variable parameters, and also with the necessity of taking into account continuously varying random disturbances.

Examples of such systems of automatic adjustment are systems of stabilization of the motion of aircraft, systems of industrial adjustment with a constant delay (for example, systems of automatic adjustment of the thickness of rolled material), systems of automatic tracking of target by radar, when into the error signal enter besides the useful signals also interferences, systems of automatic adjustment and control with many interconnected regulated magnitudes (boiler, paper-making machine, complex power installation, etc).

On the other hand, the acceleration of processes in objects of adjustment (intensification of production) leads to increase of the influence of small parameters and nonlinearity of the regulator and object. In these cases the systems are described by nonlinear differential equations of a high order, for whose solution, in general, analytic methods are still not developed. With the analogous position one also comes into contact during investigation of automatic control and adjustment, systems which are based on new principles (for example, optimizing control and self-adjusting systems).

The use of the methods and means of simulation at a definite level of the analytic investigation of systems of automatic adjustment allows one to obtain solutions for the enumerated problems, reducing thereby to a minimum the required expenditure of time.

The essence simulation consists of replacement of the whole control system or certain of its elements with a model, in its properties to greater or lesser extent reproducing properties of the initial system or its separate parts. Then in the system, containing the model, there appear processes analogous to those which take place in the real system. These processes one can observe, record, check for their conformity to the results of theoretical analysis, replace the analytic calculations of the transient with its direct observation, test, and adjust the system in laboratory conditions.

Thus, simulation allows one also to solve the basic problems of experimental investigation.

At present there are distinguished two basic methods of simulation:

1. Physical simulation.
2. Mathematical simulation.

Physical simulating is based on study of phenomena on models of the same physical nature as the original. Examples are tests of aircraft models in wind tunnels, the replacement of huge synchronous generator by a synchronous generator of smaller dimensions.

Inasmuch as the physical nature of the process is retained, the model reproduce whole complex of phenomena, characterizing the investigated process. In this complex enter or can enter, in particular, also such aspects of the phenomena or process, which do not yield to mathematical description and cannot be considered in the equations of the process. Therefore, physical simulation permits one to deepen the knowledge regarding the complex of the occurring phenomena, to refine and to facilitate the mathematical description of separate processes. Methods of mathematical simulation, which reproduce the investigated process only in the frames of the given equations are partly devoid of these possibilities.

Physical simulation has been applied in technology for a long time, mainly in aero- and hydrodynamics and in construction technology; it finds application also in many cases of investigations of control systems.

The method of physical simulation of control systems has the following merits:

a) the properties of the control system are reproduced more fully than in mathematical simulation, resting on an idealized mathematical description of the object;

b) the regulating equipment can be joined to the model without conversion devices, introducing additional errors and distortion.

At the same time physical simulation has also substantial deficiencies:

a) during investigation of each new process it is necessary to create a new model;

b) variation of the parameters of the simulated object usually causes labor-consuming alterations of the model or even its replacement;

c) models of complicated objects (boilers, various power plants) usually are very expensive.

Determining the place of physical simulation, one should note that this method, without doubt, is less versatile than the method of mathematical simulation, but in a number of cases it turns out to be very effective; for example, for the investigation of processes of adjustment and various non-stationary regimes in power systems, separate units of chemical and metallurgic productions, in investigation of automation of electric drives, in investigation of pneumatic regulators, etc.

The theory and practice of physical simulation is sufficiently well-developed in domestic and foreign works*.

During the investigation of systems of automatic control during the last few years methods of mathematical simulation received wide application, based on the identity of differential equations describing the phenomena in the original and the model. They permit one to accomplish with the help of one device the solution of a whole group of problems, provide speed and ease of transition from one problem to another, the possibility of introduction of variable parameters and various initial conditions, almost complete removal of the influence of its own parameters of the model equipment on accuracy of the solution, simplicity of introduction of various kinds of systematic and random disturbances, possibility of simulating systems of automatic control by elements.

Equally with this with mathematical simulation it is possible comparatively simple to change the parameters of separate elements of the investigated system and explain the influence of these changes on the performance of the system on the whole.

In many cases it is useful to combine installations of physical and mathematical simulation in a single system, allowing one to combine the advantage of both methods.

In mathematical simulation the original is the mathematical description of the process, and the mathematical models themselves can be considered the devices realizing the given mathematical relationships, i.e., computers.

---

*See in Russian the works of M. V. Kirpichev and M. A. Mikheyev [1], M. P. Kostenko [1], V. A. Venikov [1].

In each computer the mathematical operations, assigned by the initial equations, are executed on certain so-called machine quantities. Each quantity, participating in the initial relationships, can be, thus, placed in conformity with one or several of the machine quantities.

Depending upon the method of representation of the initial quantities in the machine there are distinguished two classes of computers:

1) digital computers,

2) analog computers.

In digital computers the instantaneous value of each initial quantity is represented as several machine quantities, whose weight, or value, is determined by their spatial or temporary location (the position code of the representation of the initial quantity). In purely digital machines the transition from the initial quantity to its representation is accomplished by quantization of the first one by level. Elementary mathematical operations, executed on the representing quantities, are limited here to the addition composition and shift. Since each mathematical operation requires a certain number of additions of numbers, represented in position codes, then to obtain a result always requires a finite time. By force of this the selection of the values of initial continuous quantities is executed discretely (with quantisation in time).

Anlog computers are characterized by the fact that to each instantaneous value of the initial quantity is placed in conformity an instantaneous value of a machine quantity, often differing in physical nature and scale factor.

Important is the fact that each elementary computing element in the machine executes a strictly defined elementary mathematical operation on machine quantities.

To this operation, as rule, there corresponds a certain physical law, establishing required mathematical dependences between the physical quantities at the input and output of the computing element. As such laws it is possible, for example, to indicate the laws of Ohm and Kirchhoff for electric circuits, the expression for the Hall effect, Lorentz force and so forth.

The division accepted herein of computers, by the method of representation of quantities, into analog and digital, in distinction from the general-usage division (by the nature of signals circulating in the machine) into continuous and discrete, in the opinion of the author, reflects better the most essential laws of the work of a computer.

Processes in computers of both classes can be described by mathematical relationships (although different), which are analogous to the mathematical relationships of the initial problem, and in this sense from the most general positions computers can be considered means of mathematical simulation and one can preserve for electronic analog computers their former name—electronic simulating devices.

In accordance with these determinations in Fig. 1 is depicted the classification of means of mathematical simulation.

Peculiarities of representation of initial quantities and construction of separate computing elements in analog computers to a significant measure predetermine their comparatively high speed of work, simplicity of programming and setting-up, limiting, on the other hand, the dynamic range and accuracy of the obtained result.

As compared with digital computers analog devices differ also by less versatility in the sense that during transition from one class of problems to another it is necessary not only to change the relationship between the number of linear and nonlinear computing elements, but also to supplement the installation in principle with new elements.

The limitation of accuracy in most cases is not a substantial obstacle to use of these devices, since systems of automatic adjustment and control, as a rule, are crude, in the sense of Andronov, dynamic systems whose parameters are known with an accuracy not exceeding 10 to 20%.

The impossibility of solving any problem by the same equipment should also not be considered a substantial limitation for analog technology, considering its application for the solution of problems of the dynamics of adjustment and control. These problems, basically reducible to the class of ordinary differential equations, can be solved with success within the limits of one, sufficiently great quantity of computing elements, installation which remains still comparatively simple and reliable.

It is impossible to compare digital computers to analog ones. Both classes of computers have an independent value and their own sufficiently clearly outlined region of application.

Future progress of analog technology, apparently, will be connected with the penetration of digital methods. For example, one should point out the development of so-called digital models, for which separate computing elements execute mathematical operations on increments of variables, represendted in one of the digital codes, and where the transfer of results from block to block is carried out just as in analog computers. During paralled carrying out of separate arithmetical operations it is possible to reach a comparatively high speed of operation and accuracy, avoiding the necessity of labor-consuming programming of the problem. The skeleton diagram of the joining of two blocks of such a device is shown in Fig. 2*.

Still greater prospects are promised by the construction of combined analog-digital devices, in which to increase accuracy of solution the part of the operations, mainly nonlinear placed on digital devices or in which the analog installation solves the problem simultaneously with the digital, where to the analog installation is entrusted solution of the problem in increments, and to the digital—seeking the solution, corresponding to unperturbed motion. Here the error of the analog part will be an error of the second order of smallness as compared to the error of

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*K. V. Diprose [1]. Analogous principle of construction of machines for integration of differential equations was offered independently by Prof. L. I. Gutenmakher

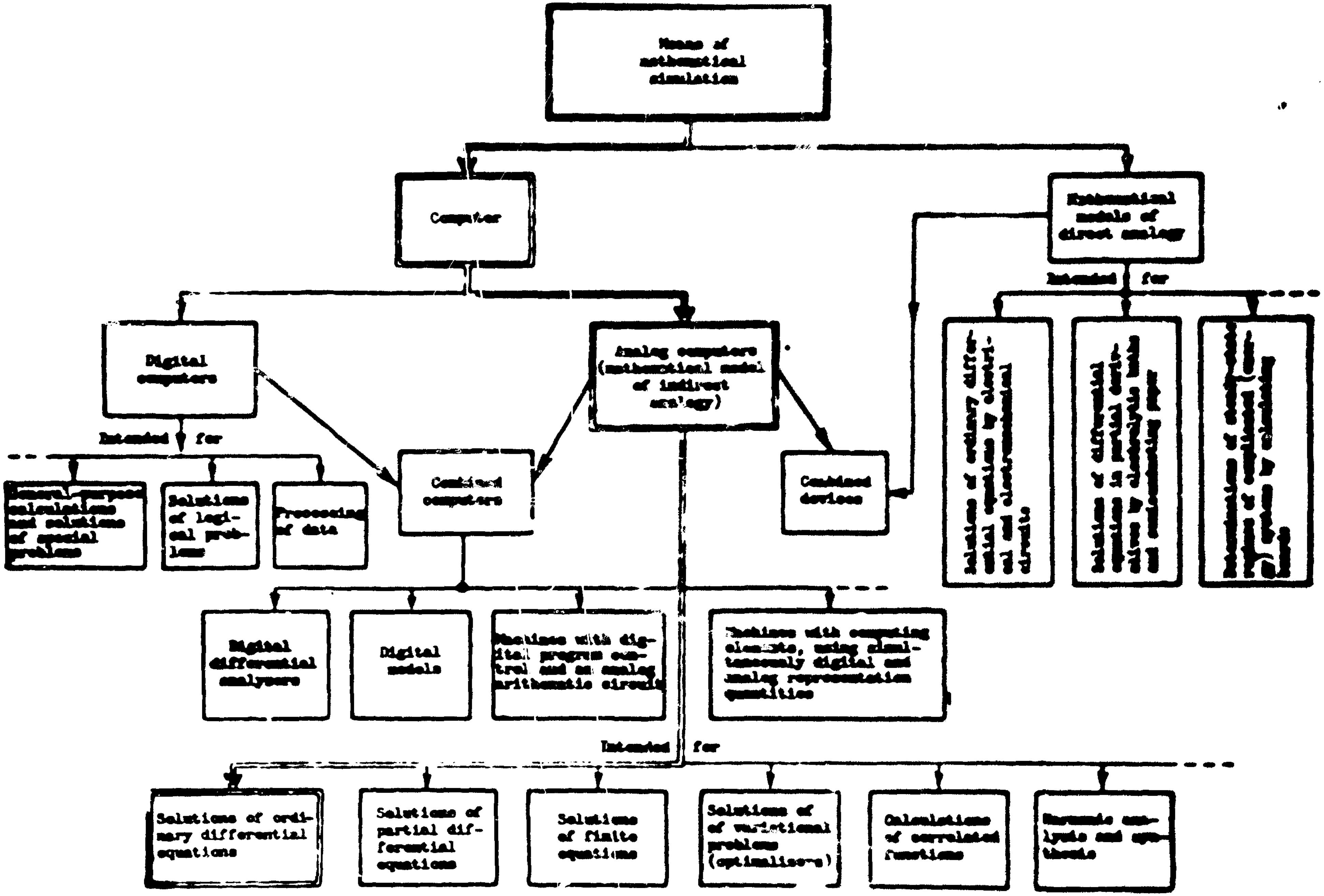
Means of mathematical simulation
Computer
Mathematical models of direct analogy
Intended for
Digital computers
Analog computers (mathematical model of indirect analogy)
Intended for
General-purpose calculations and solutions of special problems
Solutions of logical problems
Processing of data
Combined computers
Combined devices
Solutions of ordinary differential equations by electrical and electromechanical circuits
Solutions of differential equations in partial derivatives by electrolytic baths and semiconducting paper
Determinations of steady-state regimes of complicated (energy) systems by calculating boards
Digital differential analyzers
Digital models
Machines with digital program control and an analog arithmetic circuit
Machines with computing elements, using simultaneously digital and analog representation quantities
Intended for
Solutions of ordinary differential equations
Solutions of partial differential equations
Solutions of finite equations
Solutions of variational problems (optimalizers)
Calculations of correlated functions
Harmonic analysis and synthesis

Fig. 1. Classification of means of mathematical simulation.

determination of the unperturbed motion.

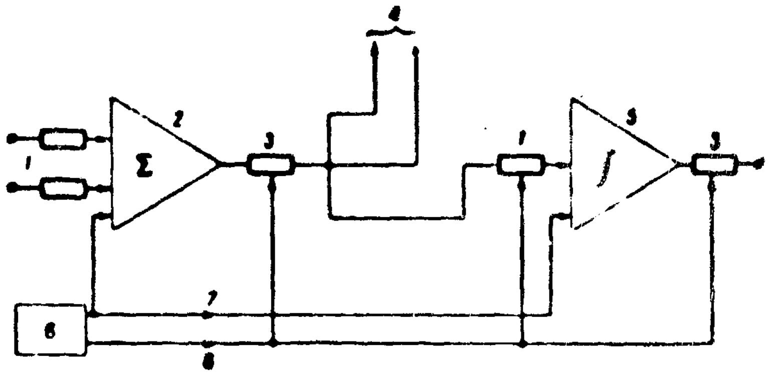

Fig. 2. Skeleton diagram of electronic model with digital elements. 1—input memory units; 2—arithmetical element (adder); 3—output memory units; 4—transmission of signal to other units; 5—arithmetical element (integrator); 6—control element; 7—order to start calculations; 8—order to transmit the result.

It is natural that in such combined systems devices connecting the analog part with the digital part acquire great significance.

To the analog computer technique and related disciplines is devoted a great number of works of domestic and foreign scientists. Thus, in the book by I. S. Bruk [1] there is a description of the mechanical integrator created under his leadership and method of its use is presented.

Regarding the theory of the work and construction of the mechanical computers light is thrown upon the subjects in books of N. I. Pchel'nikov [1], A. A. Birshteyn [1], A. Svoboda [1], S. O. Dobrogurskiy [1], S. O. Dobrogurskiy and V. K. Titov [1].

Questions of the general theory of accuracy of mechanical computers and dynamic accuracy of electric circuits are discussed in books by N. G. Bruyevich [1] and M. L. Bykhovskiy [3]. In the book by I. I. Etterman [1] there is considered a number of mathematical aspects of preparation of problems for solution on analog computers, and also there are shown certain approaches to appraisal of the accuracy of solution and there is composed a set of test problems.

Mechanical and electromechanical computers is the subject of books by B. I. Stanislavskiy [1], Ya. V. Novosel'tsev and A. N. Lebedev [1], a monograph by

N. Ye. Kobrinskiy[1], a textbook by N. G. Bruyevich and B. G. Dostupov [1] and summaries of lectures by G. M. Zhdanov [1, 2]. In these books are touched also questions of construction of electronic linear and nonlinear computing elements.

G. L. Shnirman [1] in a work devoted to the development of differentiating and integrating devices for vibrographs, first systematically expounds the analysis of electric differentiating and integrating circuits and indicates the expediency of their use for the construction of electronic amplifiers.

Theory and results of the development of the original electronic integrator with the use of a-c amplifiers, working with artificial iteration of processes, and also an electrointegrator for solution of partial differential equations, carried out on electric nets, are presented in books by L. I. Gutenmakher [1. 2]. Theories and techniques of electronic analog-computers with iteration of processes are developed also in books by R. Tomovic [1], R. Tomovic and W. Karplus [1].

Principles of the theory of electric and electronic analog computers, besides the monograph by N. Ye. Kobrinskiy and the mentioned books by L. I. Gutenmakher, were developed also in books by F. J. Murray [1] and F. H. Raymond [1]. In the latter the account is conducted with reference to d-c linear electronic integrators, and a number of theoretical computations based on results, published in the book by I. Gutenmakher [2] mentioned above. In the book by F. H. Raymond is discussed experience of the French firm SEA.

Questions of mathematical simulation are most completely considered in the book by W. W. Soroka [1]. Here are the description and principle of work of mechanical, electromechanical, electric and electronic computing elements, devices for solution of algebraic linear and nonlinear equations, description of a mechanical integrator and principles of construction of electronic integrators. In the book there are also expounded the principles of construction of models on the basis of analogies and devices, reproducing partial differential equations and finite differences. The book reminds one in many respects of the work of L. I. Gutenmakher [2]. Principles of construction of models on the basis of analogies from passive elements and the application of the theory of similitude to simulation are presented in the book by I. M. Tetel'baum [1].

D-c electronic analog computers received the most detailed discussion in books by G. Korn and T. Korn [3], A. S. Jackson [1] and the four volumes by S. Fifer [1]. Here are questions regarding the method of solution of problems, principles of construction of separate computing elements and description of certain types of electronic models, produced in the United States of America. The books of C. L. Johnson [1], G. Smith and R. Wood [1], I. N. Warfield [1], A. E. Rogers and T. W. Connoly [1], Danloux-Dumesnils [1] are good guides and training aids on application of electronic analog computers. Among training aids one must also mention the summary of lectures by V. S. Tarasov [1], devoted to linear computing elements.

At the same time there appeared books in which the authors sought to embrace the whole field of computer technique, both analog and digital. Among these works first of all one should consider the books by A. A. Fel'dbaum [5] and M. Pelegrin [1]. The encyclopedicness and extraordinarily wide scope of questions, naturally, could not allow the authors to allot sufficient place to questions of analog computer technique.

Problems of mathematical simulation were also considered in a number of books, devoted to the theory and practice of automatic control: T. N. Sokolov [1], A. A. Fel'dbaum [1], F. E. Nixon [1].

Books of the overwhelming majority of foreign authors basically are devoted to questions of technique of analog computers and specifically their application for solution of concrete problems.

Together with the extensive literature on questions of techniques of analog computers, produced recently intense scientific and design developments both of machines as a whole, and also their separate elements have not stopped. In the United States, France, England, Yugoslavia, Norway, Belgium and especially in recent years in Japan many firms, including aviation firms, are occupied with the development of these machines.

Original constructions of analog computers of various type, built in our country under the leadership of L. I. Gutenmakher, I. V. Korol'kov, V. A. Trapeznikov, A. A. Fel'dbaum, V. B. Ushakov, G. M. Petrov, L. N. Fitsner, I. M. Vittenberg, T. N. Sokolov, P. P. Gorayev, O. V. Kirillov, L. V. Yamshanov, V. A. Kotel'nikov, G. L. Polisar, A. V. Shishkin and others, were successfully used for the solution of problems of automatic adjustment and control.

SECTION I

ELECTRONIC ANALOG COMPUTERS AND THEIR ELEMENTS

CHAPTER I

METHODS OF MATHEMATICAL SIMULATION

1. Simulation on the Basis of Analogies

Mathematical simulation in recent years is developing into two basic directions: construction of models by direct analogy on the basis of known systems of analogies and construction of computers (digital and analog).

In the construction of models of the first type there are used systems of analogies between phenomena of a different physical nature, for example the analogy between mechanical and electric phenomena, between electric and acoustic phenomena, between electric and thermal phenomena. This allows one to transfer the study of phenomena in the original to the models of a physical nature different from the original. Transition from one region of physical phenomena into another here pursues the goal of simplifying cheapening manufacture of models, making method easier and increasing the accuracy of measurement of the desired quantities. Thus, for example, the motion of a mechanical pendulum (Fig. 3) near the position of equilibrium with viscous friction under the influence of a perturbing influence F(t) can be described by differential equation of this form

$$J_0 \frac{d^2\varphi}{dt^2} + h\frac{d\varphi}{dt} + mgl\varphi = F(t). \qquad (1.1)$$

Motion of charges in a circuit (Fig. 4) with lumped constants (L, R and C), to which there is applied an electromotive force (emf) E(t), is described analogously by the equation:

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}q = E(t), \qquad (1.2)$$

where q is the electric charge.

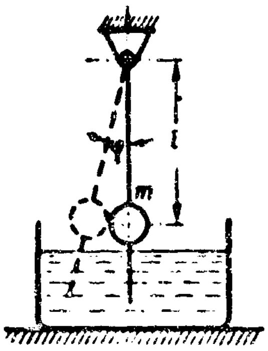

Fig. 3. Physical pendulum.

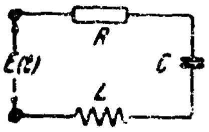

Fig. 4. Oscillatory circuit.

The identity of differential equations (1.1), (1.2) allows one with the corresponding selection of constants to conduct the study of mechanical oscillations on an electric model and vice versa.

An example of the application of analogy for investigation of systems of automatic control is an electrodynamic device*. At the base of the device is an electrodynamic analog (Fig. 5), a system of five electromagnets and five frames, located in the air gaps of the cores of the electromagnets.

The first four frames are placed in radial gaps of the electromagnets; the fifth frame is disposed in an uniform field, formed between the plane-parrallel poles of the core of the last electromagnet. In the absence of current in the frames of the analog the plane of the fifth frame is perpendicular to the direction of the magnetic flux. Each frame has an outlet from the center point. In the instrument there is an indicator of the angle of rotation of the frame (the critical angle is ± 45°) and a limiter of current in the frames, triggered with turning of the mobile part at the critical angle. In the lower part of the mobile system are affixed loads on a rod for application of the required moment of inertia.

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*A device of this type for solution of equations of the second order was offered by N. F. Minorsky [1] (see also K. A. Ludeke [1], [2]). V. V. Soldovnikov [1] applied it in 1939 for investigation of a system of automatic adjustment of a hydroturbine. Results of later works with an electrodynamic analog are described by V. A. Karabanov [1].

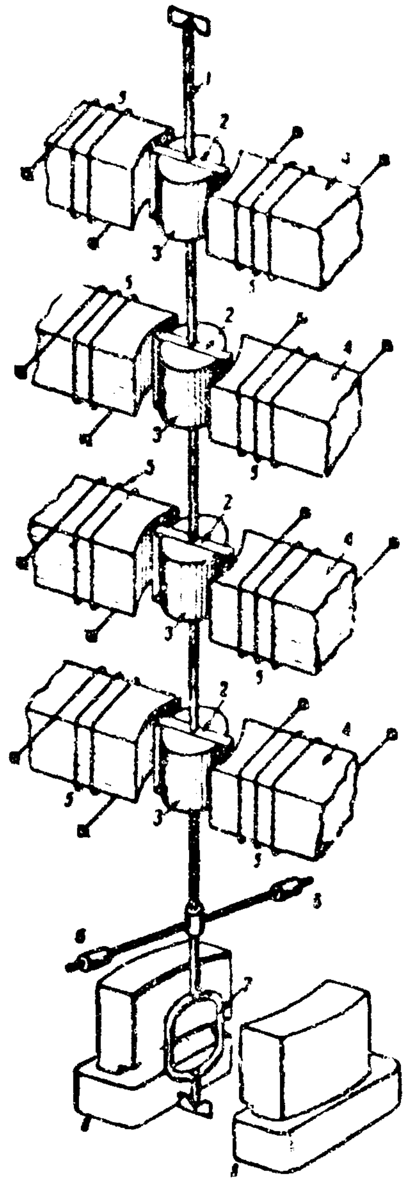

Fig. 5. Device of an electrodynamic analog. 1—dural shaft; 2—frames of electromagnets with a radial gap; 3—permalloy cylinders; 4—fixed part of the magnetic circuit; 5—windings of electromagnets with radial gap; 6—loads, movable on the rod; 7—frames of electromagnet with plane-parallel gap; 8—electromagnet winding.

The equation of motion of the mobile part of the analog can be written thus:

$$J\frac{d^2\varphi}{dt^2} = \sum_{1}^{n} M_i. \qquad (1.3)$$

where J — the moment of inertia of the mobile part, φ — the angle of rotation of the mobile part, counted off from the initial position, $\sum_{1}^{n} M_i$ — the sum of moments of all forces, acting on the mobile system.

The totality of moments of all forces, acting on the mobile system, can be divided into two groups: mechanical moments and electromagnetic moments. Disregarding the moment of dry friction, mechanical moments can be reduced to the sum of the moment, caused by elasticity of the suspension and the moment of friction with air:

$$M^{(\text{мех})} = -\left(c_1\varphi + b_1\frac{d\varphi}{dt}\right). \qquad (1.4)$$

where c_1 and b_1 — constants.

Electromagnetic moments, developed by frames, can be in turn divided into motive and retarding for each type of magnetic system, utilized in the instrument.

For a magnetic system with radial gap the torque caused by interaction of current in the frame with a magnetic flux in the gap, will be

$$M^{(\text{ш})} = kI_p\Psi\cos\varphi. \qquad (1.5)$$

where $\Psi = wDlB = w\Phi(t)$ — the total number of flux linkages (D — frame width, B — magnetic flux density in gap, l — frame length), I_p — current intensity in winding of the loop, k — proportionality factor, considering dimension, w — number of turns of the loop winding.

In cases, when φ is small

$$M^{(э.д)} = kI_p\Psi. \tag{1.6}$$

During motion of the frame in the radial magnetic field in the winding of the frame is induced an emf of rotation, which in the variable flux will be determined by the expression

$$\mathcal{E} = -w\frac{d[\Phi(t)\sin\varphi]}{dt}. \tag{1.7}$$

For small φ we obtain:

$$\mathcal{E} = -w\left(\varphi\frac{d\Phi(t)}{dt} - \Phi(t)\frac{d\varphi}{dt}\right). \tag{1.8}$$

If the resistance of the circuit of the frame is equal to r + R, then current in the frame under the influence of this emf will be

$$I_\varphi = \frac{\mathcal{E}}{r+R}$$

and the retarding moment can be found by the formula

$$M^{(э.д)}_{\text{тор}} = -\frac{kw^2}{r+R}\left[\varphi\Phi(t)\frac{d\Phi(t)}{dt} + \Phi^2(t)\frac{d\varphi}{dt}\right]. \tag{1.9}$$

For the fifth frame, moving in a plane-parallel field, the torque will be

$$M_5^{(э.д)} = k\Psi_5(t)I_{p5}(t)\sin\varphi; \tag{1.10}$$

for small angles φ

$$M_5^{(э.д)} = k\Psi_5(t)I_{p5}(t)\varphi.$$

The retarding moment, caused by the interaction of currents from the emf of the rotation of the fifth frame and the magnetic flux, can be found from the relationship

$$M^{(9.1)}_{\text{top.}\,5} = -\frac{kw_5^2}{r_5 + R_5}\left[\Phi_5(t)\frac{d\Phi_5(t)}{dt}\cos\varphi - \sin\varphi\,\Phi_5^2(t)\frac{d\varphi}{dt}\right]\sin\varphi;$$

for small angles φ

$$M^{(9.1)}_{\text{top.}\,5} = -\frac{kw_5^2\varphi}{r_5 + R_5}\left[\Phi_5(t)\frac{d\Phi_5(t)}{dt} - \varphi\Phi_5^2(t)\frac{d\varphi}{dt}\right]. \qquad (1.11)$$

The general equation of motion of the analog for small angles of rotation of the mobile system will have the form:

$$J\frac{d^2\varphi}{dt^2} + \left[b_1 + \sum_1^4 \frac{kw_i^2}{R_i + r_i}\cdot\Phi_i^2(t) - \frac{kw_5^2}{r_5 + R_5}\varphi^2\Phi_5^2(t)\right]\frac{d\varphi}{dt} +$$

$$+\left[c_1 + \sum_1^4 \frac{kw_i^2}{r_i + R_i}\Phi_i(t)\frac{d\Phi_i(t)}{dt} + kw_5\Phi_5(t)I_{p5}(t) + \right.$$

$$\left. + \Phi_5(t)\frac{kw_5^2}{r_5 + R_5}\frac{d\Phi_5}{dt}\right]\varphi = \sum_1^4 kw_i\Phi_i(t)I_{pi}(t).$$

Depending on the nature of change in time of magnitudes $\Phi_i(t)$, $\Phi_5(t)$, $I_{pi}(t)$ and $I_{p5}(t)$ the analog can reproduce different linear and nonlinear second order equations.

For an example, let us consider the solution, by an electrodynamic analog, of a linear second order differential equation. The circuit diagram of separate electrodynamic instruments of the analog for this case is shown in Fig. 6.

By adjusting the analog we establish:

$$\Phi_1(t) = (\Phi_1)_0 = \text{const},\ \ \tilde{\Phi}_2 = \Phi_2\max\sin\omega t,\ \ \Phi_3(t) = (\Phi_3)_0 = \text{const},$$

$$\Phi_4(t) = (\Phi_4)_0 = \text{const},\ \ \Phi_5(t) = (\Phi_5)_0 = \text{const};$$

$$R_1 = \ ,\ I_{p1} = 0,\ I_{p2} = \pm A\frac{d\varphi}{dt},\ I_{p3} = I_{p3}(t),\ I_{p4} = (I_{p4})_0 = \text{const}.$$

The second frame serves to record and, therefore does not participate in the formation of moments.

For selected values of fluxes and currents the equation of the mobile system will be

$$J\frac{d^2\varphi}{dt^2}+\left[b_1+\frac{kw_3^2}{r_3+R_3}(\Phi_3)_0^2+\frac{kw_4^2}{r_4+R_4}(\Phi_4)_0^2-\frac{kw_5^2}{r_5+R_5}\varphi^2(\Phi_5)_0^2\right.$$
$$\left.\mp Akw_3(\Phi_3)_0\right]\frac{d\varphi}{dt}+[c_1+kw_5(\Phi_5)_0 I_p]\varphi=kw_4\Phi_4 I_p(t). \qquad (1.12)$$

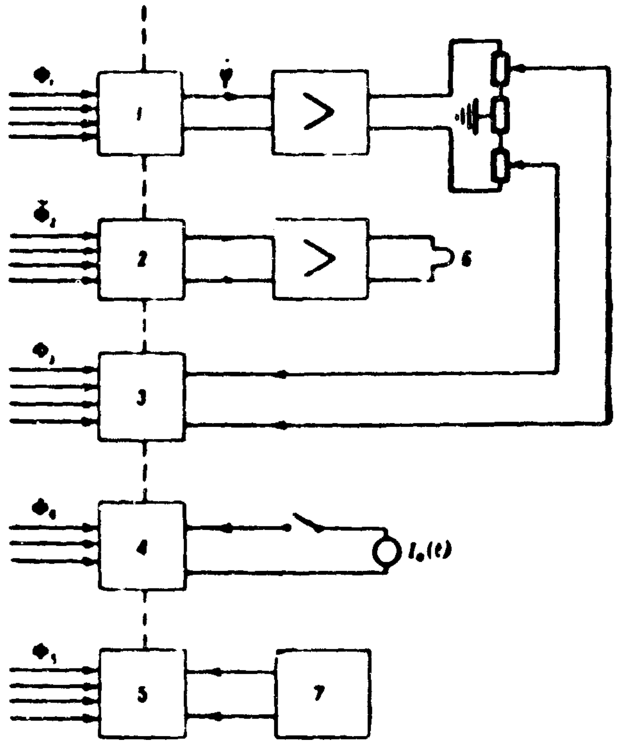

Fig. 6. Diagram for the solution of linear differential second order equation by an electrodynamic analog. 1—frame for measurement of speed of the mobile system; 2—frame for recording results; 3—frame for creation of damping moment; 4—loop for creation of perturbing forces; 5—frame for reproduction of moments; 6—go-and-return circuit; 7—device, ensuring constancy of current.

With sufficiently great R_5 and small φ component $(\Phi_5)_0^2\frac{kw_5^2}{r_5+R_5}\varphi^2$ can be disregarded as compared to the remaining ones and then the motion of the system will be described by a nonuniform linear differential second order equation. Initial conditions for angle are assigned by the initial setting of the angle of rotation of the mobile system, the initial conditions for speed — by supplying a current pulse to one of the windings of a frame, moving in the radical gap.

The considered electrodynamic instrument gives the possibility also to solve nonlinear differential equations.

Another example of simulation of systems of automatic control on the basis of analogies is investigation of banking of a neutral aircraft with the help of a d-c electric motor. The equation of isolated motion of banking of an aircraft, as is known (V. S. Vedrov [1]), has the form

$$J_x \frac{d^2\varphi}{dt^2} = M_x\left(v, \frac{d\varphi}{dt}, \delta_э\right). \qquad (1.13)$$

where φ — the angle of banking (Fig. 7), $\delta_э$ — angle of displacement of ailerons.

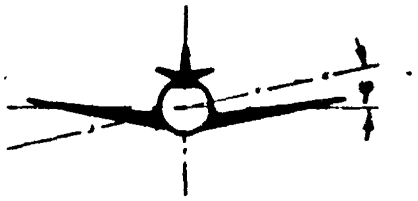

Fig. 7. Investigation of the process of stabilisation of banking of an aircraft.

If one were to consider the speed of flight v constant and consider the deflection of the aircraft from the equilibrium regime (the regime in which all parameters, characterising motion of aircraft, have steady-state values), then the linear differential equation of motion of banking will have the form

$$J_x \frac{d^2\varphi}{dt^2} + \left(\frac{\partial M_x}{\partial\varphi}\right)_0 \frac{d\varphi}{dt} = -\left(\frac{\partial M_x}{\partial\delta_э}\right)_0 \delta_э. \qquad (1.14)$$

where J_x — the moment of inertia of the aircraft relative to the longitudinal axis, $\left(\frac{\partial M_x}{\partial\varphi}\right)_0$ — the moment of high-speed resistance (determines the moment of air friction), $\left(\frac{\partial M_x}{\partial\delta_э}\right)_0$ — the coefficient, determining the aerodynamic effectiveness of the ailerons.

We will use now a d-c electric motor for simulating the isolated motion of banking of an aircraft (Fig. 8.). The equation of motion of a d-c electric motor in the absence of load on the shaft and disregarding losses in iron and reaction of the armature will be

$$J\frac{d\omega}{dt} = M_m, \quad M_m = f(\omega, U_s).$$

where J — the moment of inertia of the motor.

For the case of a linear mechanical characteristic of the motor, with constant magnetic flux, disregarding inductance of the armature circuit, we will obtain:

$$M_{дв} = c_м \Phi I_я \text{ and } I_я = \frac{U_я - c_e \Phi \omega}{R_я}.$$

The equation of the motor here will be

$$J\frac{d\omega}{dt} + \frac{c_e c_м \Phi^2}{R_я}\omega = \frac{c_м \Phi}{R_я} U_я.$$

or

$$\frac{d^2\alpha}{dt^2} + \frac{1}{T_p}\frac{d\alpha}{dt} = \frac{c_м \Phi}{i_p R_я J} U_я.$$

where Φ is the flux of excitation, $\omega = i_p \frac{d\alpha}{dt}$ — the angular velocity of rotation of the motor, i_p — the transmission ratio of the reductor, $T_p = \frac{R_я J}{c_e c_м \Phi^2}$ — the time constant of acceleration of the electric motor, α — angle of rotation of the potentiometer.

The converted equation of isolated motion of banking of an aircraft can be written in the form

$$\frac{d^2\gamma}{dt^2} + \left(\frac{\partial M_x}{\partial \dot\gamma}\right)_0 \frac{1}{J_x}\frac{d\gamma}{dt} = \left(\frac{\partial M_x}{\partial \delta_э}\right)_0 \frac{1}{J_x}\delta_э. \tag{1.15}$$

Thus, if one were to select parameters of motor in such a manner that these equalities were sustained

$$\frac{1}{T_p} = \frac{1}{J_x}\left(\frac{\partial M_x}{\partial \dot\gamma}\right)_0, \quad \frac{c_м \Phi}{i_p R_я J} k_u = \left(\frac{\partial M_x}{\partial \delta_э}\right)_0 \frac{1}{J_x}, \quad U_я = - k_u \delta_э,$$

then the angle of rotation of the cursor of potentiometer P-1 will depict the angle of bank. Here the processes of change of angle of bank of the aircraft and the angle of rotation of the axis of the potentiometer will proceed in time equally. The general diagram of simulation is presented in Fig. 8. One of the main advantages of this method of simulation consists of the fact that the parameters of the model, for example the moment of inertia, can be used as equivalents of corresponding parameters of the original. Among the deficiencies should be mentioned, in the first place comparatively great error of reproduction of initial equations, since friction, moment of losses, etc., are not considered; the possibility of reproduction by one analog of only an equation of the second order; the difficulties of combination of several such analogs; the difficulties of investigation of processes with non-sero initial conditions, especially for the first and second derivative.

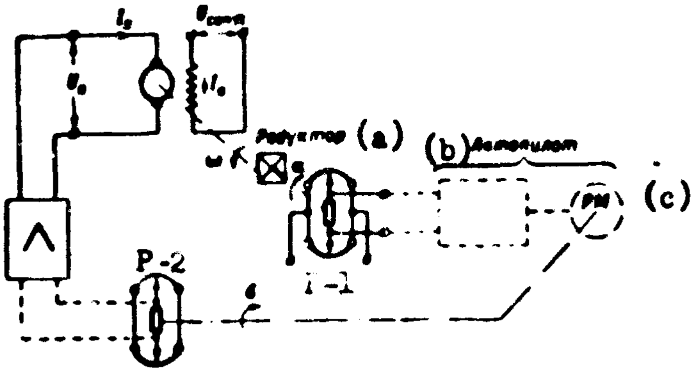

Fig. 8. General diagram of simulation of the isolated motion of banking of an aircraft.
KEY: (a) Reducer; (b) Automatic pilot; (c) Computing point.

In simulation of a system with many degrees of freedom to achieve the required accuracy it is necessary to pay special attention to accuracy of manufacture of equipment, or apply special means to combat the harmful influence of dry and high-speed friction, clearances and inertial masses. The last significantly raises the cost and complicates the construction, simultaneously lowering reliability work.

2. Mathematical Simulation by Analog Computers

Analog computers, built from separate computing elements, are based on carrying-out of elementary mathematical operations such as addition, subtraction, multiplication, division, differentiation and integration. From models, made on the basis of direct analogy, they differ by absence of a direct physical analogy between the quantities, characterising the studied phenomenon, and quantities, obtained as the result of carrying-out separate mathematical operations. Such an analogy does not exist between parameters of the studied physical system and the parameters of the installation.

Let us consider a method of solving linear differential equations with constant coefficients by an installation, built from separate computing elements. Such an installation should have in its composition computing elements, conditionally designated by the rectangles in Fig. 9a. For example let us "set up" with the help of these computing elements the above-considered equation of motion of aircraft for banking.

The equation can be reduced to the form

$$\frac{d^2\gamma}{dt^2} = B_1\frac{d\gamma}{dt} - B_2\lambda_1. \qquad (1.16)$$

Here $\lambda_1(t)$. B_1 and B_2 are given quantities, and $\gamma(t)$ is the sought dependent variable.

As follows from equation (1.16), for locating $\gamma(t)$ it is necessary to subject the sum $-\left(B_1\frac{d\gamma}{dt} + B_2\lambda_1\right)$ to double integration. The unknown component $-B_1\frac{d\gamma}{dt}$ can be is formed by means of multiplication of the quantity, received after the first integration and the coefficient — B_1. This component by feedback is fed to the adder (Fig. 9b).

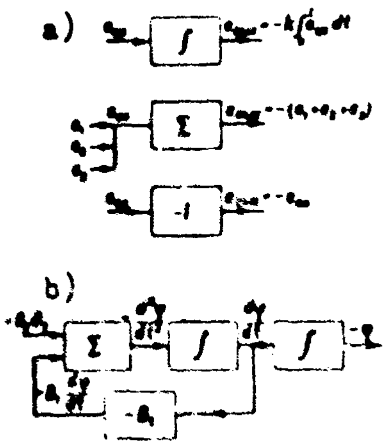

Fig. 9 Basic computing elements and a diagram of their connection for solution of the equation of motion of an aircraft for banking.

Application of analog computers in automation is extraordinarily varied.

The basic problems solved by these means computer technique, can be boiled down to the following:

1. Analysis of dynamics of systems of control and adjustment.

Here the given equations of an object and the system of control are solved in a selected time scale on installations (Fig. 10a) for the purpose of explaining the meaning of the main parameters, ensuring the required flow of the process. Application of analog computers gives in this

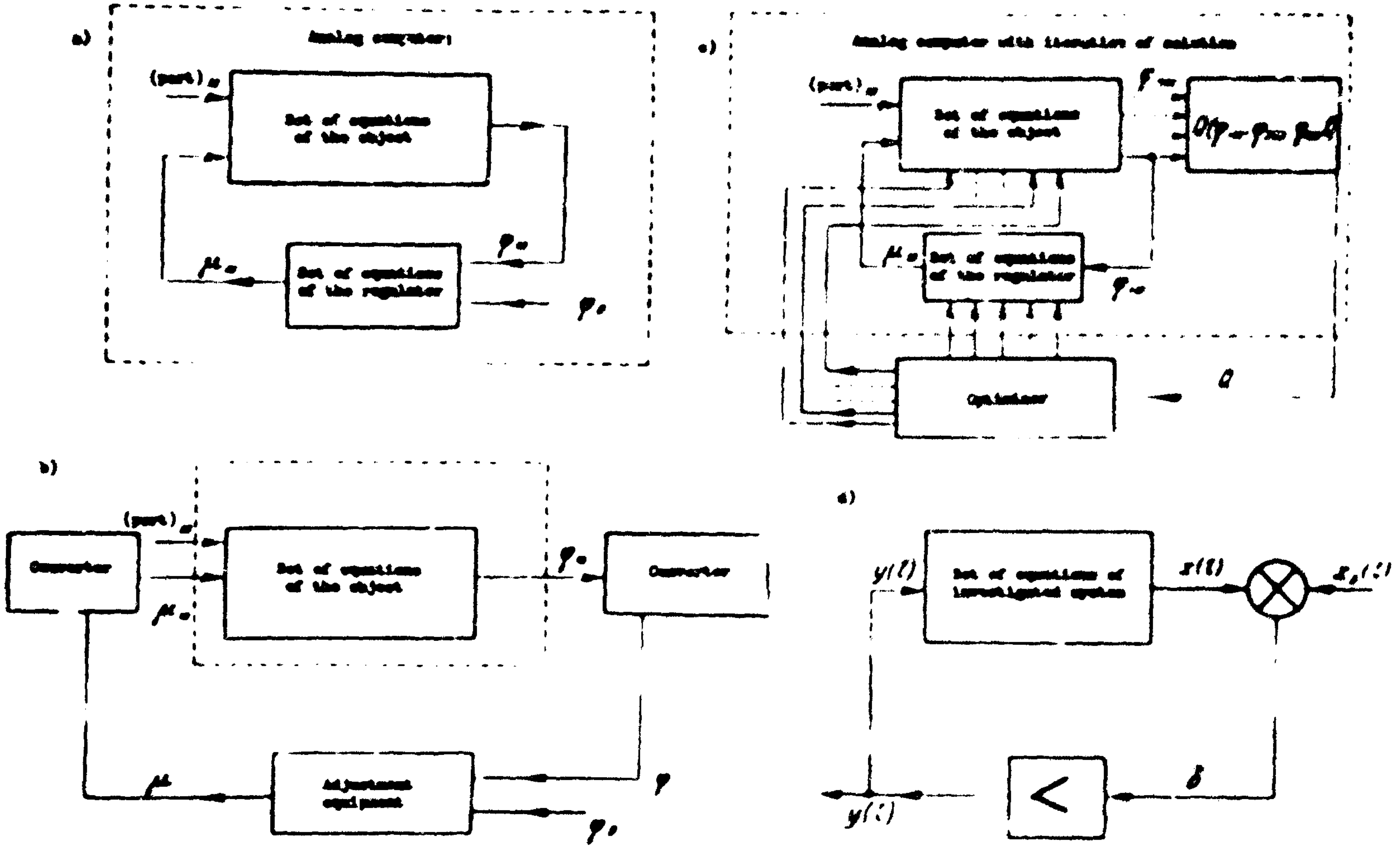
a)
Analog computer:
(part)$_m$
Set of equations of the object
φ_m
μ_m
Set of equations of the regulator
φ_0
c)
Analog computer with iteration of solution
(part)$_m$
Set of equations of the object
φ_m
$Q(\varphi_m - \varphi_{0m})$
μ_m
Set of equations of the regulator
φ_{0m}
Optimizer
Q
b)
Converter
(part)$_m$
μ_m
Set of equations of the object
φ_m
Converter
μ
Adjustment equipment
φ
φ_0
d)
$y(t)$
Set of equations of investigated system
$x(t)$
$x_0(t)$
δ
$y(t)$

Fig. 10. Basic applications of electronic analog computers in automation.

case a sharp reduction of the time required to carry out calculations on the first stages of projection, and also exceptional graphicness of the results obtained.

2. Experimental investigation of the behavior of a system with control and adjustment equipment in laboratory conditions.

With such experimental investigations equipment of control or adjustment is supplemented by an analog computer, reproducing in full time scale for the given equations the behavior of that part of the system of control or adjustment, whose work for one or another reason cannot be reproduced in laboratory conditions.* Coupling of the analog computer with the control or adjustment equipment in most cases is carried out by a special converter** (Fig. 10b).

3. Solution of problems of synthesis of control and adjustment systems.

Problems of this type come down to selection by the given specifications of the structure of the changed part of the system, of the required form of functional dependences and values of the basic parameters. The final result is found usually by means of multiple test solutions with appraisal of them in accordance with the accepted criterion of proximity (Fig. 10c). Problems of this type very often can be reduced to locating the extremum of a certain functional (A. A. Fel'dbaum [1]).

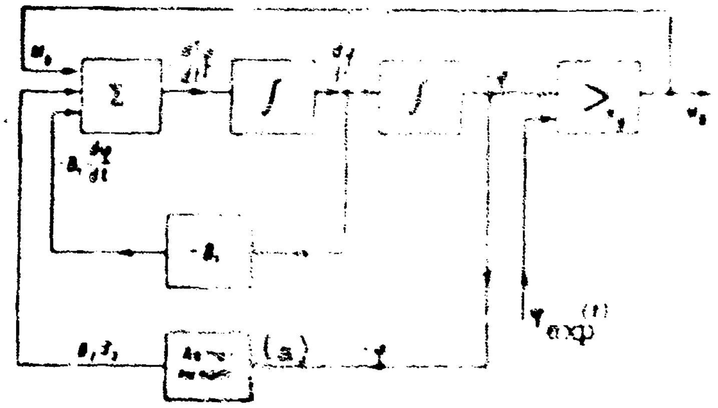

Fig. 11. Fundamental diagram of solution of the inverse problem on an electronic model. KEY: (a) Automatic pilot.

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*Method of simulation with the addition of sections of the control loop was offered by V. V. Solodovnikov, author's certificate No. 67093 of November 10, 1940.

**Three-stage platforms, electro-hydraulic converters, etc.

4. Solution of problems of determining perturbations or the useful signal, acting on the systems.

During solution of this type of problem by a given system of differential equations, describing the dynamic system, by values of initial conditions and the nature of change of the output coordinate known from experiment is determined the value of the perturbation or the useful signal at the input. In Fig. 10d is brought a functional diagram of the solution of a very simple problem of this type, based on the imposition of additional negative feedback. This method of application of analog computers can be used during making of instruments, automatically recording perturbations and producing a control signal depending upon the nature and quantity of these perturbations. The fundamental circuit of solution of the inverse problem for the above-considered case of stabilization of banking with a real automatic pilot is shown in Fig. 11.

Besides these problems, connected basically with investigation of a system of control and adjustment, analog computers find application also as elements and units of complicated systems of automation. Here they are used for:

a) calculation of the value of a certain combined parameter of adjustment (efficiency, power, productivity, etc.) (see V. A. Trapeznikov, B. Ya. Kogan [2]).

b) for working out optimum adjustments in the process of work of a dynamic system (D. P. Eckman, I. Lefkovich [1]);

c) for working out correcting signals by carrying out advance analysis of the dynamics of the control system (H. Ziebolz, H. M. Paynter [1]);

d) for creation of optimum in high speed of operation control systems by application of prognosticators (H. Chestnut, W. Sollecito, P. Troutman [1]).

3. Combining Methods of Mathematical Simulation

The classification of methods and devices of simulation is, in a known measure, conditional, since often in one device it is possible to observe the application simultaneously of several different principles of simulation.

Combining different principles of mathematical simulation is especially effective when simulating with real regulators, whose sensitive elements require for bringing into action comparatively great power. Application in these cases of models, made on the basis of analogy, for reproduction of the motion of the output coordinate of the object of adjustment, lets one have an input quantity of the regulator and an output quantity of the model of the object of the same physical nature, which gives the possibility of directly joining the model of the object with the regulator.

As example let us consider the model of the system of automatic adjustment of the speed of a big diesel engine (see I. Ya. Krichevskiy [1]). This system consists of a motor, with whose shaft there is coupled a centrifugal speed regulator, acting on the control mechanism of the rods of fuel pumps (Fig. 12). The simulating installation should give the possibility to adjust and to check new constructions of speed regulators in order to reduce to a minimum the time of their finishing on the object.

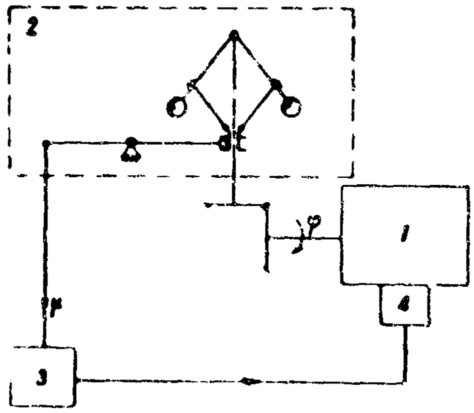

Fig. 12. Skeleton diagram of a system of automatic adjustment of the speed of a diesel engine. 1—engine; 2—centrifugal regulator; 3—control mechanism of rods of fuel pumps; 4—fuel pumps

Equations, subject to simulation, can be written in the form

$$T_a \frac{d\varphi}{dt} = \lambda - \lambda_{\text{дв}} = \Delta\lambda. \tag{1.17}$$

$$\lambda_{\text{дв}} = \mu(t - \tau) - a(\varphi)\varphi. \tag{1.18}$$

[Ed. Note. Subscript ДВ = motor.]

where T_a is the time of acceleration of the regulated motor, $a(\varphi)$ — the coefficient of self-levelling, φ — the relative change of the regulated parameter, μ — the relative change of position of the regulating element, τ — total delay in transmission of the regulating influence to the object, $\Delta\lambda$ — excess moment.

So that the physical nature of the output coordinate of the φ model coincides with the physical nature of the input coordinate of the regulator, it is expedient to place the reproduction of equation (1.17) on the model, made on the basis of analogies, and reproduce equation (1.18) by separate computing elements.

In Fig. 13 is brought the fundamental circuit of one of the possible variants of such an installation. The d-c motor here plays the role of the dynamic model of the object. Indeed, the equation of motion of the motor armature can be written in this form

$$J\frac{d\omega}{dt} = M_{\text{дв}} - M_{\text{н}} = \Delta M, \tag{1.19}$$

where J is the moment of inertia of the motor, ω — is the angular velocity, $M_{\text{дв}}$ the moment, developed by the motor, $M_{\text{н}}$ — the moment of load, ΔM — the excess moment on the armature.

Thus, by form equation (1.19) coincides with the equation of the object (1.17).

For reproduction of equation (1.18) there are used separate computing elements. With this aim the coordinate of the regulating unit μ is converted with the help of a potentiometer into voltage U_μ. This voltage is passed through the block of delay and is summed at the input of an amplifier with a large amplification factor K_y with components, reproducing the term $f(U_\varphi)$, characterising self-levelling, load U_λ and excess moment $U_{\Delta\lambda}$.

For a very great amplification factor of the amplifier K_y, voltage $U_{\text{вх}}$ will be minute. Disregarding it as compared with its separate components, we will receive:

$$U_\mu(t-\tau) + f(U_\varphi) + U_{\Delta\lambda} - U_\lambda = 0. \tag{1.20}$$

Separate components in equation (1.20) can be found from expressions

$$U_{\Delta\lambda} = -c_1 I_{\text{я}} R, \quad f(U_\varphi) = f(k_1\omega).$$

As is known, excess moment on the armature of a motor is proportional to the change of current of armature

$$\Delta M = c\Phi I_{\text{я}}.$$

where c is constant, Φ is the flux of the motor, I_a — the increment of armature current.

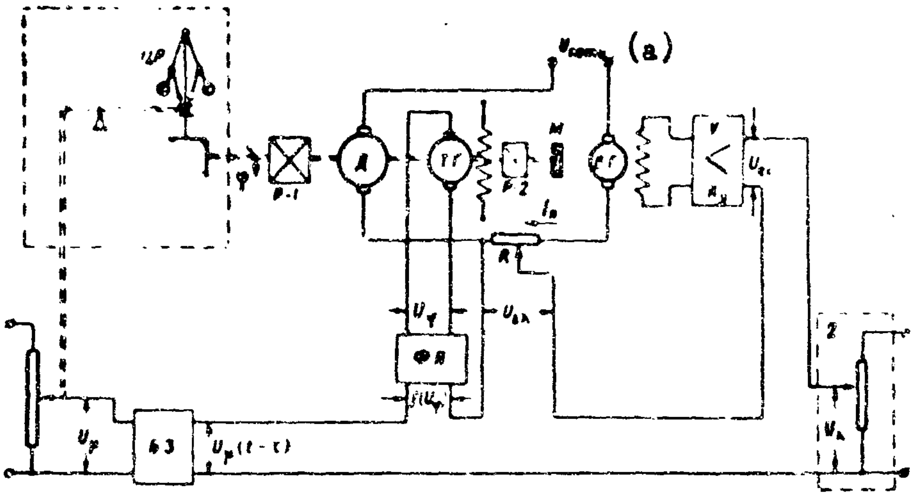

Fig. 13. Fundamental circuit of simulation of the system of automatic adjustment of a speed of diesel engine. Д — d-c motor; ТГ — tachogenerator; ВГ — booster generator; ЦР — centrifugal regulator; БЗ — delay block; ФП — functional converter; Р-1, Р-2 — reducer; У — amplifier. 2 — load assigner, М — flywheel.
KEY: (a) Net.

Therefore

$$U_{\Delta\lambda} = -\frac{c_1 R}{c\Phi}\Delta M. \tag{1.21}$$

We will introduce equations of scale conversions, connecting initial variables of the problems with variables of the model:

$$\Delta M = m_{\Delta\lambda}\cdot\Delta\lambda,$$
$$\omega = m_\varphi\cdot\varphi,$$
$$U_\mu = m_\mu\cdot\mu$$

and, substituting them in (1.19) and (1.20), we receive equations of the model, reduced to the initial variable:

$$J\frac{m_\varphi}{m_{\Delta\lambda}}\cdot\frac{d\varphi}{dt} = \Delta\lambda \tag{1.22}$$

and

$$\Delta\lambda = \lambda + \frac{m_\mu}{m_{\Delta\lambda}}\cdot\frac{c\Phi}{c_1 R}\mu(t-\tau) + \frac{c\Phi}{c_1 R m_{\Delta\lambda}} f(k, m_\varphi\varphi). \tag{1.23}$$

Comparison of expressions (1.17) and (1.18) with equations (1.22) and (1.23) obtained for the model gives the following relationships between parameters, ensuring identity of the indicated equations:

$$J\frac{m_{\varphi}}{m_{\lambda\lambda}} = T_{a}, \quad \frac{m_{\rho}}{m_{\lambda t}} \cdot \frac{c\Phi}{c_1 k} = -1.$$

$$a(\varphi)\varphi = \frac{c\Phi}{c_1 k m_{\lambda t}} f(k_r m_{\varphi} \varphi).$$

Methods of simulation, especially when simulating with real elements of the closed control loop, can also be considered a special variety of experimental methods of investigation of systems of automatic control.

Simulation should not be compared to analytic investigation. Simulation is impossible without analytic investigation, since it anticipates certainly knowledge of the system of differential equations, describing the behavior of the simulated control loop, and requires generalization of a large number of particular solutions.

CHAPTER II

LINEAR COMPUTING ELEMENTS

1. Computing Elements, Necessary for Solving Linear Differential Equations

As is known, the theory of automatic control is most fully developed for linear systems. However for determination of the nature of transients in a linear system and explanation of the influence of separate parameters on stability and quality of processes it is necessary to execute labor-consuming numerical calculations or graphic constructions. With increase of the order of the equations, describing the control system, the labor-consumption of the above-indicated calculations continually increases. Therefore it is expedient after deriving initial differential equations, describing the behavior of the system, to apply for their solution simulating devices. The latter in this case will be called as mathematical computers.

Differential equations of motion of systems of automatic control can be set up on analog computers in varying form:

1. In the form of one equation, written for the investigated coordinate, usually the controlled variable:

$$a_0\frac{d^nx}{dt^n}+a_1\frac{d^{n-1}x}{dt^{n-1}}+\dots+a_{n-1}\frac{dx}{dt}+a_nx=$$
$$=b_0\frac{d^my}{dt^m}+b_1\frac{d^{m-1}y}{dt^{m-1}}+\dots+b_{m-1}\frac{dy}{dt}+b_my,\quad y=F(t), \tag{2.1}$$

where $a_0, a_1, \dots, a_n$; $b_0, b_1, \dots, b_x$ are coefficients, x — the controlled variable

F(t) — the external disturbance.

2. In the form of a system of first order differential equations of this type:

$$b_i \frac{dx_i}{dt} + \sum_{k=1}^{n} a_{ik} x_k + F_i(t) = 0, \tag{2.2}$$

where b_i, a_{ik} are coefficients, $x_1, x_2, \dots, x_i, \dots, x_n$ — coordinates of the system, $F_i(t)$ — disturbances acting on the system.

3. In the form of a system, broken down into the equation of the controlled member

$$a_0 \frac{d^n\varphi}{dt^n} + a_1 \frac{d^{n-1}\varphi}{dt^{n-1}} + \dots + a_n\varphi + k\mu = b_0 \frac{d^m y}{dt^m} + b_1 \frac{d^{m-1}y}{dt^{m-1}} + \dots + b_m y. \tag{2.3}$$

$$y = F(t)$$

and the equation of the regulator.

With a static regulator the equation of the regulator will be

$$c_0 \frac{d^s\mu}{dt^s} + c_1 \frac{d^{s-1}\mu}{dt^{s-1}} + \dots + c_{s-1}\frac{d\mu}{dt} + \mu = g_n\varphi + g_{n-1}\frac{d\varphi}{dt} + g_{n-2}\frac{d^2\varphi}{dt^2} + \dots + g_1 \frac{d^{n-1}\varphi}{dt^{n-1}}.$$

with an astatic regulator —

$$c_0 \frac{d^s\mu}{dt^s} + c_1 \frac{d^{s-1}\mu}{dt^{s-1}} + \dots + c_{s-1}\frac{d\mu}{dt} = g^*_n\varphi + g^*_{n-1}\frac{d\varphi}{dt} + \dots + g^*_1 \frac{d^{n-1}\varphi}{dt^{n-1}}.$$

where φ is the controlled variable, μ — the controlling variable, F(t) — disturbance acting on the controlled member, $a_0, a_1, \dots, a_n$; $b_0, b_1, \dots, b_n$; $c_0, c_1, \dots, c_n$; $g_1, \dots, g_n$; $g_1^*, \dots, g_n^*$ — coefficients, where g_n — the static amplification factor of the regulator.

4. In the form of equations of dynamic units. If, for example, there are solved equations of a one-circuit system of automatic control, containing one oscillatory, two inertial and one integrating units (Fig. 14), then equations of motion can be

written in the form

$$\left.\begin{aligned} T_1\frac{dx_2}{dt}+x_2&=k_1x_1,\\ T_2\frac{dx_3}{dt}+x_3&=k_2x_2,\\ T_3T_4\frac{d^2x_4}{dt^2}+T_4\frac{dx_4}{dt}+x_4&=k_3x_3\\ \frac{dx_5}{dt}&=k_4x_4,\\ x_1&=y(t)-x_5,\end{aligned}\right\} \tag{2.4}$$

where x_1, x_2, x_3, x_4, x_5 are coordinates of the system, $y(t)$ — the external manipulated variable, T_1, T_2, T_3, T_4 — time constants of separate units, k_1, k_2, k_3, k_4 — amplification factors of separate units.

5. In the form of the initial system of differential equations of the investigated physical object. As an example we will bring the linearized equations of the system of stabilization of course of an aircraft (V. S. Vedrov [1]):*

equation of the object

$$\left.\begin{aligned} \frac{d\Theta}{dt}&=A_1\beta+A_2\delta+\frac{F_B}{mv},\\ \frac{d^2\psi}{dt^2}+A_6\frac{d\psi}{dt}+A_4\beta+A_5\delta&=\frac{M_B}{J_y},\\ \Theta&=\psi-\beta.\end{aligned}\right| \tag{2.5}$$

equation of the automatic pilot

$$T_p\frac{d^2\delta}{dt^2}+n\frac{d\delta}{dt}+\delta=i\left(\psi+T_1\frac{d\psi}{dt}+T_2\frac{d^2\psi}{dt^2}\right). \tag{2.6}$$

where ψ is the angle of the aircraft heading, β — angle of side slip, δ — rudder deflection angle, A_1, A_2, A_3, A_4, A_5, A_6 — constants of the aircraft, T_p, n, i, T_1, T_2 — constants of automatic pilot, M , F_B — disturbing moment and disturbing force, and J_y — moment of inertia of the aircraft.

Whatever the form of notation of the differential equations of motion of the system, their reproduction with the help of computing elements can be in principle carried out either by increasing the order of the derivative, or by lowering the order of the derivative.

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*It is assumed that the movement of the course is isolated, i.e., there is not considered the influence of the motion of banking on the course movement of the aircraft.

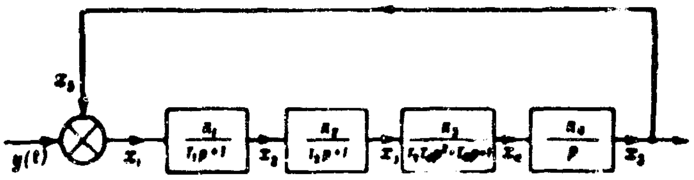

Fig. 14. Skeleton diagram of a one-circuit system of automatic control.

In the first case the diagram of set-up of computing elements is constructed on the principle of successive differentiation with summation (taking into account sign) at the input of the first unit of quantities after each differentiation. In Fig. 15 is brought as an example the functional diagram of the set-up of the linear second order differential equation

$$a_0\frac{d^2x}{dt^2}+a_1\frac{dx}{dt}+a_2x=y(t). \qquad (2.7)$$

During set-up by the method of increasing the order of the derivative equation (2.7) is solved for coordinate x:

$$x=-\frac{a_0}{a_2}\frac{d^2x}{dt^2}-\frac{a_1}{a_2}\frac{dx}{dt}+\frac{1}{a_2}y(t). \qquad (2.8)$$

Let us assume that to the integrating unit there are passed all addends of the right side of equation (2.8); then at the output of the unit we will obtain quantity x. Subjecting this quantity to double differentiation and multiplying the result of each differentiation with the help of multiplication units by the corresponding constants $\left(-\frac{a_0}{a_2}\text{ and }-\frac{a_1}{a_2}\right)$. we will recieve the necessary components for the first ingetrating unit. In this method of setting-up a problem the basis of the device is composed of differentiating decision elements.* During set-up of the problem by lowering the order of the derivative the installation is constructed on the principle of successive integration with summation of quantities after each integration at the input of the integrating unit. An example of set-up of a linear second order differential equation by lowering the order of the derivative is shown in Fig. 16. For composition of the functional diagram the equation given for solution is solved for the higher derivative, where all components of the right side, besides independent variables, are introduced to the integrating unit by feedbacks.

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*It is necessary to turn attention to impracticality of such method of setting-up of the problem, since during series differentiation interferences, always existing in the input signal, can increase impermissibly.

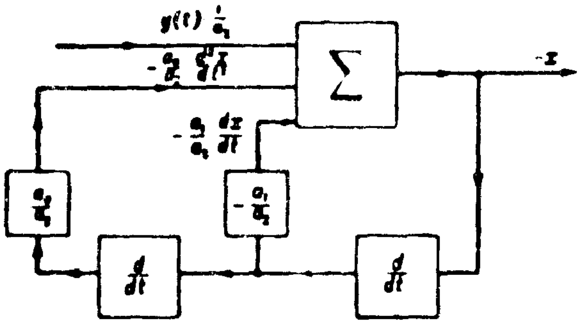

Fig. 15. Functional diagram of set-up of a linear differential equation of the second order by increasing the order of the derivative.

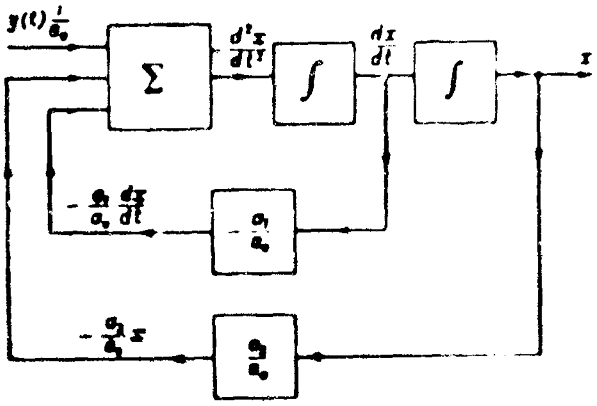

Fig. 16. Functional diagram of set-up of a linear differential equation of the second order by lowering the order of the derivative.

As can be seen from Fig. 16, the basis of the installation in this method of set-up of a problem is composed of integrating decision elements.

The considered examples show that for solution of linear differential equations it is necessary to have the following computing elements; adding devices, integrating devices and devices for multiplication and division by a constant (including by a quantity less than zero). The enumerated computing elements are called linear, since the connection between quantity at their output and input is lines .

Linear computing elements are constructed on the most varied principles. By their structure they can be divided into:

a) computing elements of open type;

b) computing elements with parametric compensation;

c) computing elements of closed type (with negative feedback).

2. Linear Computing Elements of Open Type

As an example of a computing element of open type let us consider the friction wheel integrator (Fig. 17).

The frictional wheel integrator consists of a revolving disk Д, connected by friction with wheel 1. The disk can move relative to the wheel owing to movement of carriage 2 along guides. If one were to designate the radius of the wheel by r, then from the condition of equality of linear speed of the disk and wheel at point a, we will receive:

$$kx\omega_z = r\omega_y \tag{2.9}$$

or

$$kx\frac{dz}{dt} = r\frac{dy}{dt},$$

whence

$$y = \frac{k}{r}\int_{z_0}^{z} x\,dz. \tag{2.10}$$

With a constant angular velocity $\omega_z = \frac{dz}{dt} =$ const we obtain:

$$y = \frac{k\omega_z}{r}\int_0^t x\,dt. \tag{2.11}$$

Thus, the considered mechanical integrator can execute integration not only for independent variable t, but also for any variable, for example z in equation (2.10).

In this lies the great merit of the considered integrating device. However accuracy of work of such a mechanical computing element in many respects depends on accuracy of construction and the amount of load on the output shaft.

Usually to decrease load on the output shaft we use additional amplifiers of moments (I. S. Bruk [1]) or electromechanical servo systems (V. A. Bush and C. H. Coldwell [1]). Main deficiencies of mechanical computing devices are their comparatively low speed of work, cumbersomeness, large labor-consumption of manufacture (thus, for example, grinding of guide prisms, by which the carriage is transferred should be carried out with accuracy up to 1 micron) and consequently, high cost. In spite of these deficiencies, mechanical computing devices still have not lost their value.

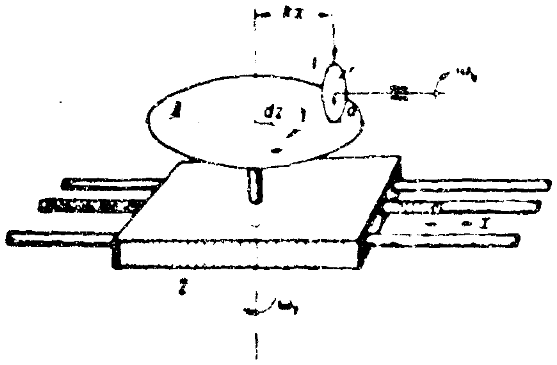

Fig. 17. Friction wheel integrator.

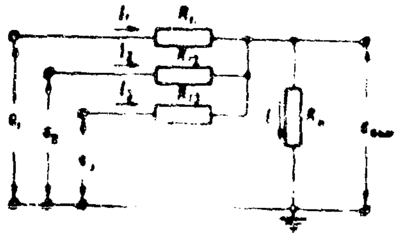

Fig. 18. Electric circuit of summation of three voltages.

As another example of a computing element of open type let us consider an electric circuit designed for summation of three voltages (Fig. 18). Values of currents flowing in the circuit are determined from the relationships:

$$\left.\begin{aligned} I &= e_{out} \cdot Y_н \\ I_1 &= (e_1 - e_{out}) Y_{11} \\ I_2 &= (e_2 - e_{out}) Y_{12}. \\ I_3 &= (e_3 - e_{out}) Y_{13}. \end{aligned}\right\} \qquad (2.12)$$

where $Y_н$, Y_{11}, Y_{12}, Y_{13} are corresponding values of conductance.

Since $I = I_1 + I_2 + I_3$, then

$$e_{out} = e_1 \frac{Y_{11}}{\sum_{1}^{3} Y_{1i} + Y_{н}} + e_2 \frac{Y_{12}}{\sum_{1}^{3} Y_{1i} + Y_{н}} + e_3 \frac{Y_{13}}{\sum_{1}^{3} Y_{1i} + Y_{н}}$$

or

$$e_{out} = \frac{\sum_{1}^{3} e_i Y_{1i}}{\sum_{1}^{3} Y_{1i} + Y_{н}} \qquad (2.13)$$

From the formula it follows that the result of summation will depend on the quantity of load and change of the number of components. With a finite $R_{н}$ correct summation will be guaranteed only if the number of components and the quantity of load are constant. When $R_{н} \rightarrow 0$, and $Y_{н} \rightarrow$ the dependence of the result of summation on change of the number of components and the quantity of load decreases. However, here it is impermissibly to sharply decreases the absolute value of the sum (output quantity).

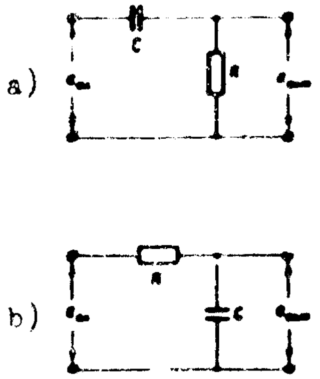

Fig. 19. Passive electric circuits for construction: a) of integrating and b) differentiating devices.

Passive electric circuits may also be used for construction of differentiating and integrating devices (G. L. Shm[illegible] [1]). Here as elements of these circuits are selected ohmic resistances and capacitors (Fig. 19).* Possibility of realisation of operations of differentiation and integration is based on the property of a capacitor to accumulate a charge q during application to its plates of a difference of potentials e.

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*Self-inductors practically do not find application as elements of such a circuit, since creation of inductance with minute ohmic resistance presents a problem significantly more difficult than creation of capacitance with minute leak.

Indeed,

$$q \qquad e. \qquad (\ .14)$$

The current through the capacitor here,

$$I_c = \frac{dq}{dt} \qquad \frac{de}{dt}$$

can serve as a measure of the derivative of the difference of potentials applied to its plates. On the other hand, the difference of potentials on capacitor plates, connected in an electric circuit, is a measure of the integral in time of the current flowing through the capacitor. These properties of a capacitor in practice cannot be realized in pure form. During construction of an integrator on the basis of a capacitor accurate integration of the input signal can be received only in an idealised circuit (Fig. 20), when the capacitor is fed from an ideal source of current.*

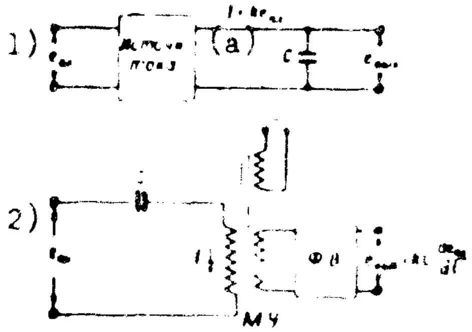

Fig. 20. On the principle of obtaining accurate 1) integration and 2) differentiation. MY — magnetic amplifier, PB — phased rectifier.
KEY: (a) current source.

Approximation to conditions of obtaining accurate differentiation can be obtained in the circuit of Fig. 20b with series coupling of the capacitor with control coil of the magnetic amplifier, possessing a very low impedance.

It is natural that during use of a passive electric circuit with R and C the result of the executed operation of integration or differentiation will be obtained with distortion. We will estimate the magnitude of these distortions and their dependence on parameters of the circuit.

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*Ideal source of current is such a source of electric energy, which creates in circuits a given current independently of the resistance of the load.

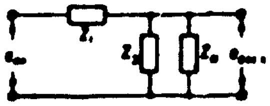


Fig. 21. On appraisal of the magnitude of distortions during differentiation and integration.

For this let us consider a more general case, where the circuit consists of two impedances Z_1 and Z_3, and load is impedance Z_H (Fig. 21). We will find the relationship between output and input voltages for the considered circuit. This relationship in operator form* gives the transfer function of the circuit:

$$\frac{\bar{e}_{\text{вых}}}{\bar{e}_{\text{вх}}} = \frac{Y_1(p)}{Y_3(p)} \frac{1}{\frac{Y_1(p)+Y_{\text{н}}(p)}{Y_3(p)}+1}. \tag{2.15}$$

where $Y_1(p) = \frac{1}{Z_1(p)}$ and $Y_3(p) = \frac{1}{Z_3(p)}$ are operator conductances of circuit, $Y_H(p) = \frac{1}{Z_H(p)}$ — operator conductance of load. If conductance of circuit and load are selected so that in the operational range of frequencies of input signals it is possible to disregard the quantity

$$\left|\frac{Y_1(j\omega)+Y_{\text{н}}(j\omega)}{Y_3(j\omega)}\right|$$

as compared with unity**, then expression (2.15) can be presented in the form

$$\bar{e}_{\text{вых}} = \frac{Y_1(p)}{Y_3(p)} \bar{e}_{\text{вх}}. \tag{2.16}$$

Let us consider a particular case, where the circuit consists of series coupling of resistance R_1 and capacitance C_3, shunted by load resistor R_H. In this case

$$Y_3(p) = C_3 p. \quad Y_1(p) = \frac{1}{R_1}. \quad Y_{\text{н}}(p) = \frac{1}{R_{\text{н}}}.$$

On the basis of expression (2.16) we obtain $\bar{e}_{\text{вых}} = \frac{1}{R_1 C_3 p} \bar{e}_{\text{вх}}$. if in the operating range of frequencies of input signals

$$\left|\frac{\frac{1}{R_1}+\frac{1}{R_{\text{н}}}}{C_3 j\omega}\right| \ll 1. \tag{2.17}$$

---

*Here and henceforth are taken the designations: $p = s \pm j\omega$ is a complex variable $\bar{e}_{\text{вых}}, \bar{e}_{\text{вх}}$ representations of quantities $e_{\text{вых}}(t), e_{\text{вх}}(t)$.

**As is known, to obtain a frequency response for the transfer function of a system one must replace in the latter $p = j\omega$.

Changing to originals, we obtain:

$$e_{out} = \frac{1}{R_1 C_3} \int_0^t e_{ex} dt \tag{2.18}$$

Thus, the considered passive electric circuit executes operation of integration. To satisfy condition (2.17) it is necessary to select the time constant of the circuit ($T_0 = R_1 C_3$) and of the load ($T_H = R_H C_3$) as large as possible.

Let us consider another particular case, where the circuit consists of capacitor with capacitance C and resistance R_3, shunted by load R_H. In this case, if in the operating range of frequencies of input signals

$$\left| \frac{Cp + \frac{1}{R_H}}{\frac{1}{R_1}} \right| \ll 1, \tag{2.19}$$

then

$$\bar{e}_{out} = R_1 C p \bar{e}_{ex}.$$

Changing to originals, we obtain:

$$e_{out} = R_3 C \frac{de_{ex}}{dt}. \tag{2.20}$$

Expression (2.20) shows that the considered circuit executes the operation of differentiation. To observe condition (2.19) it is necessary to select the time constant of the circuit $T_0 = R_3 C$ and relationship $\frac{R_1}{R_H}$ as small as possible.

From analysis of conditions (2.17) - (2.19) it follows that accuracy of fulfillment by a passive electric circuit of a given mathematical operation will be higher the less the voltage taken from the output of this circuit. We will estimate error, introduced by a computing element, made in the form of a passive electric circuit.

Error of such a computing element let us arbitrarily choose to call the difference between instantaneous values of the output quantity in real conditions and during ideal fulfillment of the given mathematical operation for the same value of the input quantity

$$\Delta e_{\text{вых}} = e_{\text{вых р}} - e_{\text{вых и}} \qquad (2,21)$$

On the basis of preceding the ideal value of the output quantity will be

$$e_{\text{вых и}} = -\frac{Y_1(p)}{Y_3(p)} e_{\text{вх}}$$

and real value of the output quantity —

$$e_{\text{вых р}} = -\frac{Y_1(p)}{Y_3(p)} \frac{1}{\frac{Y_1(p)+Y_н(p)}{Y_3(p)}+1} e_{\text{вх}} \qquad (2,22)$$

If in the operating range of frequencies of input signals

$$\left|\frac{Y_1(j\omega)+Y_н(j\omega)}{Y_3(j\omega)}\right| \ll 1,$$

then it is possible approximately to write:

$$\frac{1}{1+\frac{Y_1(j\omega)+Y_н(j\omega)}{Y_3(j\omega)}} \approx 1 - \frac{Y_1(j\omega)+Y_н(j\omega)}{Y_3(j\omega)}.$$

Therefore with accuracy up to sign

$$\Delta e_{\text{вых}} = \frac{Y_1(p)+Y_н(p)}{Y_3(p)} \cdot \frac{Y_1(p)}{Y_3(p)} e_{\text{вх}} \qquad (2,23)$$

Formula (2,23) is a general expression for representation of absolute error of a computing element.

As an example let us consider error, introduced by such a computing element during work in conditions of an integrator.

In this case

$$Y_1(p) = \frac{1}{R_1}, \quad Y_н(p) = \frac{1}{R_н}, \quad Y_3(p) = \frac{1}{\frac{1}{C_3 p}} = C_3 p.$$

$$\Delta e_{\text{вых}} = \left(\frac{1}{R_1} + \frac{1}{R_н}\right) \frac{1}{C_3 p} \cdot \frac{1}{C_1 R_1 p} e_{\text{вх}}$$

For e_{cx}, given in the form of a step function, and zero initial conditions, changing to originals, we will receive

$$\Delta e_{вых} \ldots \frac{R_н}{R_1 + R_н} \ldots$$

Thus, absolute error during a step change of the input signal grows with increase of the time of integration and with decrease of the time constant of the circuit and load resistor.

Usually as the criterion of accuracy is used the quantity of relative error, the relation of absolute error to maximum or the current ideal value of the output quantity. In the first case for maximum error we obtain:

$$\delta e_{вых\,max} = \left(\frac{\Delta e_{вых}}{e_{вых\,max}}\right)_{max} = \frac{1}{\frac{R_1 R_н}{R_1 + R_н} C_3} - \frac{1}{R_1 C_3}\, \frac{t^2}{2}\, \frac{e_{вх\,max}}{e_{вых\,max}}$$

Considering that $e_{вых\,max} = \frac{1}{R_1 C_3} t e_{вх\,max}$ we will receive:

$$\delta e_{вых\,max} = \frac{1}{2}\, \frac{1}{\frac{R_1 R_н}{R_1 + R_н} C_3}\, t$$

In the second case, considering that $e_{вых} = \frac{1}{R_1 C_3} t e_{вх}$ we obtain:

$$\delta_1 e_{вых} = \frac{\Delta e_{вых}}{e_{вых\,и}} = \frac{1}{2}\, \frac{1}{\frac{R_н}{R_н + R_1} R_1 C_3}\, t \qquad (2,24)$$

Thus, relative error grows in time linearly and will be less than the greater the time constant of the circuit $R_1 C_3$. Presence of load $R_н$ decreases the time constant of the circuit and therefore increases the error of fulfillment of the operation of integration. Hence it directly follows that union of such computing elements together and with other equipment leads to build-up of error of each element. So that error $\delta_1 e_{вых}$ does not exceed a given value for selected values of the time constant and load, it is necessary to limit the total operating time of this integrating device.

Thus, for example, when $\delta_1 e_{вых} = 1\%$, $R_1 C_3 = 10$ sec, and $R_н = \infty$ $t_p = 2 R_1 C_3$ $\delta_1 e_{вых} = 0.2$ sec. With a sinusoidal variable input voltage the error in a given mathematical operation is conveniently expressed in the form of the error of reproduction of amplitude and the error of reproduction of phase of the output voltage. Expressions for these values of error can easily be obtained after replacing p for jω in expression (2.22). Performing this replacement, we find:

$$\delta e_{out} = \frac{\Delta e_{out}}{e_{out\,max}} = \frac{1}{2\frac{R_2}{R_1+R_2}R_1C_3\omega}; \qquad \operatorname{tg}\delta\varphi = \frac{1}{\frac{R_2}{R_1+R_2}R_1C_3\omega}$$

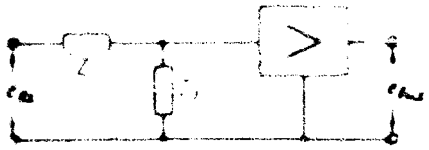


Fig. 22. Use of an electronic amplifier in combination with a passive electric circuit.

From these expressions it follows that amplitude and phase error with a sinusoidal variable input signal decrease with increase of the frequency of the signal and with increase of the time constant of the circuit $T_0 = R_1C_3$. Increase of time constant T_0, as follows from expression (2.18), leads to decrease of the magnitude of the output signal. This contradiction is possible to avoid, if in series with the considered passive network one were to connect an electronic amplifier (G. L. Shnirman [1]) (Fig. 22). The connection between output and input quantities now will be

$$e_{out} = k_y\frac{Y_2(p)}{Y_1(p)}\,\frac{1}{\frac{Y_2(p)}{Y_3(p)}+1}\,e_{in}$$

If we were to select the relationship of parameters so that in the operating range of frequencies $\frac{Y_2(p)}{Y_3(p)} \ll 1$ then we will receive finally:

$$e_{out} = k_y\frac{Y_2(p)}{Y_1(p)}e_{in} \qquad (2.25)$$

With the help of amplifier here we succeed, with sufficiently low value of $\frac{Y_2(p)}{Y_3(p)}$ determined by requirements of accuracy, in obtaining the necessary quantity of output voltage and simultaneously in unloading the passive network.

The considered principle of construction of a computing element of open type, based on combination of a parametric system with an amplifier, also has a number of deficiencies:

1. Special selection of parameters of the computing element is necessary, with which in the operating range of frequencies this condition is met $\frac{Y_1(p)}{Y_3(p)} \ll 1$.

2. The amplifier should possess a sufficiently high amplification factor (of the order of 200 or greater), where for removal of error due to variation of the parameters of the amplifier the amplification factor of the latter should be stabilized with a high degree of accuracy.

3. There is possible appearance of "drift" of zero of the amplifier due to the charge of the capacitor of the passive network at the input by the grid current of the first cascade during operation of the device as an integrator or differentiator.

4. Permissible time of work as an integrator is comparatively small due to the difficulty of obtaining a large time constant for the passive network.

3. Linear Computing Elements with Parametric Compensation

Connection of passive electric circuits with electronic amplifiers opens also a number of new possibilities of improving computing elements. One of these possibilities is compensation of error, introduced by the passive electric circuit, with the help of positive feedback in the amplifier. The idea of this principle of construction of a computing element follows from analysis of the equation for the passive electric circuit. Indeed, the connection between the output and input quantities of the circuit (without considering the load) can be presented in the form

$$e_{вых}\frac{Y_1(p)}{Y_3(p)} + e_{вых} = \frac{Y_2(p)}{Y_3(p)}e_{вх} \qquad (2.26)$$

From comparison of equation (2.26) with equation (2.16) it follows that the term $e_{вых}\frac{Y_1(p)}{Y_3(p)}$ determines error of the circuit. If we could manage to add to the input voltage a component, proportional to the output voltage, then, obviously, the problem would be solved. Indeed, let the new value of input voltage be

$$e'_{вх} = e_{вх} + \gamma e_{вых}$$

Then, substituting $e'_{вх}$ in place of $e_{вх}$ in expression (2.26), we will receive:

$$\bar{e}_{вых} = \frac{Y_1(p)}{Y_3(p)} e_{вх} + (1 - a)\bar{e}_{вых} \frac{Y_2(p)}{Y_3(p)}.$$

If one were to select a, close to 1, then we will receive: $\bar{e}_{вых} \approx \frac{Y_1(p)}{Y_3(p)} \bar{e}_{вх}$.

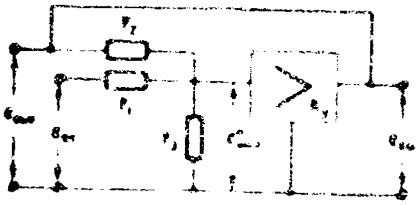


Fig. 23. The principle of construction of a computing element with parametric compensation of errors.

With an amplifier connected with the passive electric circuit, this idea can easily be realised, if with the help of positive feedback (Fig. 23) we carry out addition to the input signal of a component, proportional to the output signed. Indeed, for the diagram of Fig. 23 we obtain on the basis of (2.13):

$$\bar{e}'_{вых} = \frac{Y_1(p)}{Y_1(p) + Y_2(p) + Y_3(p)} \bar{e}_{вх} + \frac{Y_2(p)}{Y_1(p) + Y_2(p) + Y_3(p)} \bar{e}_{вых}. \tag{2.27}$$

The equation of amplifier is:

$$\bar{e}_{вых} = K_y \bar{e}'_{вых}. \tag{2.28}$$

Hence

$$\bar{e}_{вых} = \frac{Y_1(p)}{Y_1(p) + Y_2(p) + Y_3(p) - K_y Y_2(p)} K_y \bar{e}_{вх}. \tag{2.29}$$

If one were to select parameters of system Y_2 and K_y in such a way that in the operating range of frequencies $|Y_1(j\omega) + Y_2(j\omega) - K_y Y_2(j\omega)| \ll |Y_3(j\omega)|$. that is is possible with sufficient accuracy to write the equation of this computing element in the form

$$\bar{e}_{вых} \approx K_y \frac{Y_1(p)}{Y_3(p)} \bar{e}_{вх}. \tag{2.30}$$

Thus, the considered computing element executes the same conversion as the preceding one, but without the limitations placed on the parameters of the converter.

In order to reduce to zero the left part of the above-mentioned inequality, it is necessary to select an amplification factor of the amplifier equal to

$$K_y = \frac{Y_1(p)}{Y_2(p)} + 1. \qquad (2.31)$$

In particular when $Y_1(p) = Y_2(p)$, $K_y = 2$.

This principle of construction of a computing element is offered by L. I. Gutenmakher [1]. It is placed in somewhat modified form at the basis of the construction electrointegrators of type ELI-12 and ELI-14 (L. I. Gutenmakher, N. V. Korol'kov, I. A. Vissonov, L. S. Klabukov, G. K. Kus'minok [1]).

In these electrointegrators thanks to application of multiple automatic iteration of the solution computing elements turned out to be possible to construct from a-c amplifiers. The fundamental circuit of such a computing element is shown in Fig. 24. Along with total external positive feedback, here there is provided also negative feedback in the amplifier to stabilize the amplification factor.

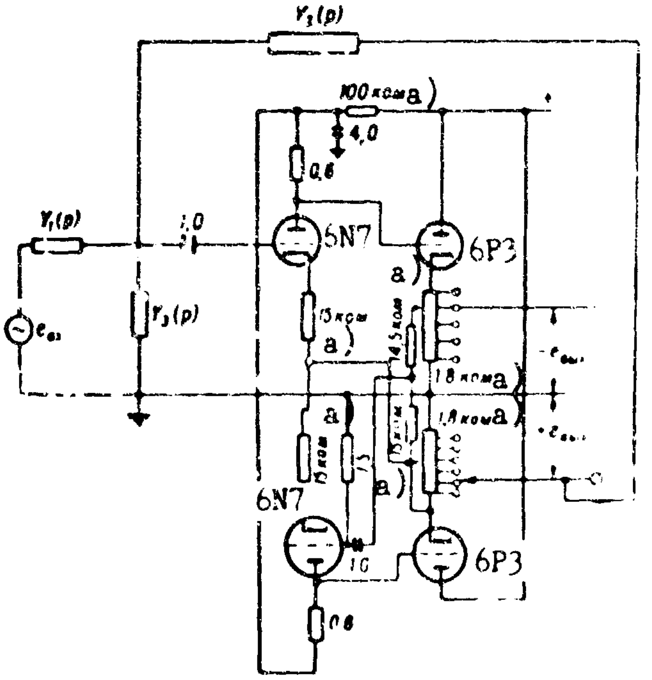


Fig. 24. Diagram of an electronic amplifier with parametric compensation of error (ELI).
KEY: (a) k

Compared with the case of application of a passive network with an amplifier this method of construction of computing elements has the advantage that it does not require an amplifier with a large amplification factor and special selection of parameters of the network. An essential deficiency of the considered method of construction of a computing element is the dependence of its transfer function on the amplification factor which leads to the necessity of stabilisation of the amplification factor and to selection for operation of a limited band of frequencies, in which the parameters of the amplifier do not occasion a noticeable influence. Furthermore, the condition of complete compensation practically is unacceptable, since it can lead to unstable operation of the computing element even with slight variation of parameters. Indeed, let the computing element work as an integrator when $Y_1(p) = \frac{1}{R_1}$, $Y_2(p) = \frac{1}{R_2}$, and $R_1 = R_2$, $Y_3 = Cp$. Then on the basis of expression (2.29) we will have

$$\bar{e}_{вых} = \frac{1}{\frac{1}{K_y} R_1 C p + \left(\frac{2}{K_y} - 1\right)} \bar{e}_{вх}. \tag{2.32}$$

To satisfy conditions of compensation one must choose an amplification factor of the amplifier equal to $K_y = 2$. If after setting this value for the amplification factor values of resistances R_1 and R_2 slightly change, the condition of compensation will be violated. Let us assume that $R_1 = 1.05\ R_2$. In this case

$$\bar{e}_{вых} = 2 \frac{\bar{e}_{вх}}{R_1 C p - 0{,}05}. \tag{2.33}$$

The presence of a negative sign in the denominator of expression (2.33) testifies to the unstable regime of the computing element. With a constant ratio $\frac{R_1}{R_2}$ change of the amplification factor can lead to these results. In connection with this for obtaining stable operation it is necessary to depart from conditions of complete compensation (in the considered case take $K_y < 2$) and thereby knowingly allow qualitative and quantitative distortion of the results of the mathematical operation, executed by the computing element.

This method of construction of computing elements can be compared with the method known in electric machines of parametric compensation (compounding), which ensures the required machine regime only with its strictly constant parameters.

The principle of parametric compensation can be used also during construction of electromechanical integrating devices. As is known, for construction of such computing elements there can be used following relationships for rotation:

$$J\frac{d\omega}{dt} = M, \quad \frac{d\varphi}{dt} = \omega, \tag{2.34}$$

where J is the moment of inertia, ω — the angular velocity, M — the total moment, φ — the angle of rotation.

If one were to use the first relationship, then the angular velocity can serve as a measure of the integral from total moment M, which is the input quantity. With use of the second relationship the angle of rotation φ will serve as a measure of the integral in time of the angular velocity.

When the input quantity is an electric voltage, it is necessary to supplement the device by a converting link, carrying out the conversions M = kU and $\omega = k_1 U$.

Technically it is quite simple to carry out the conversion M = kU with the help of an ordinary d-c or a-c motor. The conversion $\omega = k_1 U$ is usually carried out with the help of a servo system. Influence of the parameters of the servo system distorts this conversion and lowers accuracy of the executed mathematical operation. In this case $\bar{\omega} = \frac{K_1}{D(p)}\bar{U}$. where D(p) is a ploynomial, whose coefficients are determined by the parameters of the servo system.

For these reasons they prefer to build electromechanical integrators (A. E. Kharybin [1]) on the basis of the relationships J $\frac{d\omega}{dt}$ = M and M = kU.

Let us consider as an example the fundamental circuit of construction of an integrator on the basis of a d-c motor (Fig. 25). Voltage of the input signal moves through amplifier to clamps of the armature of a d-c motor with independent excitation. The equation of motion of the motor without considering the moment of resistance on the shaft and the moment of armature losses will be

$$J\frac{d\omega}{dt} + \frac{M_n}{\omega_0}\omega = \frac{c\Phi}{R_a}U.$$

where J is the total moment of inertia, brought to the shaft of the motor; $M_n = c\Phi I_n$ — the starting moment of the motor, $U_я$ — armature voltage of the motor, $R_я$ — armature resistance of the motor.

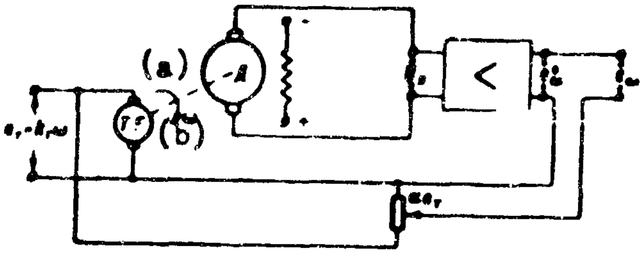


Fig. 25. Diagram of an electromechanical integrator with parametric compensation of error.
KEY: (a) Motor; (b) Tachogenerator.

If the tachogenerator is disconnected, then $e'_{вх} = e_{вх}$, and then

$$J\frac{d\omega}{dt} + \frac{M_n}{\omega_0}\,\omega = \frac{c\Phi}{R_я}\,K_y e_{вх}. \tag{2.35}$$

Equation (2.35) indicates that acceleration, developed by the motor, depends not only on the input signal, but also on the speed of rotation of the motor shaft. If to the input signal one adds a component, proportional to the output signal (in the given case, angular velocity ω), then, as in the preceding case, it is possible to compensate error, introduced by the anti-electromotive force. Indeed, if voltage of the tachomachine is added to the input voltage, then

$$e'_{вх} = e_{вх} + \alpha k_т\omega. \tag{2.36}$$

By joint solution of equation (2.36) and the equation of motion of the motor (2.35) we will receive:

$$J\frac{d\omega}{dt} + \left(\frac{M_n}{\omega_0} - \frac{c\Phi}{R_я}\,K_y\alpha k_т\right)\omega = \frac{c\Phi}{R_я}\,k_y e_{вх}.$$

If select parameters of the system K_y and α so that the expression in parentheses turns into zero, then the considered electromechanical device will ideally execute the operation of integration:

$$J\frac{d\omega}{dt} = \frac{c\Phi}{R_я}\,K_y e_{вх}.$$

whence

$$\omega = \frac{c'\Phi}{R_a J} K_y \int_0^t e_{a1}\, dt. \qquad (2.37)$$

The required value of K_y here can be found from the expression

$$K_y = \frac{c'\Phi}{k_1} \qquad (2.38)$$

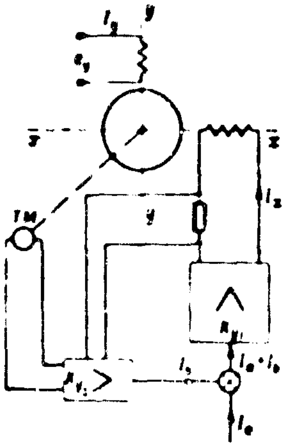


Fig. 26. Use of an asynchronous two-phase motor for construction of an electromechanical integrator.

It is obvious that by considerations of stability one must not approach conditions of complete compensation too closely, and therefore the value of K_y should be taken somewhat smaller than that which was found from expression (2.38).

A more accurate electromechanical integrator can be created on the basis of the considered principle with the help of an asynchronous two-phase motor (Fig. 26). In this case the moment of resistance on the shaft can be made insignificantly small, since brushes are absent, however the amplifier is complicated here due to the nonlinear dependence of the emf of the rotation on current in the windings.

The equation of motion of the motor can be written in the form

$$J\frac{d^2\theta}{dt^2} + f\frac{d\theta}{dt} = M\left(I_x, I_y, \frac{d\theta}{dt}\right), \qquad (2.39)$$

where J is the total moment of inertia brought to the motor shaft; θ — the angle of rotation of the motor; f — coefficient of viscous friction against air; M — electromagnetic torque on the rotor, I_x, I_y — currents in coils of the motor stator.

The moment of resistance and moment of losses we will disregard. As is known from the theory of two-phase asynchronous motors (I. M. Sadovskiy [1]), the expression for electromagnetic torque can with accuracy sufficient for practice be written in the form

$$M = k\left(\omega_c I_x I_y - \frac{I_x^2 + I_y^2}{2}\frac{d\Theta}{dt}\right). \tag{2.40}$$

This expression is correct, if we disregard electromagnetic transients in coils of the motor, attenuating significantly faster than electromechanical transients, caused by inertia of the motor rotor.

In expression (2.40) there is designated: k — constant for the given frequency of the network, which is a function of the impedances of the windings, ω_c — the synchronous speed of the motor.

On the basis of (2.39) and (2.40), if quantity f is negligible, we find:

$$J\frac{d^2\Theta}{dt^2} + \frac{k}{2}(I_x^2 + I_y^2)\frac{d\Theta}{dt} = k\omega_c I_x I_y \tag{2.41}$$

From expression (2.41) it follows that acceleration of the motor depends not only on current I_x in the control coil, but also on the rpm of the motor. If one were to use method, mentioned above in the example of a d-c motor, then, obviously, for compensation of error of such a computing element one must also introduce positive feedback, but this feedback should be nonlinear and its coefficient should depend on the square of the control current I_x. Technically this is possible with the help of application of an a-c tachomachine and a special nonlinear electronic amplifier (R. L. Cosgriff [1]). For the diagram of Fig. 26 it is possible to write additionally the following relationships:

$$I_x = K_y(I_a + I_b), \quad I_b = K_{y_1} k_T \frac{d\Theta}{dt},$$
$$K_{y_1} = c(I_x^2 + I_y^2),$$
$$I_a = k_1 e_{ex},$$

where c, k_1 are proportionality factors, Hence

$$I_x = K_y\left[k_1 e_{ex} + c(I_x^2 + I_y^2) k_T \frac{d\Theta}{dt}\right], \tag{2.42}$$

After substituting expression (2.42) in (2.41), wc will receive:

$$J\frac{d^2 w}{dt^2} + $$ [illegible]

If one were to select amplification factor K_{y_1} on the condition that $K_{y_1} = \frac{1}{\Omega_0 J_y k_1}$, then the equation of the computing element can be given in the form

$$J\frac{d^2 w}{dt^2} = k\omega_0 J_y K_y k_1 e_{01}$$

or

$$\frac{dw}{dt} = \Omega - \frac{k\omega_0 J_y K_y k_1}{J}\int_0^t e_{01}\,dt \tag{2.43}$$

As in the preceding example, to guarantee stability of the system it is necessary to depart somewhat from conditions of ideal compensatio

4. Computing Elements of Closed Type (witr Negative Feedback).

Lately in construction of computing elements more and more there are used principles, placed at the basis of closed systems of automatic control. Under certain conditions it turns out that accuracy of operation of such elements does not depend on variation of the parameters of the main channel, converting the signal, but is determined only by the magnitude and stability of parameters of the feedback circuit and the input circuit.

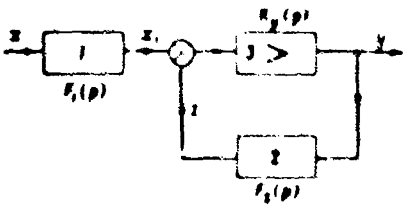


Fig. 27. Skeleton diagram of a very simple system of automatic control.

Let us consider the diagram of a very simple linear system of automatic control, consisting of three units (Fig. 27). Let the equations of these units have the form:

$$x_1 = F_1(p)x,$$
$$y = K_y(p)(x_1 - z),$$
$$z = F_2(p)y$$

where $F_1(p)$ is the transfer function of the input unit, $F_2(p)$ — the transfer function of the feedback, unit $K_y(p)$ — the transfer function of the amplifier.

From these equations it follows:

$$y = \frac{K_y(p)F_1(p)}{1 + K_y(p)F_2(p)}x \tag{2.44}$$

If one were to select so a large value of the amplification factor of the third unit that for values of ω, at which the system works this inequality is satisfied,

$$|K_y(j\omega)F_2(j\omega)| \gg 1, \tag{2.45}$$

then the expression, connecting the output quantity y with the input x can be, with accuracy sufficient for practice, presented in the form

$$\bar{y} = \frac{F_1(p)}{F_2(p)}\bar{x} \tag{2.46}$$

Thus, with a sufficiently great amplification factor of the main unit of the system of automatic control the connection between output and input quantities is determined only by parameters of the feedback circuit and the input circuit.

Depending upon the form of transfer function $F_1(p)$ and $F_2(p)$ the system of automatic control can execute various mathematical conversions of the input quantity. In general it is possible to consider that a system of automatic control, having a very great amplification factor in the open state, allows one to solve differential equation of the form

$$F_2(p)\bar{y} = F_1(p)\bar{x} \tag{2.47}$$

Let us consider several examples of construction of computing elements on the basis of systems of automatic control:

a) Electromechanical integrator. The fundamental circuit of the integrator is shown in Fig. 28. The input signal is added to the signal from the tachomachine with the help of an electric circuit, consisting of resistances R. Voltage ε at point Σ is a signal of mismatch or error of the servo system. The amplified voltage of mismatch is applied to the armature of a d-c motor and determines the change of speed of rotation of the armature in such direction and enough to decrease the appearing mismatch. Therefore this system with accuracy up to ϵ ensures proportionality between the speed of rotation of the d-c motor and the input voltage.

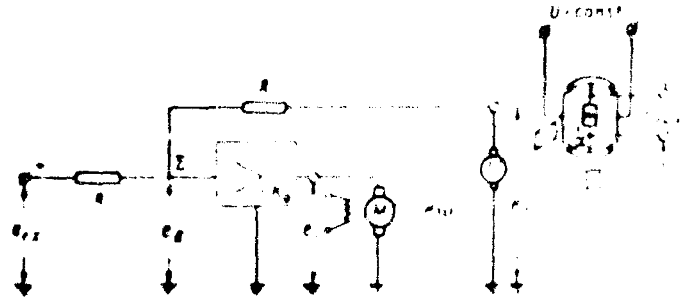


Fig. 28. Fundamental circuit of the device of an electromechanical integrator.

Indeed,

$$\left.\begin{aligned} \bar e_\Sigma &= f_1(p)\, e_{in} + f_2(p)\, \bar e_1 \\ e_\Sigma &= -\frac{\bar e}{K_0} \\ \bar e_1 &= k_1\omega \end{aligned}\right| \qquad (2.48)$$

where $f_1(p) = \frac{\bar e}{e_{in}} = \frac{1}{2}$; $f_2(p) = \frac{e}{e_1} = \frac{1}{2}$; $\bar e$ is the output voltage of the amplifier; k_1 -- the transmission factor of the tachomachine, K_0 -- amplification factor of the amplifier. Since $e_{1\,max}$ from conditions of physical feasibility of the amplifier has a finite value, then for a value of K_0 very large in modulo the quantity of mismatch e_Σ will be minute.

If, for example, $e_{1\,max} = 100$ v, and $K_0 = 50{,}000$ then $e_\delta = \frac{100}{50000} = 2$ mv. Thus, the integrating point obtains a potential differing little from the potential of the ground, i.e., as it is said, it is potentially grounded.

Thus, with a very large amplification factor, it is possible to disregard magnitude e_Σ in expression (2.48) as compared with the remaining terms, and then

$$k_1\omega = -e_{in}$$

or

$$\omega = -\frac{1}{k_1} e_{in} \qquad (2.49)$$

To the shaft of the motor through lowering reducer with a transmission ratio there is joined potentiometer П, by which the angle of rotation of the output shaft will be converted into output voltage. Considering that $\omega = k\frac{d\alpha}{dt}$, we will recieve:

$$e_{out} = k_{\Pi}\alpha = \frac{k_{\Pi}}{k_{1}}\int_{0}^{t} e_{in}\,dt, \tag{2.50}$$

where k_{Π} is the proportionality factor.

Consequently, the output voltage or the angle of rotation of the output shaft can serve as a measure of the integral in time of the input voltage.

Let us consider in somewhat more detail, how the parameters of the system will influence accuracy of the executed mathematical operation. If one were not to disregard the quantity of mismatch and not consider loss in the motor armature, friction and load on its shaft, and also consider the amplification factor of amplifier K_0 = const in the operating range of frequencies, then it is possible for the motor-amplifier unit to write the relationship

$$\bar{\omega} = \frac{K_0 k_M}{Tp + 1}\bar{\varepsilon}_1, \tag{2.51}$$

where T is the time constant of acceleration of the motor, k_M — the amplification factor of the motor.

Substituting value $\bar{\varepsilon}_1$ from equation (2.48) in equation (2.51), we will receive:

$$\bar{\omega} = \frac{K_0 k_M}{2(Tp + 1) + K_0 k_1 k_M}\bar{e}_{in} \tag{2.52}$$

or

$$\bar{e}_{out} = \frac{k_{\Pi}}{k_1}\cdot\frac{1}{p}\cdot\frac{1}{2(Tp + 1)\frac{1}{K_0 k_M k_1} + 1}\bar{e}_{in}. \tag{2.53}$$

If in the operating range of frequencies

$$\left|\frac{2(Tp + 1)}{K_0 k_M k_1}\right| \ll 1,$$

then

$$\bar{e}_{out} \approx -\frac{k_{\Pi}}{k_1}\cdot\frac{1}{p}\bar{e}_{in}$$

and the system executes the operation of integration without error. Error of operation of such a device will, obviously, depend on the absolute value of the expression $\frac{2(T_1\mu + 1)}{K_0 k_M k_1}$ and the stability of accuracy of set-up of the ratio $\frac{k}{k_1}$.

Decrease of the absolute value of expression $\frac{2(T_1\mu + 1)}{K_0 k_M k_1}$ can be attained by increase of the total amplification factor of the servo system $K_0\ k_M\ k_1$, selection of low-inertia elements, for example motors with a low time constant of acceleration, and finally, by limitation of the operating range of frequencies. Increase of amplification factor K_0 of such a system usually prevents loss of stability, caused by presence of such neglected factors as dry friction, the gap in kinematic circuit, equivalent constant delay, caused the neglected small inertnesses of the system and so forth. By force of this to obtain a given accuracy it is necessary to decrease the band of frequencies of operation of the system, i.e., in essence to limit circle of problems solved with the help of such electromechanical computing elements.

The passband width of electromechanical computing elements for the low-inertia magnetic clutches proposed by P. F. Klubnikin (see Ye. K. Krug and O. Minina [1]), possessing an equivalent time constant of acceleration T = 0.028 sec, does not exceed 4 to 5 c.

The required dynamic stability and fast attenuation of natural motions of the computing element can be attained comparatively simply in a wide range of operating frequencies with use as the computing elements of systems of automatic control, composed of low-inertia units. Special advantages in this respect are presented by d-c amplifiers with negative feedback (I. R. Ragazzini, R. H. Randall, F. A. Russell [1]).

b) Electronic amplifier with negative feedback as a computing element. An operational amplifier (Fig. 29) can be considered as a servo system, reacting to several (in the general case, n) input signals.

Role of the controlled member here is played by the same d-c amplifier, the role of the regulator — by a unique mismatch indicator, a multiterminal network, composed of input impedances $z_{11}, z_{12}, \ldots, z_{1n}$, feedback resistors z_2 and input resistance of the d-c amplifier z_3. Since feedback in the considered amplifier is negative, output voltage of this multiterminal network e_4 can be considered the error or mismatch of the servo system. Considering the linearity of elements, forming the multiterminal network of the mismatch indicator, it is possible to present the total voltage of error in the form of sum, in which every addend is determined by the value of the voltage, applied to the given input pole.

Indeed:

$$\bar{e}_4 = f_{11}(p)\bar{e}_1 + f_{12}(p)\bar{e}_2 + f_{13}(p)\bar{e}_3 + \ldots + f_{1n}(p)\bar{e}_n + f_2(p)\bar{e}_{\text{вых}} \tag{2.54}$$

where, as we know (K. A. Krug [1]), transfer functions $f_{11}(p)$, $f_{12}(p)$, ..., $f_{1n}(p)$ and $f_2(p)$, expressed by the conductance of the corresponding circuits, have the values:

$$f_{11}(p) = \frac{Y_{11}(p)}{\sum\limits_{1}^{n} Y_{1i}(p) + Y_2(p) + Y_3(p)}, \tag{2.55}$$

$$f_{12}(p) = \frac{Y_{12}(p)}{\sum\limits_{1}^{n} Y_{1i}(p) + Y_2(p) + Y_3(p)}, \tag{2.56}$$

$$f_{13}(p) = \frac{Y_{13}(p)}{\sum\limits_{1}^{n} Y_{1i}(p) + Y_2(p) + Y_3(p)}, \tag{2.57}$$

. . . .

$$f_{1n}(p) = \frac{Y_{1n}(p)}{\sum\limits_{1}^{n} Y_{1i}(p) + Y_2(p) + Y_3(p)}, \tag{2.58}$$

$$f_2(p) = \frac{Y_2(p)}{\sum\limits_{1}^{n} Y_{1i}(p) + Y_2(p) + Y_3(p)}. \tag{2.59}$$

Here

$$Y_{11} = \frac{1}{Z_{11}}, \quad Y_{12} = \frac{1}{Z_{12}}, \ldots, \quad Y_{1i} = \frac{1}{Z_{1i}}, \ldots$$

$$Y_{1n} = \frac{1}{Z_{1n}}, \quad Y_3 = \frac{1}{Z_3}, \quad Y_2 = \frac{1}{Z_2}.$$

With a sufficiently great amplification factor of the amplifier (in the operating range of frequencies) and limited maximum value of output voltage of the amplifier, the voltage of error e_ε is very low. Here, as in the preceding example, it turns out that integrating point Σ is as if potentially grounded. If we disregard in expression (2.54) quantity $\bar{e}_\varepsilon$ as compared with the remaining terms, then it is possible to find the connection between the output and input voltage of the operational amplifier:

$$\bar{e}_{\text{вых}} = -\frac{\sum_{1}^{n} f_{1i}(p)\bar{e}_i}{f_2(p)}$$

or after expression of transfer functions through the conductances of the corresponding circuits:

$$\bar{e}_{\text{вых}} = -\frac{\sum_{1}^{n} Y_{1i}(p)\bar{e}_i}{Y_2(p)} \qquad (2.60)$$

From equation (2.60) it follows that accuracy of mathematical operations, executed by the computing element, does not depend on parameters of the actual amplifier

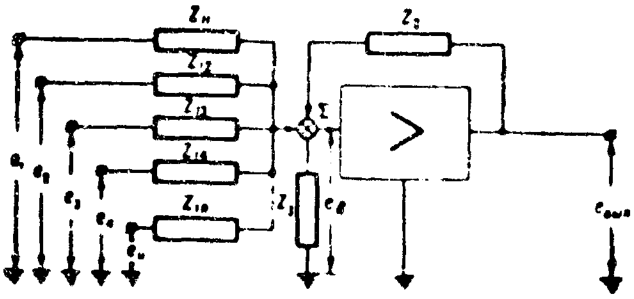


Fig. 29. Operational amplifier.

if its amplification factor is sufficiently great, but depends on accuracy of set-up and stability of values of the conductances of the input circuits and feedback circuit.

Let us consider several particular regimes of an operational amplifier.

Let the number of inputs n = 1, $Y_{11} = \frac{1}{R_1}$, $Y_2 = \frac{1}{R_2}$ then on the basis of (2.60) we will receive:

$$e_{\text{вых}} = -\frac{R_2}{R_1} e_{\text{вх}} \qquad (2.61)$$

The operational amplifier executes here the operation of multiplication by the constant $-\frac{R_2}{R_1}$.

Let there by n inputs $Y_{1i} = \frac{1}{R_{1i}}$, $Y_2 = \frac{1}{R_2}$ then

$$e_{\text{вых}} = -\sum_{1}^{n} \frac{R_2}{R_{1i}} e_i. \tag{2.62}$$

The operational amplifier executes here algebraic summation of n input signals with multiplication of every component by a given constant $-\frac{R_2}{R_{1i}}$.

If $R_{11} = R_{12} = \ldots = R_{1n} = R_2$, then here is carried out ordinary algebraic summation.

If in the feedback circuit we connect a capacitance, and on the input —, an ohmic resistance, then when n = 1 we will receive:

$$\bar{e}_{\text{вых}} = -\frac{1}{RCp}\bar{e}_{\text{вх}}. \tag{2.63}$$

Changing from representations to originals, we will have:

$$e_{\text{вых}} = \frac{1}{RC}\int_0^t e_{\text{вх}}\,dt. \tag{2.64}$$

Thus, with these resistances on input and in the feedback circuit* the amplifier executes operation of integration in time of the input quantity. If the number of input signals is n and at the input are connected ohmic resistances R_{11}, R_{12}, ..., R_{1n}, then here is executed the operation of integration of the sum of input signals:

$$\bar{e}_{\text{вых}} = -\frac{1}{p}\sum_{1}^{n}\frac{1}{CR_{1i}}\bar{e}_i.$$

whence

$$e_{\text{вых}} = \int_0^t \sum_{1}^{n}\frac{1}{CR_{1i}} e_i\,dt. \tag{2.65}$$

---

*Here and henceforth for brevity of speech by term "feedback circuit" is designated a circuit, connected between integrating point $\sum$ and the output terminal of the amplifier. In reality a feedback circuit is the totality of this circuit and the circuit connected to the amplifier input.

Finally, with connection at the input of capacitors, and in the feedback of resistance R_2 we will receive when n = 1:

$$\bar{e}_{\text{вых}} = R_2 C p e_{\text{вх}}$$

or

$$e_{\text{вых}} = R_2 C \frac{de_{\text{вх}}}{dt}. \qquad (2.66)$$

The operational amplifier here works as a differentiator.

Thus, depending upon the values of conductances of input circuits and feedback circuits the operational amplifier can execute various mathematical operations. The expression

$$\frac{Y_{1i}(p)}{Y_2(p)}$$

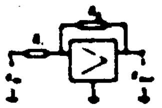
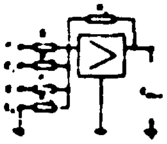
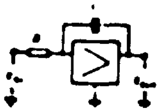
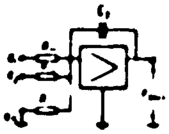
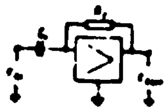
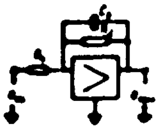
carries the name of the transfer function of a computing element for the i-th input and henceforth will be designated by letter [illegible] (p) in distinction from the static transfer ratio K, equal for an integrator to $K = \frac{1}{RC}$, for a differentiator $K = RC$, for an adder for the i-th input -- $K_{1i} = \frac{R_2}{R_{1i}}$.

In table I are shown the basic mathematical operations executed by such an operational amplifier.

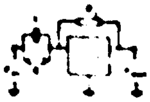
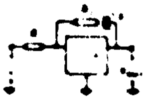
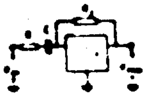
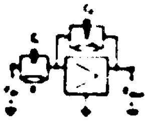
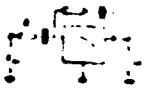
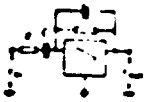
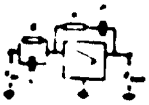
On the basis of the theory of electric circuits at the end of the book (in Appendix I) there is given a more general derivation of equations of the operational element and there are brought cases of obtaining combined linear operations with the help of one operational amplifier, at whose input and feedbacks there are connected networks of various types.

It is necessary to indicate that the examples in Table I of multiplication and division by constants [illegible] and [illegible] can also be used for realization of operations of multiplication and division of two variables, if the diagram is supplemented with a servo system. Besides the mathematical operations enumerated in Table I and Appendix I, by an operational amplifier with connection of a feedback circuit and at the input connection of nonlinear resistances there may also be carried out functional

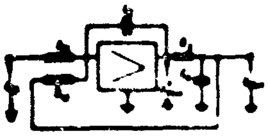
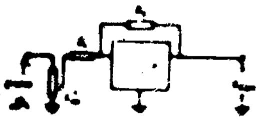
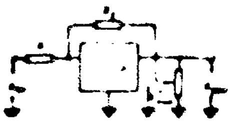
Table I

Fundamental circuit diagram of input and feedback circuits of operational amplifier	Conductance of feedback circuit	Conductance of input circuit	Equation of operational amplifier	Mathematical operation carried out by operational amplifier
a	b	c	d	e
1. Elementary operations				
	$Y_2(p) = \frac{1}{R_2}$	$Y_{11}(p) = \frac{1}{R_1}$ $Y_{12}(p) = 0$ . . . . $Y_{1n}(p) = 0$	$e_{out} = -\frac{R_2}{R_1} e_{in}$	Change of sign and multiplication by a constant
	$Y_2(p) = \frac{1}{R_2}$	$Y_{11}(p) = \frac{1}{R_{11}}$ $Y_{12}(p) = \frac{1}{R_{12}}$ . . . . $Y_{1n}(p) = \frac{1}{R_{1n}}$	$e_{out} = -R_2 \sum_{1}^{n} \frac{1}{R_i} e_{ini}$	Summation of several independent variables
	$Y_2(p) = C_2 p$	$Y_{11}(p) = \frac{1}{R_{11}}$ $Y_{12}(p) = 0$ . . . . $Y_{1n}(p) = 0$	$e_{out} = -\frac{1}{R_{11} C_2 p} e_{in}$	Integration with respect to time of one variable. If we connect to the integrating point additional signal $\dot{e}_{in}$ through capacitor C_1, then unit will execute operation $e_{out} = -\left(\frac{1}{R_{11} C_2 p} e_{in} + \frac{C_1}{C_2} \dot{e}_{in}\right)$
	$Y_2(p) = C_2 p$	$Y_{11}(p) = \frac{1}{R_{11}}$ $Y_{12}(p) = \frac{1}{R_{12}}$ . . . . $Y_{1n}(p) = \frac{1}{R_{1n}}$	$e_{out} = -\frac{1}{C_2 p} \sum_{1}^{n} \frac{1}{R_i} e_{ini}$	Integration of sum of several variables with respect to time
	$Y_2(p) = \frac{1}{R_2}$	$Y_{11}(p) = C_1 p$ $Y_{12}(p) = 0$ . . . . $Y_{1n}(p) = 0$	$e_{out} = -R_2 C_1 p e_{in}$	Differentiation with respect to time of one variable
2. Reproduction of equations of separate units of systems of automatic control				
	$Y_2(p) = C_2 p + \frac{1}{R_2}$	$Y_1(p) = \frac{1}{R_1}$	$e_{out} = -\frac{R_2}{R_1} \frac{e_{in}}{R_2 C_2 p + 1}$	Solution of the equation of a circuit, consisting of an inertial unit

Continuation of Table I

Fundamental circuit diagram of input and feedback circuits of operational amplifier	Conductance of feedback circuit	Conductance of input circuit	Equation of operational amplifier	Mathematical operation carried out by operational amplifier
1	2	3	4	5
	$Y_0(p) = \frac{1}{R_0}$	$Y_1(p) = C_1 p + \frac{1}{R_1}$	$\bar{e}_{out} = -\frac{R_0}{R_1}(R_1 C_1 p + 1)\bar{e}_{in}$	Solution of the equation of a circuit, consisting of a phase-shifting network
	$Y_0(p) = \frac{1}{R_0 + \frac{1}{C_0 p}}$	$Y_1(p) = \frac{1}{R_1}$	$\bar{e}_{out} = -\frac{R_0 C_0 p + 1}{R_1 C_0 p}\bar{e}_{in}$	Solution of the equation of a circuit, consisting of series-connected integrating and phase-shifting networks
	$Y_0(p) = \frac{1}{R_0}$	$Y_1(p) = \frac{1}{R_1 + \frac{1}{C_1 p}}$	$\bar{e}_{out} = -\frac{R_0 C_1 p}{R_1 C_1 p + 1}\bar{e}_{in}$	Solution of the equation of a circuit, consisting of series-connected differentiating and inertia networks
	$Y_0(p) = C_0 p + \frac{1}{R_0}$	$Y_1(p) = C_1 p + \frac{1}{R_1}$	$\bar{e}_{out} = -\frac{(R_1 C_1 p + 1) R_0}{(R_0 C_0 p + 1) R_1}\bar{e}_{in}$	Solution of the equation of a circuit, consisting of series-connected phase-shifting and inertia networks
	$Y_0(p) = \frac{1}{R_0 + \frac{1}{C_0 p}}$	$Y_1(p) = \frac{1}{R_1 + \frac{1}{C_1 p}}$	$\bar{e}_{out} = -\frac{(R_0 C_0 p + 1) C_1}{(R_1 C_1 p + 1) C_0}\bar{e}_{in}$	Solution of the equation of a circuit, consisting of series-connected phase-shifting and inertia networks
	$Y_0(p) = C_0 p + \frac{1}{R_0}$	$Y_1(p) = \frac{1}{R_1 + \frac{1}{C_1 p}}$	$\bar{e}_{out} = -\frac{R_0 C_1 p}{(R_0 C_0 p + 1)(R_1 C_1 p + 1)}\bar{e}_{in}$	Solution of the equation of a circuit, consisting of three series-connected networks — one differentiating and two inertial
	$Y_0(p) = \frac{1}{R_0 + \frac{1}{C_0 p}}$	$Y_1(p) = \frac{1}{R_1} + C_1 p$	$\bar{e}_{out} = -\frac{(R_1 C_1 p + 1)(R_0 C_0 p + 1)}{R_1 C_0 p}\bar{e}_{in}$	Solution of the equation of three series-connected networks — one integrating and two phase-shifting

Continuation of Table I

Fundamental circuit diagram of input and feedback circuits of operational amplifier	Conductance of feedback circuit	Conductance of input circuit	Equation of operational amplifier	Mathematical operation carried out by operational amplifier
a	b	c	d	e
	$Y_2(p) = C_2 p$	$Y_{11}(p) = \frac{1}{R_{11}}$ $Y_{12}(p) = \frac{1}{R_{12}}$	$\dot{e}_{out} = -\frac{1}{C_1 p}\left(\frac{e_{01}}{R_{11}} + \frac{e_{out}}{R_{12}}\right)$, $e_{out} = \frac{1}{R_1 C_2 p + 1}\dot{e}_{out}$, $e_{out} = -\frac{R_{12}/R_{11}}{R_{12}R_1C_1C_2p^2 + C_2R_{12}p + 1}e_{01}$	Solution of the equation of a circuit, consisting of an oscillatory circuit on the condition that [illegible]
	$Y_2(p) = \frac{1}{R_2}$	$Y_1(p) = \frac{1}{R_1}$	$e_{out} = -\frac{R_2}{R_1}\dot{e}_{01}$, $\dot{e}_{01} = \beta_1 e_{01}$, $e_{out} = -\beta_1\frac{R_2}{R_1}e_{01}$	Multiplication by $\beta_1 < 1$
	$Y_2(p) = \frac{1}{R_2}$	$Y_1(p) = \frac{1}{R_1}$	$\dot{e}_{out} = -\frac{R_2}{R_1}e_{01}$, $\dot{e}_{out} = \beta_2 e_{out}$, $e_{out} = -\frac{R_2}{\beta_2 R_1}e_{01}$	Division by $\beta_2 < 1$

conversion and there may be reproduced static characteristics of basic type nonlinearities of systems of automatic control (gap, dry friction, zone of insensitivity, limitation of coordinates by modulo, etc).*

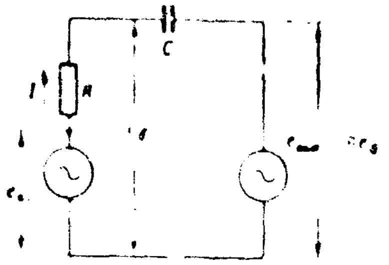


Fig. 30. Toward explanation of the work of an operational amplifier as an integrator.

We will give a physical explanation of the work of an operational amplifier as an integrator. For this let us consider the equivalent diagram of Fig. 30. In this diagram the amplifier is replaced by an equivalent generator with voltage $e_{\text{вых}}$ and a capacitor is connected through resistance R to a voltage, equal to $e_{\text{вых}} + e_{\text{вх}}$ Let us consider the case when $e_{\text{вх}}$ is applied in the form of a step function. By force of this fact that the voltage on the plates of the capacitor cannot change instantaneously (assume $e_c = 0$ at moment t = 0), at the moment of switching on the input signal through the circuit will leak current

$$I_0 = \frac{e_{\text{вх}}}{R}.$$

The voltage of error

$$e_\varepsilon = e_{\text{вх}} - I_0 R = 0$$

It is obvious that with passage of time, by measure of the charge of the capacitor; current in the circuit will fall, and the voltage of error will grow. If the time constant of charge is very great, then practically within a certain interval of time it is possible to consider the current constant. So that through capacitor there flows a current of unchanging strength, it is necessary that the voltage on its plates grows linearly in time.

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*See Ch. VIII and XIII.

Indeed, $I = C\frac{de_c}{dt}$; if I = const, then

$$\frac{de_c}{dt} = \text{const} \quad \text{and} \quad e_c = \frac{I}{C}t \tag{2.67}$$

Since with a large amplification factor of the amplifier voltage is minute, then the linear build-up of voltage on the capacitor signifies linear build-up of voltage on output. If one were to consider the input signal constant, then it will become evident that the output signal is a measure of the integral of the input signal with respect to time.

We will find the law of change of current in the equivalent circuit of Fig. 30 with application of an input signal in the form of a step function. Using Kirchhoff's law, we will receive:

$$e_{вх} - e_{вых} = IR + \frac{1}{C}\int I\,dt.$$

but

$$e_{вых} = -K_y e_1 = -K_y(e_{вх} - IR);$$

hence

$$e_{вх}(1 + K_y) = I(1 + K_y)R + \frac{1}{C}\int I\,dt. \tag{2.68}$$

From relationship (2.68) it follows that the process of change of current, flowing through the capacitor of an operational amplifier, working as an integrator, is equivalent to the process of change of current in a passive electric circuit, consisting of a series connected capacitor of the same capacitance and ohmic resistance, amplified (1 + K_y) times under the condition that to this circuit is connected an input signal amplified (1 + K_y) times.

Solution of differential equation (2.68) is

$$I = \frac{e_{вх}}{R}e^{-\frac{1}{(1+K_y)RC}t}. \tag{2.69}$$

By expression (2.69) it is possible to calculate the maximum speed of decrease of current, flowing through the capacitor:

$$\left(\frac{dI}{dt}\right)_{max} = \frac{e_{BX}}{R}\,\frac{1}{T(1+K_y)} \tag{2.70}$$

If e_{BX} = 10 v, R = 1 megohm, T = RC = 1 sec, K_y = 50,000, then

$$\left(\frac{dI}{dt}\right)_{max} = 0.2\cdot 10^{-9}\,\text{a/sec} = 0.2\cdot 10^{-3}\,\mu\text{a/sec}.$$

Knowing the law of change of current through the capacitor, it is easy to find the law of change of output voltage.

Indeed,

$$e_1 = e_{BX} - IR = e_{BX}\left[1 - e^{-\frac{1}{(1+K_y)RC}t}\right]$$

and

$$e_{out} = -K_y e_1 = -K_y e_{BX}\left[1 - e^{-\frac{1}{(1+K_y)RC}t}\right] \tag{2.71}$$

Expression (2.71) can also be obtained formally as the solution of the differential equation of the circuit (Fig. 30), written in operator form

$$\bar{e}_{out} = -\bar{e}_{BX}\,\frac{K_y}{(1+K_y)RCp+1} \tag{2.72}$$

For a passive electric circuit, consisting of R and C, we had:

$$e_{out} = e_{BX}\left(1 - e^{-\frac{1}{RC}t}\right) \tag{2.73}$$

Thus, in case of application of an operational amplifier as an integrator the output voltage changes also by exponential law, but with a time constant and steady-state value of the output quantity amplified $(1 + K_y)$ times. This gives an increase of the interval of time in which output voltage grows linearly in time, and consequently, increase of the interval of time, in which the process of integration of the input signal is carried out correctly.

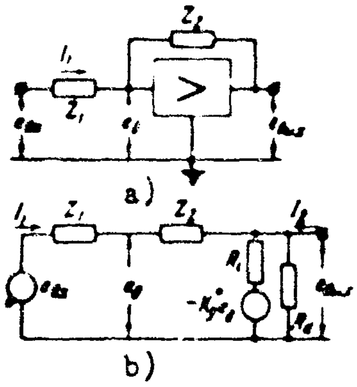

Fig. 31. Diagram for determination of a) input and b) output impedances of an operational amplifier.

We will find values of input and output impedances of the operational element, determining the possibility and convenience of its combination with other devices.

By definition input impedance can be found as

$$Z_{вх} = \frac{e_{вх}}{I_1}. \qquad (2.74)$$

On the basis of 31a, using the earlier relationships, we will write $I_1 = \frac{e_{вх} - e_1}{Z_1}$, and $e_1 = -\frac{e_{вых}}{K_у}$.

After substituting these expressions in (2.74) we will receive finally:

$$Z_{вх} = Z_1 + \frac{Z_2}{1 + K_у}. \qquad (2.75)$$

From expression (2.75) it follows that input impedance of the operational amplifier consists of two components: one, equal to the resistance connected at the input, and the other, equal to the resistance of the feedback, decreased by $(1 + K_y)$.

With a large amplification factor the second component is negligible. Indeed, let

$$Z_1 = R_1 = 10 \text{ ком}, \quad Z_2 = R_2 = 1 \text{ мом}, \quad K_у = 5 \cdot 10^4,$$

then

$$Z_{вх} = R_{вх} = 10^4 + 20 \approx 10^4 \text{ ohm}.$$

Thus, input impedance of an operational amplifier with accuracy sufficient for practice can be considered equal to the resistance connected to the input. With connection at the input of a capacitor the input impedance, obviously will decrease with increase of frequency.

For determination of the output impedance of the operational amplifier we will turn to the equivalent diagram in Fig. 31b. Considering this diagram as a quadripole, we will write the equation for currents in the poles:

$$I_1 = Y_{11}e_{\text{вх}} + Y_{12}e_{\text{вых}},$$
$$I_2 = Y_{21}e_{\text{вх}} + Y_{22}e_{\text{вых}}$$

Output impedance of the amplifier according to definition will be:

$$Z_{\text{вых}} = \frac{1}{Y_{22}} = \left(\frac{e_{\text{вых}}}{I_2}\right)_{e_{\text{вх}}=0}. \tag{2.76}$$

From the equivalent diagram it follows that

$$I_2 = \frac{e_{\text{вых}}}{R_a} + \frac{e_{\text{вых}} + K_y^* e}{R_i} + \frac{e_{\text{вых}} - e_1}{Z_2}, \quad e_1 = e_{\text{вых}}\frac{Z_1}{Z_1 + Z_2}.$$

After simple conversions for a very large K_y^* we obtain

$$Z_{\text{вых}} \approx \frac{R_i}{K_y^*}\left(1 + \frac{Z_2}{Z_1}\right). \tag{2.77}$$

It follows from this that the output impedance is smaller, the larger the K_y^* -- the amplification factor of the amplifier without accounting for the plate load of the output cascade -- and the less the transmission factor $\frac{Z_2}{Z_1}$, set on the computing element.

During work of the operational amplifier as an integrator with increase of frequency the output impedance slightly decreases, approaching $R_{\text{вых}} \approx \frac{R_i}{K_y^*}$. In operation as a differentiator the output impedance slightly increases.

We will define the order of magnitude of the output impedance of the computing element. Let $\frac{Z_2}{Z_1} = 1$, $R_i = 22$ ком, $K_y^* = 5 \cdot 10^4$. With these conditions

$$Z_{\text{вых}} = R_{\text{вых}} = \frac{22 \cdot 10^3 \cdot 2}{5 \cdot 10^4} = 0.88 \text{ ohm}.$$

Thus, output impedance of the operational amplifier is minute, which ensures simplicity of interconnection of such amplifiers and connection with other equipment.

5. Comparison of Various Types of Integrators.

The merits and deficiencies of various types of linear computing elements (Type I — with a passive network at the input of the amplifier, type II — with parametric compensation of error and type III — with negative feedback) already were considered above in sufficient detail.

It is of interest to compare these devices in the most critical regime — when carrying out the operation of integration. As the criterion for appraisal here is expedient to select the least permissible frequency of the sinusoidal input signal and the maximum permissible time of integration of the step input signal. These quantities determine the possibility of simulation with real control equipment and the possibility of use of integrating elements in the composition of control equipment.

These quantities are determined by the totality of properties of the computing element. However for comparative appraisal it is sufficient to determine these quantities, proceeding from the permissible quantity of systematic error and limitation of the dynamic range of voltages of the devices.* Calculation of drift of the amplifier, imperfection of the integrating capacitor and grid current is brought in Chapter III.

We will give an appraisal of the least permissible frequency of input signals, proceeding from accuracy of fulfillment of the operation of integration.

On the basis of earlier material the equations of the considered integrators in operator form can be presented in the form:*

---

*By the dynamic range of voltages K_1 here is understood the ratio of the value of the output signal at the boundary of linearity to the minimum value of the input signal distinguished from interferences.

$$\left.\begin{aligned} \tilde{e}_{\text{вых}} &= \frac{K_{I}}{Tp+1}e_{\text{вх}} \\ \tilde{e}_{\text{вых}} &= \frac{\frac{1}{\alpha}K_{II}}{\frac{1}{\alpha}Tp+1}e_{\text{вх}} \\ \tilde{e}_{\text{вых}} &= \frac{K_{III}}{(1+K_{III})Tp+1}e_{\text{вх}} \end{aligned}\right| \tag{2.78}$$

where α is the coefficient, taking into account the degree of compensation of error, $0<\alpha<1$; K_{I}, K_{II}, K_{III} — amplification factors of amplifiers of integrators I, II, III types without feedback, T = RC — the time constant of the passive network, taken identical for all three types of devices.

With a sinusoidal input signal for peak values we have:

$$\left.\begin{aligned} e_{\text{вых}} &= \frac{K_{I}}{T\omega}\frac{e_{\text{вх}}}{\sqrt{1+\frac{1}{(T\omega)^2}}} \approx \frac{K_{I}}{T\omega}\left[1-\frac{1}{2}\frac{1}{(T\omega)^2}\right]e_{\text{вх}} \\ e_{\text{вых}} &\approx \frac{K_{II}}{T\omega}\left[1-\frac{\alpha^2}{2}\frac{1}{(T\omega)^2}\right]e_{\text{вх}} \\ e_{\text{вых}} &\approx \frac{K_{III}}{(1+K_{III})T\omega}\left[1-\frac{1}{2}\frac{1}{(1+K_{III})^2(T\omega)^2}\right]e_{\text{вх}} \end{aligned}\right\} \tag{2.79}$$

The second addend in the square brackets of these three expressions determines the systematic error of the operation of integration. Designating it Δ we will receive the expression for the minimum permissible frequency of the processes

$$\left.\begin{aligned} \omega_{\min I} &= \frac{1}{T}\sqrt{\frac{1}{2\Delta}}, \\ \omega_{\min II} &= \frac{\alpha}{T}\sqrt{\frac{1}{2\Delta}}, \\ \omega_{\min III} &= \frac{1}{T(1+K_{III})}\sqrt{\frac{1}{2\Delta}} \end{aligned}\right\} \tag{2.80}$$

From comparison of values of $\omega_{\min}$ it follows that for the same T and Δ the least permissible ω will be in an integrator type III. However with a sinusoidal input signal, besides error in amplitude, there occurs phase error.

We will find the minimum permissible values of φ proceeding from the given permissible phase error $\Delta\varphi$. With the help of expressions (2.79) we find:

---

*Signs $\tilde{e}_{\text{вых}}$ in expressions (2.78) are omitted, since for the given consideration they do not have essential meaning.

$$\left.\begin{aligned} \omega^{*}_{\min I} &= \frac{1}{T}\,\mathrm{tg}\left(\frac{\pi}{2}-\Delta\varphi\right), \\ \omega^{*}_{\min II} &= \frac{a}{T}\,\mathrm{tg}\left(\frac{\pi}{2}-\Delta\varphi\right), \\ \omega^{*}_{\min III} &= \frac{1}{(1+K_{III})T}\,\mathrm{tg}\left(\frac{\pi}{2}-\Delta\varphi\right). \end{aligned}\right\} \tag{2.81}$$

Since $1 < \frac{1}{a} < (1+K_{III})$, then for given $\Delta\varphi$ and T the least permissible ω will be in an integrator of type III.

Let us consider the case of ideal integration. Here, according to (2.79) we obtain

$$\left.\begin{aligned} e_{\text{вых}} &= \frac{K_I}{T\omega}\,e_{\text{вх}}, \\ e_{\text{вых}} &= \frac{K_{II}}{T\omega}\,e_{\text{вх}}, \\ e_{\text{вых}} &= \frac{1}{T\omega}\,e_{\text{вх}}. \end{aligned}\right| \tag{2.82}$$

Assuming that $\frac{e_{\text{вых}\,\max}}{e_{\text{вх}\,\min}} = K_a$ is a finite quantity, we will find the minimum permissible ω from expression (2.82):

$$\omega^{**}_{\min I} = \frac{K_I}{TK_a}, \quad \omega^{**}_{\min II} = \frac{K_{II}}{TK_a}, \quad \omega^{**}_{\min III} = \frac{1}{TK_a}. \tag{2.83}$$

Since $K_I > K_{II} > 1$, it follows that an integrator of type III gives in this case too the least permissible value of ω .

Thus, for each type of device there are obtained three conditions for determination of $\omega_{\min}$. Obviously, the determining condition will be the one for which the frequency is higher. Which of the three conditions will be the determining one, depends on the taken values of Δ, $\Delta\varphi$, T, a, K_a, K_I, K_{II} and K_{III}

In Table II are brought, for example, results of calculation for values of the enumerated parameters, often met in existing computing elements.

Thus, for integrators of types I and II the quantity $\omega_{\min}$ is determined by the necessity of sustaining the given phase error, and for a device of type III — by the finiteness of quantity K_a. This last is limited in by output voltage by the limits of linearity of the amplifier, and in input signal — by its minimum value, which it is still possible to distinguish in the output signal from interference, i.e., the quantity of permissible error. In many cases it is possible to consider $e_{\text{вых max}} = 100$, $e_{\text{вх min}} = 0.1$ and, consequently, $K_д = 1000$

During integration of a step input signal output voltages of integrators will change by these laws:

$$\left.\begin{aligned}
e_{\text{вых}} &= K_I e_{\text{вх}}\left(1 - e^{-\frac{t}{T}}\right) \approx K_I e_{\text{вх}} \frac{t}{T}\left(1 - \frac{t}{2T}\right),\\
e_{\text{вых}} &= \frac{K_{II}}{\alpha} e_{\text{вх}}\left(1 - e^{-\frac{\alpha t}{T}}\right) \approx K_{II} e_{\text{вх}} \frac{t}{T}\left(1 - \frac{\alpha t}{2T}\right),\\
e_{\text{вых}} &= K_{III} e_{\text{вх}}\left(1 - e^{-\frac{t}{(1+K_{III})T}}\right) \approx\\
&\approx \frac{K_{III}}{1+K_{III}} e_{\text{вх}} \frac{t}{T}\left(1 - \frac{t}{2(1+K_{III})T}\right)
\end{aligned}\right| \qquad (2.84)$$

Comparison of these formulas shows that error of integration, expressed by the second addend in parentheses, will increase most slowly for integrators of the third type.

Table II

(a) Тип устройства	(b) Условия определения ω_{min}			(g) Примечание
	(c) погрешность		(f) конечное значение динамического диапазона $K_д$ = 1000	
	(d) амплитудная	(e) фазовая		
I	22.4	57.3	0,01	$K_I = 10$ $T = 1$
II	0,224	0,573	0,002	$K_{II} = 2$ $\alpha = 0{,}01$ $T = 1$
III	$4{,}5\cdot10^{-4}$ $4{,}5\cdot10^{-6}$	$1{,}15\cdot10^{-3}$ $1{,}15\cdot10^{-5}$	$1\cdot10^{-5}$	$K = 5\cdot10^{4}$ $K_{III} = 5\cdot10^{6}$ $T = 1$

KEY: (a) Type of device; (b) Conditions of determination of ω_{min}; (c) error; (d) Amplitude error; (e) Phase error; (f) Finite value of dynamic range; (g) Note.

Appraisal of the maximum permissible duration of the process of integration can be conducted just as in the preceding case, proceeding from the given systematic error of the process of integration and the finite value of dynamic range of voltages of the instrument.

In the first case obtain:

$$t_{\max I} = 2T\Delta \quad t_{\max II} = \frac{2T\Delta}{\varepsilon}, \quad t_{\max III} = 2T\Delta(1 + K_{III}). \tag{2.85}$$

Proceeding from the finite value of $K_д$ in ideal integration we have:

$$t_{\max I} = \frac{K_д}{K_I}T, \quad t_{\max II} = \frac{K_д}{K_{II}}T, \quad t_{\max III} = K_д T. \tag{2.86}$$

In Table III are the results of calculation of $t_{\max}$ for the most frequently encountered values of parameters.

Table III

(a) Тип устройства	(b) Условия определения $t_{\max}$		
	(c) Погрешность $\Delta = 0{,}001$	(d) Конечное значение $K_д = 1000$	(e) Примечание
I	0,002 сек	100 сек	$K_I = 10$ $T = 1$ сек
II	0,2 сек	500 сек	$K_{II} = 2$ $T = 1$ сек
III	100 сек 10 000 сек	1 000 сек	$K_{III} = 5 \cdot 10^4$ $T = 1$ сек $K_{III} = 5 \cdot 10^6$ $T = 1$ сек

KEY: (a) Type of device; (b) Conditions of determination of $t_{\max}$; (c) error; (d) finite value; (e) note.

Data of Table III indicate that for the most frequently encountered values of parameters and permissible error of $\Delta = 0.1\%$ determining factors for finding the maximum permissible duration of integration are: systematic error for integrators of types I, II and III (when $K_{III} = 5 \cdot 10^4$) and the finite value of the dynamic range of voltages for devices of type III when $K_{III} = 5 \cdot 10^6$. Devices of type III as compared with devices of types I and II, other conditions being equal, ensure a time of integration many orders greater.

For further increase of the permissible time of integration of a stabilized operational amplifier ($K_{III} = 5 \cdot 10^6$), as follows from these computations, it is necessary to increase the dynamic range of voltages of the instrument. This is possible to carry out both by decrease of error and by expansion of the linear range of change of output voltage. The last was very shrewdly realized in the step integrator offered by I. V. Korol'kov and I. A. Bubnov [1] (Fig. 32).* The basic idead of this integrator consists of the fact that the operational amplifier integrates only for the duration of permissible time $t_{max\,III}$.

After e'_{out} reaches the value $e_{out\,max}$ = ± 100 v on the divider of reversible stepping selector // the cursor moves one lamella (one step) and voltage ± ΔU is stored; simultaneously capacitor C is discharged by contacts of relay [illegible] or [illegible] and integration starts all over. Output voltage e_{out} is taken from auxiliary adder 2, where there is added the voltage from the divider of the stepping selector and voltage from the output of the integrator. This ensures a smooth curve the whole range of the output signal. Thus, after n steps on the output of the divider there will be established voltage $n\Delta U_1$, which corresponds as it were to an n time increase of the upper limit of linearity of the amplifier, and consequently, of $K_{Д}$. Therefore the maximum permissible time of integration now will be increased n times: $t_{max\,III} = nK_1T$, where n is the number of steps of the divider for voltage of the same sign. Simultaneously with increase of the dynamic range of voltages one can somewhat decrease error due to zero drift of zero due to operation with increased input signals. Thus, application of passive integrating networks with an amplifier is expedient starting with a frequency of the input signal of 10 c and higher. When indispensable to integrate signals of minute frequency it is necessary to change to operational amplifiers. The minimum permissible frequency here will be in $(1+K_{III})$ times less as compared with the case of application of passive cells. The factor determining the quantity value of ω_{min} for all three types of devices is the phase error. An exception are integrators based on stabilized operational amplifiers, for which the determining factor becomes finiteness of the quantity $K_{Д}$. Maximum permissible time of integration of a step input signal for all three types of integrators (with the exception of integrators with stabilized operational amplifiers) is determined by methodical error. Here operational amplifiers have the greatest time of integration. For integrators with stabilized operational amplifiers, the determining factors become lead of the integrating capacitor and finiteness of the dynamic range. The time of integration t_{max} for such amplifiers does not exceed 1000 sec when $K_Д = 1000$ and $R_y = 4 \cdot 10^5$ megohms. Further expansion of the permissible time of integration can be attained by application of the circuit of a step integrator with simultaneous increase of the leak resistance of the integrating capacitor.

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*See also L. N. Fitsner, L. I. Shevchenko, One method of integration of electric voltage, Instrument-making, No. 8, 1957.

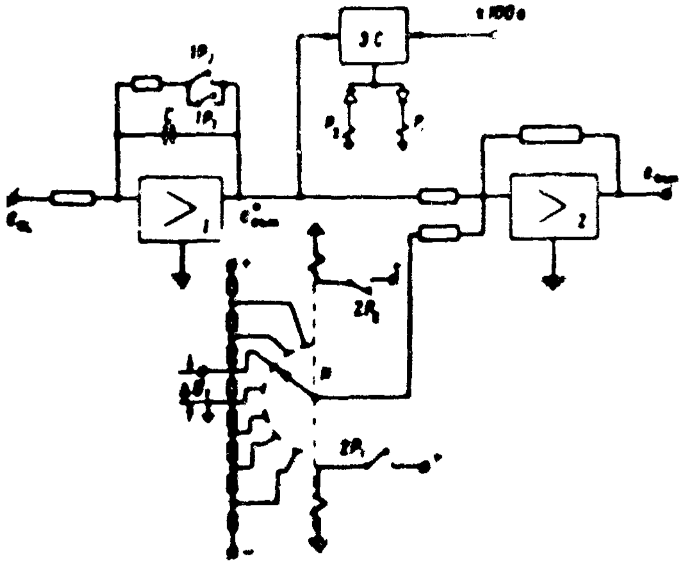

Fig. 32. Functional diagram of a step integrator.
ЭС — comparator, И — stepping selector.

CHAPTER III

ERROR OF LINEAR COMPUTING ELEMENTS

1. Basic Propositions of Error Theory

Investigation of error of a computing element pursues the goal of giving an appraisal of the accuracy of fulfillment of a given mathematical operation, of determining the main primary sources of error and their influence on total error and, finally, of establishing ways and methods of decreasing the most substantial components of error. Knowledge of error, introduced by a separate computing element, will allow one to proceed to appraisal of the error of a complex of such elements, i.e., to appraisal of error of the solution of a differential equation.

Analysis of errors of linear and nonlinear active and passive electric circuits, including computing elements, is the subject of fairly extensive literature (M. L. Bykhovskiy [1] - [3], M. A. Shnaydman [1]).

Error of a computing element is what we usually call the difference between real and ideal values of output magnitude at a given moment of time for the same value of input magnitude:

$$\Delta e_{out}(t) = e_{out\,r}(t) - e_{out\,i}(t)$$

where $e_{out\,i}(t)$ is the ideal output quantity, corresponding to a given mathematical operation, $e_{out\,r}(t)$ is the real output magnitude, obtained as a result of operation of the device.

Total error $\Delta e_{out}(t)$ can be divided into two parts: systematic and random.

The systematic part of total error is either constant, or changes by a prior known law. The random part of error is caused by accidental factors or spread of parameters within allowances for parts constituting the computing element. Random errors can change arbitrarily in time, i.e., be random functions of time or not change in time, i.e., be random parameters. Depending upon the character of processes in the computing element calculation of total error can be conducted for two regimes: steady-state and transient.

Determination of dynamic error is significantly simplified, if one were to use operational calculus (see for instance, M. I. Kantorovich [1]). Here it is possible to reduce a dynamic problem to a static one by transition to a so-called representation circuit, i.e., the circuit of a computing element, composed of operator resistances and the emf of initial conditions, assuming first that the amplifier of the computing element is ideal (its transfer function is constant). Such idealization will allow one in the beginning not to worry about questions of stability of the computing element and not consider finiteness its passband.

If the computing amplifier ideally executes the given mathematical operation, then the connection between output and input quantities will be determined by the relationship (see Chapter II, page 61)

$$\bar{e}_{\text{вых}\,и} = -\sum_{i=1}^{n} [\gamma_{1i}(p)]_и (\bar{e}_{\text{вх}\,i})_и \qquad (3.1)$$

where

$$[\gamma_{1i}(p)]_и = \frac{[Y_{1i}(p)]_и}{[Y_i(p)]_и}$$

[Ed. Note. Subscript И = ideal].

The result of the mathematical operation, executed by the operational amplifier, may differ from the ideal (3.1) due to errors in physical realization of the given transfer functions $[\gamma_{1i}(p)]$, due to the presence at the input of the amplifier of an emf Δe_1, equivalent to instability of the zero level at the output (so-called sero drift) and due to the presence in input voltages of spurious high-frequency components. Furthermore, there can be inaccurate set-up of input data, for example, in case of feeding the operational amplifier from potentiometric (or other type) transducers of the investigated equipment.

If one were to use equation (3.1 and consider the above-mentioned factors, with the exception of spurious high-frequency components of the output voltage, then the connection between output and input quantities for the operational amplifier can be presented in the form

$$e_{out} = \sum_{i=1}^{n} [W_{i}(p)]_p\, e_{i} \; [illegible]$$

$$+ e_1 \sum_{i=1}^{n} [W_i(p)] \left[\frac{[illegible]}{\sum [W_i(p)]} \right] \Delta e_{0} \qquad (3.2)$$

where

$$[W_{1i}(p)]_p = \frac{\dfrac{[W_{oi}(p)]_p}{[W_i(p)]_p}}{\left(\sum_{i=1}^{n}\dfrac{[W_{oi}(p)]_p}{[W_i(p)]_p}+1\right)\dfrac{1}{K_y}+1}$$

[Ed. Note. Subscript И and P = ideal and real, respectively].

is the transfer function for the i-th input; $\Delta e_{out\,0}$ the output voltage, caused by inaccurate setting of zero. On the basis of (3.2) the transfer function for the signal of drift is determined as

$$\sum_{i=1}^{n}[W_{1i}(p)]_{\text{И}} \quad \frac{\sum_{i=1}^{n}[W_{i}(p)]_p}{\sum_{i=1}^{n}[W_i(p)]_{\text{И}}}.$$

This expression indicates why it is necessary to bring to the input of the amplifier the signal of instability of the sero level, measured at the output of the computing element:

$$\Delta e_0 = \frac{e_{out\,0}}{\sum_{i=1}^{n}[W_i(p)]_p + \dfrac{\sum_{i=1}^{n}[W_i(p)]_p}{\sum_{i=1}^{n}[W_i(p)]_{\text{И}}}} \qquad (3.3)$$

If the error of the operational amplifier is small, then $|\varphi_{1i}(p)|_p$ differs little from $|\varphi_{1i}(p)|_н$ and in this case in the first order of approximation

$$\Delta \bar{e}_z = \frac{\bar{e}_{\text{вых } z}}{\sum_1^n |\varphi_{1i}(p)|_p + 1}. \tag{3.4}$$

Error of output voltage of the computing element can be calculated, if one were to place in equation (3.2):

$$\bar{e}_{\text{вых } p} = \bar{e}_{\text{вых } z} + \Delta \bar{e}_{\text{вых}},$$
$$|\varphi_{1i}(p)|_p = |\varphi_{1i}(p)|_н + \Delta \varphi_{1i}(p),$$
$$(\bar{e}_{\text{вх } i})_p = (\bar{e}_{\text{вх } i})_н + \Delta \bar{e}_{\text{вх } i}$$

and subtract equation (3.2) from equation (3.1).

Disregarding product of increments of variables and their squares as compared with the increments themselves, we will receive

$$\begin{aligned}\Delta \bar{e}_{\text{вых}} = -\sum_1^n \Delta \varphi_{1i}(p)(\bar{e}_{\text{вх } i})_н - \sum_1^n |\varphi_{1i}(p)|_н \Delta \bar{e}_{\text{вх } i} + \\ \mp \left[\sum_1^n |\varphi_{1i}(p)|_н + 1\right] \Delta \bar{e}_z \mp \Delta \bar{e}_{\text{вых } 0}.\end{aligned} \tag{3.5}$$

Error in the transfer functions $\Delta \varphi_{1i}(p)$ in turn depends on finiteness of the amplification factor, the presence of leaks at the input, output load and errors in parameters of elements of the input circuit and the feedback circuit. Therefore

$$\Delta \varphi_{1i}(p) = \left[|\varphi_{1i}(p)|_p - |\varphi_{1i}(p)|_н\right]_{\substack{Y_{1i}=(Y_{1i})_н \\ Y_2=(Y_2)_н}} + \frac{\partial \varphi_{1i}(p)}{\partial Y_{1i}} \Delta Y_{1i} + \frac{\partial \varphi_{1i}(p)}{\partial Y_2} \Delta Y_2. \tag{3.6}$$

Total absolute error of the unit here will be

$$\begin{aligned}\Delta \bar{e}_{\text{вых}} = -\Bigg\{\sum_1^n [\varphi_{1i}(p)_p - \varphi_{1i}(p)_н](\bar{e}_{\text{вх } i})_н + \sum_1^n \frac{\partial \varphi_{1i}}{\partial Y_{1i}} \Delta Y_{1i}(\bar{e}_{\text{вх } i})_н + \\ + \sum_1^n \frac{\partial \varphi_{1i}(p)}{\partial Y_2} \Delta Y_2 (\bar{e}_{\text{вх } i})_н\Bigg\} - \sum_1^n |\varphi_{1i}(p)|_н \Delta \bar{e}_{\text{вх } i} \mp \\ \mp \left[\sum_{i=1}^n |\varphi_{1i}(p)|_н + 1\right] \Delta \bar{e}_z \mp \Delta \bar{e}_{\text{вых } 0}.\end{aligned} \tag{3.7}$$

Thus, total error of the computing block consists of a number of primary errors, caused by inaccurate physical realisation of the diagram block (presence of leaks, leak and a finite amplification factor), errors of input quantities, errors in set-up of parameters of input impedances and feedback resistances and, finally, presence of instability of the zero level (Δe_1) and inaccuracy of its set-up $(\Delta e_{\text{вых}\,0})$.

Determination of total absolute error by expression (3.7) is possible only when the value of all primary errors are known beforehand. In reality, separate primary errors, being random variables, can take in each new cycle of work of the block different values within the full range of their change.

If a random variable has a normal distribution of probabilities, then, as we know (see, for instance, B. V. Gnedenko, A. Ya. Khinchin [1], P. L. Chebyshev [1]), the probability that its maximum deflection from the mean value exceeds the quantity 3σ (where σ is the root-mean-square devlection) is minute and constitutes 0.27%. This allows one to take as an appraisal of maximum deflection of a random variable from the mean value the quantity 3σ. If the considered random variable depends linearly on a number of mutually independent primary random variables, each of which has a normal distribution of probabilities, then its maximum deflection from mean value will, as before, lie within 3σ, but now the average and root-meat-square deflection will be determined as the sum of mean and root-mean-square deflections of the primary random variables.

If it is considered that $\Delta V_{01}, \Delta V_2, \Delta e_1, \Delta e_2$ and $\Delta e_{\text{вых}\,0}$ are random variables, then the maximum value of the random component of error can be determined from the expression

$$\Delta e_{\text{вых сл}} = |a_{\Delta e_{\text{вых}}}| + 3\sqrt{D_{e_{\text{вых}}}}, \tag{3.8}$$

where $a_{\Delta e_{\text{вых}}}$ is the mean value of error of the output voltage, and $\sqrt{D_{e_{\text{вых}}}}$ is the total root-mean-square deflection.

In the considered case the resultant mean and root-mean-square deflection can be found from these relationships:

$$a_{\Delta e_{\text{вых}}} = +\sum_{i=1}^{n}\left(\frac{\partial\varphi_{1i}}{\partial Y_{1i}}\right)(\bar{e}_{\text{вх}\,i})_n\,a_{\Delta Y_{1i}} + \sum_{i=1}^{n}\left(\frac{\partial\varphi_{1i}}{\partial Y_i}\right)(\bar{e}_{\text{вх}\,i})_n\,a_{\Delta Y_i} + \sum_{i=1}^{n}[\varphi_{1i}(p)]_n\,a_{\Delta e_{\text{вх}\,i}} + \left[\sum_{i=1}^{n}[\varphi_{1i}(p)]_n + 1\right]a_{\Delta e_{\lambda}} + a_{\Delta e_{\text{вых}\,0}}. \qquad (3.9)$$

$$\sigma_{e_{\text{вых}}} = \sqrt{D_{e_{\text{вых}}}} = \left\{\left[\sum_{i=1}^{n}\left(\frac{\partial\varphi_{1i}}{\partial Y_{1i}}\right)^2\sigma^2_{\Delta Y_{1i}} + \sum_{i=1}^{n}\left(\frac{\partial\varphi_{1i}}{\partial Y_i}\right)^2\sigma^2_{\Delta Y_i}\right](\bar{e}_i)^2_n + \sum_{i=1}^{n}[\varphi_{1i}(p)]^2_n\,\sigma^2_{\Delta e_{\text{вх}\,i}} + \left[\sum_{i=1}^{n}[\varphi_{1i}(p)]_n + 1\right]^2\sigma^2_{\Delta e_{\lambda}} + \sigma^2_{\Delta e_{\text{вых}\,0}}\right\}^{1/2}. \qquad (3.10)$$

Quantities of the mean value (expectation) and dispersion of separate errors can be found, using properties of a curve normal distribution (Fig. 33) and knowing the boundaries of the field of tolerance for the considered random variables (N. G. Bruevich [1]). In Fig. 33 on the axes of coordinates are placed possible deviations q_i and their probability density $f(q_i)$.

Indeed, if in the field of tolerance the distributive law of error is a symmetrically located curve, then

$$\sigma_i^2 = \frac{1}{9}\Delta_i^2, \qquad a_i = \frac{1}{2}(g_i - m_i). \qquad (3.11)$$

where Δ_i is half the field of tolerance, and g_i and m_i are algebraic boundaries of the field of tolerance.

Total absolute error of a computing element here will be

$$\Delta e_{\text{вых}} = \Delta e_{\text{вых. сист}} + |a_{\Delta e_{\text{вых}}}| + 3\sigma_{e_{\text{вых}}},$$

where $\Delta e_{\text{вых. сист}}$ is the systematic error.

As the criterion of accuracy of a computing element it is impossible to take the absolute value of error, since accuracy of work of the device depends not only on absolute error of its output quantity, but also on the value of the maximum output quantity. Obviously, for equal absolute values of total error the accuracy is higher for that device, for which the output quantity is greater.

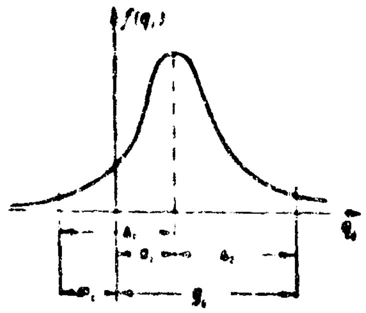

Fig. 33. Curve of normal distribution.

In measuring technology (see, for instance, Ye. G. Shramkov [1]) there is often used as the criterion of accuracy the quantity of absolute error related to the maximum value of the instrument scale — the relative error. In our case this will be the ratio of absolute error to the maximum output quantity

$$\delta e_{\text{вых}} = \frac{\Delta e_{\text{вых}}}{e_{\text{вых. max}}}, \tag{3.12}$$

where $e_{\text{вых. max}}$ is the maximum value of output voltage of the block.

When the quantity of absolute error is in turn a function of the output quantity, i.e., has different meanings at every point of the instrument scale, one should use the particular practical and integral practical criteria offered in theories of accuracy of mechanisms (N. G. Bruyevich [1]). The particular practical criterion determines accuracy of a device at a given point of the operating range:

$$\xi_{\text{частн. практ}} = \frac{\Delta e_{\text{вых}}(e_{\text{вых}})}{e_{\text{вых. max}}}, \tag{3.13}$$

and the integral practical criterion allows one to estimate accuracy of operation of a device at a definite interval of change of the output quantity

$$\sum_{\text{практ}} = \frac{\int\limits_{e_{\text{вых 1}}}^{e_{\text{вых 2}}} \xi_{\text{частн. практ}}\, de}{e_{\text{вых. max}}}. \tag{3.14}$$

It is easy to prove that both these criteria change into the generally-accepted criterion of accuracy (3.12), when $\Delta e_{\text{вых}}$ does not depend on the value of the output quantity in the operating range. For a linear computing element as the criterion of accuracy we usually take the expression

$$\delta e_{\text{вых}} = \frac{\Delta e_{\text{вых}}}{e_{\text{вых. max}}}.$$

One should give independent meaning to the absolute magnitude of the criterion of accuracy. By its nature this criterion gives only an appraisal of maximum possible error of element, and therefore allows us to compare various computing elements by their accuracy, and also to explain the most suitable methods of decreasing error. It is obvious, that that computing element for which the magnitude of the expression, taken as the criterion of accuracy takes the least value, is the best one.

2. Primary Errors of a Computing Element

Let us consider those primary errors, which determine the systematic part of error of a computing element. As was shown above, this part of total error is caused by inaccurate physical realization of the given transfer function, basically due to finiteness of the amplification factor, presence of spurious leaks and capacitive couplings in the circuit of the computing element, and also owing to internal resistance of the last cascade and the load on its output. The systematic part of error of output voltage will be determined from the relationship (3.7):

$$\Delta\bar{e}_{\text{вых.сист}} = -\sum_{1}^{n} \left\{[\varphi_{1i}(p)]_p - [\varphi_{1i}(p)]_{\text{н}}\right\} (\bar{e}_{\text{вх}\,i})_{\text{н}}.$$

Derivation of the expression for the transfer function of $[\varphi_{1i}(p)]_p$. We will definitise expression of the transfer function of $[\varphi_{1i}(p)]_p$. received in Section 1 taking into account the above-mentioned distorting factors. For this let us consider the equivalent functional circuit of an operational amplifier, shown in Fig. 34. Using the method of finding the transfer function shown in Chapter II, we will receive for the considered case: the equation for the voltage of error

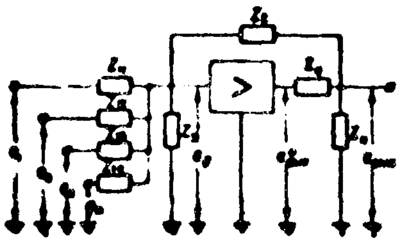

Fig. 34. Functional diagram of an operational amplifier.

$$\bar{e}_\varepsilon = f_{11}(p)\bar{e}_1 + f_{12}(p)\bar{e}_2 + \dots + f_{1n}(p)\bar{e}_n + f_2(p)\bar{e}_{\text{вых}}; \qquad (3.15)$$

here transfer functions $f_{11}(p),\ldots, f_{1n}(p)$ and $f_2(p)$ have the values:

$$\left.\begin{aligned} f_{11}(p) &= \left(\frac{\bar{e}}{\bar{e}_1}\right)_{e_2=e_3=\ldots=e_n=e_{\text{вых}}=0} = \frac{Y_{11}(p)}{\sum_1^n Y_{1i}(p)+Y_a(p)+Y_3(p)}, \\ & \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ f_{1n}(p) &= \left(\frac{\bar{e}_1}{\bar{e}_n}\right)_{e_1=e_2=\ldots=e_{n-1}=e_{\text{вых}}=0} = \frac{Y_{1n}(p)}{\sum_1^n Y_{1i}(p)+Y_a(p)+Y_3(p)}, \\ f_2(p) &= \left(\frac{\bar{e}}{\bar{e}_{\text{вых}}}\right)_{e_1=e_2=\ldots=e_n=0} = \frac{Y_3(p)}{\sum_1^n Y_{1i}(p)+Y_a(p)+Y_3(p)}; \end{aligned}\right\} \tag{3.16}$$

the equation of the amplifier taking into account internal resistance of the last cascade and of the load

$$\bar{e}_{\text{вых}} = -K_y^* \frac{Y_0(p)\left[1-\frac{Y_3(p)}{Y_{0}(p)K_y^*}\right]}{Y_0(p)+Y_н(p)+Y_3(p)}\,\bar{e}. \tag{3.17}$$

Comparison of expressions (3.14) and (3.17) with analogous expressions, found earlier, shows that taking into account internal resistance and load in the last cascade leads to a certain decrease of the total amplification factor. Designating for simplification

$$\frac{Y_0(p)\left[1-\frac{Y_3(p)}{Y_0(p)K_y^*}\right]}{Y_0(p)+Y_н(p)+Y_3(p)} = \zeta(p),$$

we receive

$$\bar{e}_{\text{вых}} = -\frac{\sum_1^n Y_{1i}(p)\bar{e}_i}{Y_3(p)}\,\frac{1}{\left[\sum_1^n Y_{1i}(p)+Y_a(p)+Y_3(p)\right]\frac{1}{Y_3(p)}\frac{1}{K_y^*\zeta(p)}+1}. \tag{3.18}$$

Here $K_y^* = K_y\frac{Y_i+Y_a}{Y_i}$ is the amplification factor of the amplifier without feedback, calculated under the condition that in the output cascade $Y_a \to 0$; K_y is the amplification factor of the amplifier without feedback, calculated taking into account the dynamic amplification factor of the output cascade, but under the condition that there is no external load, i.e., $(Y_н)_{\text{внешн}} \to 0$; Y_a is conductance of the plate load of the tube of the output cascade; $Y_i = \frac{1}{R_i}$ is the internal conductance of the tube of the output cascade.

Hence

$$|\varphi_{1i}(p)|_p = \frac{Y_{1i}(p)}{Y_2(p)} \frac{1}{\left[\sum_{1}^{n} Y_{1i}(p) + Y_3(p) + Y_2(p)\right] \frac{1}{Y_2(p)} \frac{1}{K_y^{0}\text{З}(p)} + 1} . \tag{3.19}$$

Error of the physical realization of the transfer function can be found by expression (3.19) in the form

$$\begin{aligned} \Delta\varphi_{1i}(p) = {} & \frac{\Delta Y_{1i}(p)}{Y_{1i}(p)} \frac{Y_{1i}(p)}{Y_2(p)} - \frac{\Delta Y_2(p)}{Y_2(p)} \frac{Y_{1i}(p)}{Y_2(p)} - \\ & - \left\{\left[\sum_{1}^{n} Y_{1i}(p) + Y_3(p) + Y_2(p)\right] \frac{1}{Y_2(p)} \frac{1}{K_y^{0}\text{З}(p)}\right\} \frac{Y_{1i}(p)}{Y_2(p)} . \end{aligned} \tag{3.20}$$

The systematic part of the absolute error of output valtage will be here

$$\begin{aligned} \Delta\bar{e}_{\text{вых.сист}} = {} & -\sum_{1}^{n} \frac{\Delta Y_{1i}}{Y_{1i}(p)} \frac{Y_{1i}(p)}{Y_2(p)} \bar{e}_{\text{вх}\,i} + \frac{\Delta Y_2(p)}{Y_2(p)} \sum_{1}^{n} \frac{Y_{1i}(p)}{Y_2(p)} \bar{e}_{\text{вх}\,i} + \\ & + \left\{\left[\sum_{1}^{n} Y_{1i}(p) + Y_3(p) + Y_2(p)\right] \frac{1}{Y_2(p)} \frac{1}{K_y^{0}\text{З}(p)}\right\} \sum_{1}^{n} \frac{Y_{1i}(p)}{Y_2(p)} \bar{e}_{\text{вх}\,i} . \end{aligned} \tag{3.21}$$

Relative systematic error will be

$$\begin{aligned} \delta\bar{e}_{\text{вых.сист}} = {} & \frac{\Delta\bar{e}_{\text{вых.сист}}}{\bar{e}_{\text{вых.max}}} = \\ = {} & -\sum_{1}^{n} \frac{\Delta Y_{1i}(p)}{Y_{1i}(p)} \frac{Y_{1i}(p)}{Y_2(p)} \frac{\bar{e}_{\text{вх}\,i}}{\bar{e}_{\text{вых.max}}} + \frac{\Delta Y_2(p)}{Y_2(p)} \sum_{1}^{n} \frac{Y_{1i}(p)}{Y_2(p)} \frac{\bar{e}_{\text{вх}\,i}}{\bar{e}_{\text{вых.max}}} + \\ & + \left\{\left[\sum_{1}^{n} Y_{1i}(p) + Y_3(p) + Y_2(p)\right] \frac{1}{Y_2(p)} \frac{1}{K_y^{0}\text{З}(p)}\right\} \sum_{1}^{n} \frac{Y_{1i}(p)}{Y_2(p)} \frac{\bar{e}_{\text{вх}\,i}}{\bar{e}_{\text{вых.max}}} . \end{aligned} \tag{3.22}$$

Let us consider each component of error.

Error of conductance of the input circuit. This error, as expression (3.22) shows, other conditions being equal is greater, the greater the number of component and the greater the transmission ratio $\frac{Y_{1i}}{Y_2}$. set on the computing element. Error of input conductance (systematic) usually takes place during operations of a computing element as a differentiater, when to the input is connected a capaciter, possessing a finite leak resistance R_y. Designating conductance of the leak of this capacitor by $A_y(p)$, we will receive:

$$\Delta Y_{1i}(p) = [Y_{1i}(p) + A_y(p)] - Y_{1i}(p) = A_y(p). \tag{3.23}$$

$$\delta\bar{e}_{\text{вых. сист}_1} = \frac{A_y(p)}{Y_2(p)}\,\frac{\bar{e}_{\text{вх}\,i}}{\bar{e}_{\text{вых. max}}}.$$

We will estimate the maximum possible value of this component of error. Let

$$\frac{e_{\text{вх}\,i}}{e_{\text{вых. max}}} = 1, \quad Y_2 = 1 \cdot 10^{-6}\,\text{mho}, \quad A_y = 0.2 \cdot 10^{-9}\,\text{mho}$$

(for styroflex capacitors). Then

$$\delta e_{\text{вых. сист}_1} = \frac{0.2 \cdot 10^{-9}}{10^{-6}} = 2 \cdot 10^{-4} \text{ or } 0.02\%.$$

Error of conductance in the feedback circuits. This error is caused mainly by the presence of leak in the capacitor, connected during work of the computing element as an integrator or integrator-adder. Here $\Delta Y_2 = A_y$, and

$$\delta\bar{e}_{\text{вых. сист}_2} = \frac{A_y}{Y_2}\sum_{i=1}^{n}\frac{Y_{1i}(p)}{Y_2(p)}\,\frac{\bar{e}_{\text{вх}\,i}}{\bar{e}_{\text{вых. max}}}$$

Changing to originals and assuming that $e_{\text{вх}\,i}$ changes in such a manner that $e_{\text{вх}\,i} = 0$ when t < 0 and $e_{\text{вх}\,i} = 1$ when $t \geqslant 0$, we will receive

$$\frac{d^2\delta e_{\text{вых. сист}_2}}{dt^2} = \frac{A_y}{C^2}\sum_{i=1}^{n}\frac{1}{R_{1i}}\,\frac{e_{\text{вх}\,i}}{e_{\text{вых. max}}}$$

whence for zero initial conditions we will have

$$\delta e_{\text{вых. сист}_2} = \frac{A_y}{C^2}\left[\sum_{i=1}^{n}\frac{1}{R_{1i}}\,\frac{e_{\text{вх}\,i}}{e_{\text{вых. max}}}\right]\frac{t^2}{2}. \tag{3.24}$$

Thus, error due to leak of the capacitor during work of the block as an integrator-adder will grow proportionally to the square of the time.

We will estimate the magnitude of this component of error.

Let C = 1 microfarad, $A_y = \frac{1}{5000}\left[\frac{1}{\text{megohm}}\right]$, n = 1, R_1 = 1 megohm. Then in the maximum time of solution

$$t_x = \frac{1}{\sum_{i=1}^{n}\frac{1}{CR_{1i}}\,\frac{e_i}{e_{\text{вых. max}}}}$$

we will receive

$$\delta e_{\text{вых. сист}_1} = \frac{A_y t_K}{2K}.$$

For t_K = 100 sec

$$\delta e_{\text{вых. сист}_1} = \frac{100}{2\cdot 5000} = 10^{-2} \text{ or } 1\%.$$

Error due to finiteness of the amplification factor of the amplifier. This component of error on the basis of expression (3.22) is determined by the relationship

$$\bar{\delta e}_{\text{вых. сист}_1} = \left\{\left[\sum_{1}^{n} Y_{1i}(p) + Y_a(p) + Y_2(p)\right] \frac{1}{Y_0(p)\beta(p)} \frac{1}{K_y'}\right\} \times \\ \times \sum_{1}^{n} \frac{Y_{1i}(p)}{Y_0(p)} \frac{\bar{e}_{\text{вх}\, i}}{e_{\text{вых. max}}}. \tag{3.25}$$

Here along with a finite value of the amplification factor, there is also taken into account the influence of load $\beta(p)$ and leak of the grid of the input cascade of the amplifier $Y_3(p)$. Since usually quantity on β is always less than one, the influence of load can be accounted for during analysis of error, caused by finiteness of the amplification factor, by means of a corresponding decrease of the amplification factor.

Indeed,

$$\beta(p) = \frac{Y_0(p)}{Y_0(p) + Y_a(p) + Y_2(p)}.$$

Here $Y_a(p) + Y_0(p) + [Y_a(p)]_{\text{[illegible]}}$. Since usually $Y_0 = 10^{-4}$ mho, $(Y_a)_{\text{[illegible]}} = 10^{-4}$ mho, $Y_a = 10^{-4}$ mho and $Y_2 = 10^{-6}$ mho, then the quantity Y_2 is at least two orders less than Y_0, Y_a and $(Y_a)_{\text{[illegible]}}$. During operation of a computing element in other regimes this conclusion remains correct, and therefore with sufficient accuracy it is possible to consider

$$\beta(p) = \frac{Y_0}{Y_0 + Y_a + (Y_a)_{\text{[illegible]}}}.$$

We will introduce the concept of the resultant amplification factor of an amplifier taking into account load $(K_y)_{\text{рез}} = K_y^{*}$. Since

$$K_y^{*} = K_y \frac{Y_0 + Y_a}{Y_0},$$

then

$$(K_y)_{\text{рез}} = K_y \frac{Y_0 + Y_a}{Y_0} \cdot \frac{Y_0}{Y_0 + (Y_н)_{\text{внешн}} + Y_a} = K_y \frac{1}{1 + \frac{(Y_н)_{\text{внешн}}}{Y_0 + Y_a}};$$

when $(Y_н)_{\text{внешн}} = 10^{-4}$ мо, $Y_0 = 10^{-4}$ мо and Y_a 10^{-4} мо

$$K_{\text{рез}} = \frac{K_y}{1.5} \approx 0.67 K_y.$$

Thus, the resultant amplification factor when $(R_н)_{\text{внешн}}$ = 10 kilohms is lowered 33% as compared with the case, when $(R_н)_{\text{внешн}} \to \infty$.

We will now estimate the error, appearing due to finiteness of the value of the amplification factor for the heaviest case, when the operating amplifier works as an integrator-adder, and input voltages change instantly by a constant quantity. On the basis of (3.25) when $Y_{1i} = \frac{1}{R_{1i}}$, Y_2 = Cp and Y_3 = 0 we will receive

$$\delta u_{\text{вых. ест}_1} = \left\{\left[\sum_1^n \frac{1}{R_i} + Cp\right] \frac{1}{Cp} \frac{1}{0.67 K_y}\right\} \sum_1^n \frac{1}{R_i C p} \frac{e_{\text{вх}\, i}}{e_{\text{вх}\, \max}}$$

or, changing to originals,

$$\frac{d^2 \delta u_{\text{вых. ест}_1}}{dt^2} = \left(\sum_1^n \frac{1}{R_{1i} C}\right) \sum_1^n \frac{e_{\text{вх}\, i}}{e_{\text{вх}\, \max}} \frac{1}{R_i C} \frac{1}{0.67 K_y},$$

whence, for zero initial conditions,* we will find

$$\delta u_{\text{вых. ест}_1} = \left[\frac{1}{0.67 K_y} \sum_1^n \frac{1}{R_{1i} C} \sum_1^n \frac{1}{R_i C} \frac{e_{\text{вх}\, i}}{e_{\text{вх}\, \max}}\right] \frac{t^2}{2} \qquad (3.26)$$

Thus, during work of a computing element as an integrator-adder, the relative error, caused by finiteness of the amplification factor, will be greater, the less the amplification factor of the amplifier and the greater the value of the sum of transmission ratios for separate components. Error is directly proportional to the square of the time of integration. If absolute error of such a computing element is related not to the maximum value of output voltage, but to the ideal value at the given moment,

---

*In this equation there is not considered the delta-function found in a strict resolution of the initial equation.

$$e_{\text{вых}\,n} = -\sum_{1}^{n} \frac{1}{R_{1i}C} e_{\text{вх}\,i} t,$$

as this is done in literature (M. A. Shnaydman [1], G. Korn and T. Korn [1]), then expression (3.26) will be simplified:

$$\delta e^{*}_{\text{вых. сист}} = \frac{1}{0.67 K_y} \sum_{1}^{n} \frac{1}{R_{1i}C} \frac{t}{2}. \qquad (3.27)$$

From this expression, in particular, one can determine the required value of the amplification factor so that error does not exceed a given value:

$$K_y > \frac{1}{0.67\, \delta e^{*}_{\text{вых. сист}_1}} \sum_{1}^{n} \frac{1}{R_{1i}C} \frac{t}{2}. \qquad (3.28)$$

when $t = t_k = 100$ sec, $\delta e^{*}_{\text{вых. сист}_1} = 0.01$, $n = 5$, $R_{11} = R_{12} = R_{13} = R_{14} = R_{15} = 0.5$ мгом, $C = 1\ \mu F$ we will receive

$$K_y > 75\,000.$$

Thus, to execute the operation of integration of five components during the period of 100 sec with accuracy of 1% it is necessary to have an amplification factor of the amplifier of the order of 75,000.

Limit value of the amplification factor of the operational amplifier at zero frequency. As follows from the above, increase of the amplification factor at zero frequency causes decrease of error of carrying out a given mathematical operation. This is especially important during the operation of integration. In connection with this, the wide-spread opinion that the higher the amplification factor of the amplifier at zero frequency, the higher the performance of the computing element.

However if one were to consider real properties of the integrating capacitor, then it is possible to arrive at the conclusion that there exists completely definite value of the amplification factor of the amplifier, further increase of which no longer has practical meaning.

If one were to use the relationship (3.18) and substitute in it preliminarily (taking into account leak in the integrating point and leak of the dielectric of the integrating capacitor)

$$Y_{11} = \frac{1}{R_1}, \quad Y_{12} = Y_{13} = \ldots = Y_{1n} = 0, \quad K_y(p) = K_y,$$
$$Y_3 = \frac{1}{R_3}, \quad Y_2(p) = \frac{1}{R_2} + Cp,$$

that by simple transformations we will receive:

$$\bar{e}_{\text{вых}} = -\frac{K_y e_{\text{вх}}}{\left[1 + (1 + K_y)\frac{R_1}{R_y} + \frac{R_1}{R_3}\right] + (1 + K_y) R_1 C p}$$

We will introduce the concept of the effective amplification factor of the computing element in an integrator regime:

$$(K_y)_{\text{эфф}} = \frac{K_y}{1 + (1 + K_y)\frac{R_1}{R_y} + \frac{R_1}{R_3}}.$$

Then after division of the numerator and denominator of the right side of the expression for $\bar{e}_{\text{вых}}$ by $\left[1 + (1 + K_y)\frac{R_1}{R_y} + \frac{R_1}{R_3}\right]$ we finally will receive

$$\bar{e}_{\text{вых}} = -\frac{(K_y)_{\text{эфф}}\, \bar{e}_{\text{вх}}}{(K_y)_{\text{эфф}} R_1 C p + 1}$$

Thus, the structure of the formula of an integrating operational amplifier is kept during consideration of imperfection of the integrating capacitor and leaks at the amplifier input. Only the value of the effective amplification factor changes. It is obvious that to lower error one must seek to increase $(K_y)_{\text{эфф}}$.

The limit value of $(K_y)_{\text{эфф}}$ is attained when $K_y \to$

$$(K_y)_{\text{эфф пред}} = \frac{R_y}{R_1}$$

We will determine that value of the amplification factor $K_y = K_y^*$ of the amplifier, with which $(K_y)_{\text{пр}}$ will differ from the limit value by not more than 5%. From the formula for $(K_y)_{\text{пр}}$ it directly follows tha

$$K_y^* = 19\left(\frac{R_y}{R_1} + 1 + \frac{R_y}{R_1}\right) \approx 20\,\frac{R_y}{R_1}. \tag{3.29}$$

Thus, the maximum value of the amplification factor of the amplifier, taking into account imperfection of the integrating capacitor at zero frequency, must not be taken higher than $20\,\frac{R_y}{R_1}$.

3. Random Components of Error of a Computing Element

The main components of the random part of error of a computing element are instability and inaccurate setting of the zero level of the output voltage, presence of harmonics in the output voltage and inaccuracy of setting of resistances Z_{11} and Z_2.

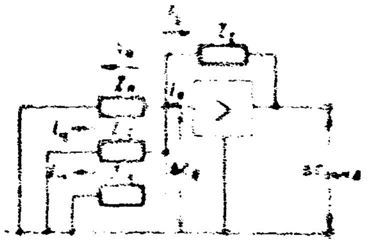

Fig. 35. On the question of zero drift of the operational amplifier.

Instability of the zero level is what we usually call variable voltage, appearing at the output of the operational amplifier with closed at a common point input impedances (Fig. 35). Appearance of voltage at the output of the operational amplifier in the absence of voltage at input can have the following causes:

1) instability of power supplies of the amplifier;

2) change of emitting properties of cathodes of electron tubes;

3) presence of grid currents;

4) variation of parameters of elements of the amplifier circuit.

Usually this false output voltage is lead to the input of the amplifier. On the basis of (3.4)

$$\Delta e_2 = \frac{\Delta e_{\text{out}\,2}}{1 + \sum_{1}^{n} [\gamma_{i1}(p)]_0} \tag{3.30}$$

For feeding operational d-c amplifiers we usually apply stabilized electronic rectifiers with an accuracy of maintenance of constancy of voltage of 0.1 to 0.01%.

Changes of the emission current of electron tubes takes place both with high frequency and also very slowly (N. A. Kaptsov [1]) and is caused both by the actual nature of thermionic emission and by oscillations of filament voltage.

For operational amplifiers slow changes of the emission current have the greatest importance. They lead to an equivalent change of grid bias of the electron tube. Thus, for example, for tube 6Ж9 the equivalent change of bias with a change of filament voltage by ± 10% is 210 millivolt (V. I. Sushkevich [1]).

As we know (see, for instance, V. F. Vlasov [1]), even with negative voltage on the grid of an electron tube in the grid circuit there can leak both positive and negative grid current. Let us consider in simplified form by the diagram of Fig. 35, the influence of grid current on the output voltage of the operational amplifier. One part of the grid current flows in the input impedances, and the other -- in the feedback resistance. Therefore

$$\bar{I}_g = \bar{I}_1 + \bar{I}_2,$$

where $\bar{I}_1 = \bar{I}_{11} + \bar{I}_{12} + \dots + \bar{I}_{1n}$ is the total current, branching in the input impedances. If the conductance of the input circuits $Y_{11}, Y_{12}, Y_{13}, \dots, Y_{1n}$, then $Y_{\text{вх}} = Y_{11} + Y_{12} + \dots + Y_{1n}$ and current $\bar{I}_1$ will be equal to

$$\bar{I}_1 = Y_{\text{вх}} \Delta \bar{e}_i. \tag{3.31}$$

Current $\bar{I}_2$ one can determine from the relationship

$$\bar{I}_2 = Y_2 (\Delta \bar{e}_i - \Delta \bar{e}_{\text{вых}\,i}). \tag{3.32}$$

Considering that $\Delta e_{\text{вых}\,i} = -K_y \Delta e_i$ we determine from equations (3.31) and (3.32) the quantity of the grid current:

$$\bar{I}_g = \Delta \bar{e}_i [Y_{\text{вх}} + Y_2 (1 + K_y)]$$

or

$$\bar{I}_g = \Delta \bar{e}_i Y_{\text{вх}} \left[1 + \frac{Y_2}{Y_{\text{вх}}} (1 + K_y) \right].$$

whence

$$\Delta \bar{e}_1 = \frac{\bar{I}_g}{Y_{\text{вх}}\left[1 + \frac{Y_2}{Y_{\text{вх}}}(1 + K_y)\right]}. \tag{3.33}$$

The presence of grid current causes the appearance of equivalent voltage on the input of the amplifier. With a large K_y, when $\frac{Y_2}{Y_{\text{вх}}}(1 + K_y) \gg 1$, it is possible to consider that

$$\Delta \bar{e}_1 = \frac{\bar{I}_g}{Y_2 K_y}. \tag{3.34}$$

Thus, with a large amplification factor grid current causes less change of the equivalent grid bias, the less the resistance of the feedback circuit and greater the amplification factor.

The absolute and relative value of error of the output voltage, caused by the presence of grid current, we will determine from expression (3.34) in the form

$$\Delta e_{\text{вых. а}} = \frac{I_g}{Y_2} \text{ and } \delta e_{\text{вых. о}} = \frac{I_g}{Y_2 e_{\text{вых. max}}}.$$

Let I_g = 0.1 microampere, $Y_2 = \frac{1}{R_2} = 10^{-6}$ mho, K_y = 50,000, $e_{\text{вых. max}}$ = 100 v, $\Delta e_1 = 2 \cdot 10^{-6}$ v, or 2 microvolt. Then

$$\Delta e_{\text{вых}} = 0.1 \text{ v}, \text{and } \delta e_{\text{вых. о}} = 0.1\%.$$

By itself voltage Δe_1 is minute. However its harmful influence can noticeably develop during work of the operational amplifier as an integrator.

Indeed, if in equation (3.34) we substitute $Y_2(p) = Cp$ and assume that grid current is constant, then we will receive

$$\Delta \bar{e}_1 = \frac{\bar{I}_g}{CpK_y}.$$

Changing to originals, we will have

$$\frac{d(\Delta e_1)}{dt} = \frac{I_g}{CK_y},$$

whence

$$\Delta e_1 = \frac{I_g}{CK_y} t.$$

With constant grid current and absence of voltage on the input the voltage at the output will linearly increase in time:

$$\Delta e_{\text{вых.2}} = \frac{I_g}{C} t.$$

The relative error of output voltage, caused by the grid current, will be

$$\delta e_{\text{вых.2}} = \frac{I_g}{C e_{\text{вых. max}}} t. \tag{3.35}$$

when $I_g = 0.1 \cdot 10^{-6}$ a, $C = 1 \cdot 10^{-6}$ f, for the solution time $t_{\text{р}} = 100$ sec and $e_{\text{вых. max}} = 100$ v, we will receive

$$\delta e_{\text{вых.2}} = 10\%.$$

These results show the necessity of maximum decrease of the quantity of grid current. This can be attained, for example, by selection of a corresponding magnitude of grid bias, introduction in the grid circuit of a corresponding compensating voltage, or application of first amplifier stages of tubes, working in an electrometric regime (L. I. Bayda and A. A. Semenkovich [1]).

The action of grid current, instability of tube emission and oscillation of power supplies can be considered the action of equivalent voltages of interferences, coupled in the grid circuit by the corresponding cascades. In the presence of negative feedback the effect of these disturbances on the output voltage will differ depending on the place of application of the disturbance (A. A. Rizkin [1]). For proof of this proposition let us consider, as an example, an operational amplifier, consisting of three cascades (Fig. 36). Following the above method of determination of the transfer function of the operational amplifier, we will write the equation of the mismatch indicator

$$e_1 = \frac{Y_1}{Y_1 + Y_2} e_{\text{вх}} + \frac{Y_2}{Y_1 + Y_2} e_{\text{вых}} \tag{3.36}$$

and the equation of the channel of amplification of the error

$$e_{\text{вых}} = -K_1 K_2 K_3 e_1 - K_1 K_2 K_3 \Delta e_1 + K_2 K_3 \Delta e_2 - K_3 \Delta e_3. \tag{3.37}$$

After substituting e_g from equation (3.36) in equation (3.37) we will receive

$$e_{\text{вых}} = -\frac{K_1K_2K_3\dfrac{Y_1}{Y_1+Y_2}}{1+K_1K_2K_3\dfrac{Y_2}{Y_1+Y_2}}e_{\text{вх}} - \frac{K_1K_2K_3}{1+K_1K_2K_3\dfrac{Y_2}{Y_1+Y_2}}\Delta e_1 + \frac{K_2K_3}{1+K_1K_2K_3\dfrac{Y_2}{Y_1+Y_2}}\Delta e_2 - \frac{K_3}{1+K_1K_2K_3\dfrac{Y_2}{Y_1+Y_2}}\Delta e_3. \tag{3.38}$$

With a very large resultant amplification factor, when $\left|K_1K_2K_3\frac{Y_2}{Y_1+Y_2}\right| \gg 1$, we will receive

$$e_{\text{вых}} = -\frac{Y_1}{Y_2}e_{\text{вх}} - \left(1+\frac{Y_1}{Y_2}\right)\left[\Delta e_1 - \frac{1}{K_1}\Delta e_2 + \frac{1}{K_1K_2}\Delta e_3\right]. \tag{3.39}$$

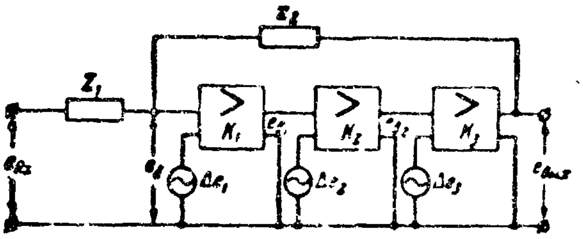

Fig. 36. Three-cascade operational amplifier.

The second addend in this expression represents the error in output voltage, caused by the emf of the disturbance in the grid circuits of separate cascades. From this expression it follows that the effect of application of disturbance is less the further from input the disturbance is applied. In connection with this with a large total amplification factor the voltage of the plate supply of the last cascade may be in general not stabilised. At the same time the greatest influence is caused by disturbances applied in the first cascade, and therefore one should take all possible steps to eliminate causes of appearance of the emf of interferences in the first cascades.

Variations of parameters of elements of the amplifier circuit usually proceed very slowly even in the case, where the circuit is assembled from resistors, and therefore they can be considered during tuning of zero every time before beginning the operation setting-up.

In electronic models setting of zero is conducted with the help of a millivoltmeter, connected at the output, and maximum absolute error does not exceed

±1 mv which, when $e_{\text{вых max}}$ = 100 v, constitutes $\delta e_{\text{вых}}$ = 0.001%. This error is more substantial during work of the operational amplifier as an integrator. Thus, for example, with a transmission factor $\frac{1}{RC} = 1$ during the time t = 100 sec error will be

$$\delta e_{\text{вых}} = 0{,}1\%.$$

Error in setting the transmission ratios of the operational element is determined depending upon the construction of the installation or by the maximum deflections in values of resistance, or error in the given method of checking transmission ratios. During operation as an integrator the accuracy of setting of transmission ratios depends also on the allowance for the capacitance of the capacitor.

This analysis of primary systematic and random errors of a computing element allows us to formulate a number of conclusions, from which directly ensue design requirements for the amplifier.

1. Greatest error is caused by the finite value of the amplification factor and instability of the zero level in the adder and, especially, the adder-integrator regimes.

2. In order to have small error due to finiteness the amplification factor, one must select an amplification factor for the amplifier as large as possible (at least 50-75 thousand) and limit the transmission ratios, set on the computing element.

3. It is necessary to turn special attention to decrease of leak between the input (grid) and output terminals of the computing element. Resistance of the insulation should be as great as possible. Here it is very expedient to separate input (to the integrating point) and output circuits by a ground shield. This can reduce leaks which exist in the circuit of the operational amplifier and which shunt the capacitor during work as an integrator, to leaks between the integrating point and the ground and the output and ground.

4. During selection of capacitors for the feedback circuit or input of the amplifier, one should use capacitors, possessing the largest leak resistance and

least dielectric after-effect. At present the most suitable for this purpose are polystyrene or styroflex capacitors. The rated capacitance of the capacitor can be sustained at present with error, not exceeding 0.1%.

5. Use as accurate methods of setting of transmission ratios as possible (for example bridge methods).

6. For stabilisation of the zero level it is necessary to provide measures of compensation of drift in the first cascade and to feed the installation from stabilised power supplies. To decrease the quantity of grid currents it is desirable that the tube of the first cascade have a low μ (that the distance from grid to cathode is as great as possible), low plate current, low voltage between the plate and cathode, lowered value of filament voltage and a negative bias at least 1.4 v.

CHAPTER IV

DIAGRAMS OF D-C OPERATIONAL AMPLIFIERS

1. Basic Requirements Presented to Operational Amplifiers

Selection of the diagram of the d-c amplifier in many respects is determined by those requirements which are presented to it as to the computer. These requirements basically can be reduced to the following.

The amplifier should be built by an asymmetric diagram with one commong pole. This simplifies setting-up the problem, since it allows one to connect the operational amplifiers in the installation, switching only one pole (lead).

The signal at the output of the operational amplifier should repeat the input signal in such a manner that with a zero input signal there is a zero output signal and with a change of sign of the input signal the sign at the output would change accordingly. In the asymmetric diagram of the amplifier this requires in the output cascade an additional source of constant voltage. The operational amplifier should consists of an odd number of cascades, changing sign, since with this it ensures the additional operation of multiplication of the input signal by — 1 and simplicity of realization of negative feedback.

The total amplification factor of the amplifier is selected depending upon the required accuracy of work. For application in general purpose simulating installations it should not be lower than 70,000. This value of the amplification factor of amplifier ensures almost complete independence of work of the computing element from variation of parameters of the amplifier and low value of the output resistance which facilitates connection of computing elements among themselves and with other equipment, and also, as analysis shows, small error (less than 1%) of work as an adder-integrator during summation of up to five components with a transmission factor K = 2 for each component.

In the diagram of the amplifier there should also be provided measures of decreasing sero drift. Zero drift constitutes the basic error of a computing element made of d-c amplifiers and, therefore, during development of the amplifier, especially its first cascade, there should be considered the requirement of obtaining minimum drift. For operational amplifiers of electronic models of general application the voltage of drift at the output must not exceed a magnitude of 1 — 2 millivolt for 10 minutes with a transmission factor of the computing element equal to unity, and during work as an integrator with a transmission factor of unity — must not exceed a magnitude of 100 millimolts for 100 sec.

Along with a high amplification factor the operational amplifier should possess a sufficiently wide passband. Thus, for example, for a general-purpose simulating installations it is desirable that, in the presence of negative feedback, a transmission factor of one and output voltage 100 v, the amplifier has a gain-frequency response with a slump of 3 db, starting from a frequency of 8 — 10 kc. The wider the passband of the amplifier, the greater possibilities the operational amplifier will possess. In particular, on d-c amplifiers with a wide passband there can be built simulating installations, working both in natural time scale (B. Ya. Kogan [1]), and with artificial repetitive operation (L. I. Gutenmakher [2]). The requirement of a high amplification factor and a wide passband are in contradiction with the requirement of stability of the amplifier in the coverage of its negative feedback. Therefore, in the amplifier diagram there should be provided correcting circuits, ensuring such a character of the gain-phase response, with which the amplifier is not self-excited in the various regimes, covered by negative feedback. Of large value in this respect is correct location of elements on the amplifier chassis, ensuring the shortest grid connections, and the least leak between various circuits.

Decrease of leaks also has essential meaning with respect to limitation of the amplitude of spurious alternating voltages at the output of the amplifier. For normal work of an amplifier as a computing element it is required that the amplitude of alternating voltage at the output does not exceed several millivolts. To insulate input circuits from the output is most simple and reliable with the help of correct shielding of wires and separate units of the circuit and grounding of the corresponding parts of the amplifier. Here leakage between input and output circuits is redistributed on leakage between the integrating point and the ground and between the output and the ground. Such a method of combatting leaks has received the name "ground insulation."

The amplifier during coverage by negative feedback should allow connection of load with the input impedance of the order of 10 kilohms and also ensure linear dependence of the output voltage on the input within ± 100 v.

The formulated requirements can be realized with the help of a three-cascade d-c amplifier, assembled by asymmetric diagram. Diagrams of operational amplifiers known from the literature have much in common. Separate diagrams are distinguished by the given method of decreasing zero drift, the principle of construction of input and output cascades, the types of tubes applied, ratings and number of stabilized power supplies. In certain cases for the purpose of increasing the total amplification factor there is used introduction of local positive feedback.

When operational amplifiers are used with a total transmission factor of one as, for example, for separation of circuits, for feeding loops of the oscillograph or for coupling the integrator with other equipment), it is possible to select a total amplification factor without feedback of the order of 2500 to 5000, which significantly simplifies the diagram.

By method of eliminating zero drift it is possible to divide all diagrams of decisive amplifiers into two groups: with parametric compensation of zero drift and automatic control of the zero level. To decrease zero drift, caused by instability of power supplies, they usually realize plate supply from electronic stabilizers, ensuring accuracy of maintenance of the constancy of voltage of the order of 0.1 to 0.01%.

2. Diagrams with Parametric Compensation of Zero Drift.

Operational amplifiers with parametric compensation of zero drift are made in two modifications. In one of them in the first cascade an auxiliary cathode follower is used, and in another the first cascade is built by a diagram with a series coupled triode. A typical diagram of an operational amplifier with compensation of drift by a cathode follower is in Fig. 37. The amplifier consists of three cascades. The first cascade is assembled with a double triode of type 6N9. The left part of the tube is used as the amplifying part, the right is connected in the circuit of the cathode follower and serves for compensation of zero drift, caused by change of filament voltage and oscillations of the emission current.

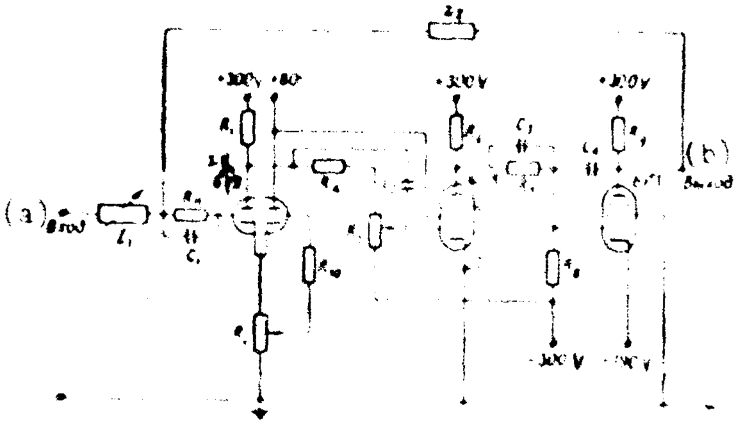

Fig. 37. Diagram of an operational amplifier with compensation of drift by a cathode follower.
KEY: (a) Input; (b) Output.

During change of filament voltage or emission current simultaneously changes the current of the right triode. Voltage drop, which here is singled out on resistance R_2 has a stabilising effect on the current of the left triode.

Depending upon the position of the cursor on resistance R_2 the coefficient of negative feedback of the right cascade changed, and consequently, the magnitude

This is a very general condition for adjustment of a diagram of cathode stabilization. If one were to assume that the right and left triodes possess identical characteristics of incadescence and identical internal resistances $R_{i_1} = R_{i_2}$, then we arrive at a more simple relationship

$$1 - (S_1 - S_2 a) R_k = 0.$$

Magnitude R_k usually is selected from calculation of permissible lowering of the amplification factor of the first cascade, and the resulting condition is satisfied by selection of the proper position of curser of the divider — magnitude a :

$$a = \frac{R_k S_1 - 1}{S_2 R_k}.$$

These expressions show that performance of the considered circuit depends on identity and stability of characteristics of both triodes. When $S_1 = S_2$ we arrive at the expression known from literature (see, for instance, L. I. Bayda and A. A. Semenkovich [1], A. M. Bonch-Bruyevich [1]):

$$R_k (1 - a) = \frac{1}{S_2}. \qquad (4.2)$$

It is necessary to note that specifics of the work of an operational amplifier, consisting of the fact that input signal e_0 is minute, facilitates compensation of drift by the considered diagram, since current I_{a_1} changes in very narrow limits, and consequently, S_1 can be considered a practically constant magnitude.

In Fig. 39 is presented an experimentally received dependence of the change of output voltage of an amplifier on change of the current of incandescence. From the figure it follows that cathode stabilization acts not quite symmetrically. If characteristics of cathodes of both halves of tubes strongly differ, it can be that one cannot satisfy conditions of cathode stabilization it is recommended to subject the cathode of the first tube to aging.

The second cascade of the amplifier is made of a pentode of type 6Zh8. Coupling between cascades is carried out with the help of a bridge circuit (Fig. 40), formed by two stabilized power supplies E_1 and E_2 and resistances R_1, R_4, and R_5 for the second tube and R_6, and R_7 and R_8 for the third (Fig. 37).

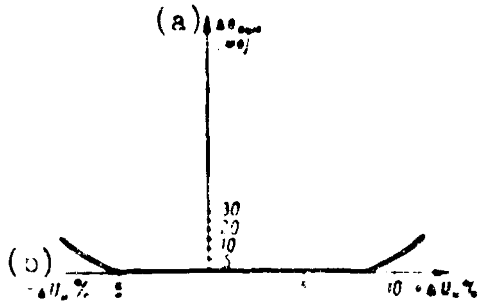

Fig. 39. Influence of change of filament voltage on change of output voltage.
KEY: (a) Change in output voltage, mv; (b) Δ load voltage, %.

Resistance R_4 and R_5 one can determine from known (L. I. Bayda and A. A. Semenkovich [1]) expressions:

$$R_4 = R_1 \frac{E_0 + e_{g1}}{E_1 - E_0\left(1 + \frac{R_1}{R_p}\right)}, \tag{4.3}$$

$$R_5 = R_1 \frac{E_2 - e_{g2}}{E_1 - E_0\left(1 + \frac{R_1}{R_p}\right)}, \tag{4.4}$$

if we know value of R_1, R_p* and E_0 and there is given magnitude e_{g2}.

Resistance R_5 is usually executed consisting of two parts: one unregulated and other adjustable. The adjustable part serves for setting the zero of the amplifier and should be sufficient in magnitude for compensation of variation of parameters of elements of the circuit (in connection with spread within allowance), and also to consider the possible range of spread of characteristics of the tubes.

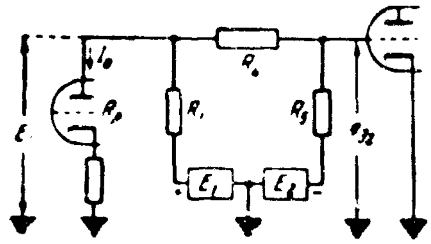

Fig. 40. Diagram of interstage coupling.

The third cascade usually is constructed on a powerful beam tetrode (6P-2, 6P-3). Obtaining of output voltage of both polarities is attained by connection in this cascade of a source of voltage of ~ 190 v. The diagram of the third cascade is presented in Fig. 41.

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*R_p — resistance of tube to direct current.

In the absence of load, output voltage can be found from the relationships:

$$e_{вых} = U_a - I_a R_a \quad I_a = \frac{U_a + U_{a_1}}{R_a + R_p}$$

Hence

$$e_{вых} = \frac{1}{R_a + R_p}(U_{a_1} R_p - U_a R_a) \qquad (4.5)$$

With an input voltage, equal to zero, the operating point is selected in such a manner that $e_{вых} = 0$

This will take place when

$$I_a = \frac{U_{a_1}}{R_a}. \qquad (4.6)$$

Limits of change of output voltage can be found, if one considers two limit cases: the tube is completely locked and completely unlocked. In the first case

$$R_p \to \infty$$

and, consequently,

$$e_{вых} \to U_a = 300 \text{ v}.$$

In the second case from characteristics of the output tube for given load R_a and U_a we find $R_p = \frac{U_0}{I_0}$

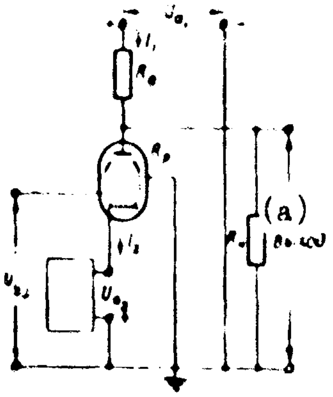


Fig. 41. Diagram of output cascade.
KEY: (a) Output.

For tube 6P3 when $U_{a_1} = 300$ v, $U_{a_2} = 190$ v, $R_a = 10$ kilohm, $U_0 \approx 25$ v, $I_0 = 40$ ma and

$$R_p = \frac{25}{0.04} = 625 \text{ ohm}.$$

$$e_{вых} = \frac{1}{10000 + 625}(300 \cdot 625 - 190 \cdot 10000) \approx -170 \text{ v}$$

During connection of external load limits of linearity accordingly narrow.

We will estimate the influence of load for the general case, when to the output is connected, besides ohmic resistance, also a source of emf with a given internal resistance. This takes place in a number of cases, when to the output of the operational amplifier is connected the circuit of a diode functional converter. Equivalent schemes of the output cascade for the two limit cases, when the output tube is completely locked or completely unlocked, are shown in Fig. 42.

In the first case we obtain

$$e_{\text{вых}} = \frac{U_{a_1} + E\frac{R_a}{R_{\text{вн}}}}{\frac{R_a}{R_{\text{н}}} + \frac{R_a}{R_{\text{вн}}} + 1}; \qquad (4.7)$$

in the second case

$$e_{\text{вых}} = -\frac{U_{a_1}\frac{R_a}{R_p} - U_a - \frac{R_a}{R_{\text{вн}}}E}{1 + \frac{R_a}{R_p} + \frac{R_a}{R_{\text{вн}}} + \frac{R_a}{R_{\text{н}}}}. \qquad (4.8)$$

Analysis of these expressions shows that introduction of load in the form of a source of emf leads to increase of the asymmetry of limit values of $e_{\text{вых}}$. The limit of positive values increases with positive E and decreases with negative. The limit of negative values of $e_{\text{вых}}$ vice versa, decreases with positive value of E and increases with negative.

The considered diagram of the output cascade extremely uneconomically expends the power of the power supplies.

Indeed, when the power consumed by the load is equal to zero, from power supplies there is put in half the maximum value of the power:

$$(P_{\text{потр}})_{e_{\text{вых}}=0} = \frac{U_{a_1}^2}{R_a} + \frac{U_a U_{a_1}}{R_a}.$$

During work at load R_H it is possible to estimate the economy of one or another diagram of the output cascade, introducing the concept of average efficiency

$$\eta = \frac{P_{\text{вых ср}}}{P_{\text{потр ср}}}. \qquad (4.9)$$

where

$$P_{вых\ ср} = \frac{\int\limits_{-e_{вых\ max}}^{e_{вых\ max}} \frac{e_{вых}^2}{R_н}\, de_{вых}}{2e_{вых\ max}} = \frac{e_{вых\ max}^2}{3R_н}$$

is the average value of power, fed to the load, and

$$P_{потр\ ср} = \frac{\int\limits_{-e_{вых\ max}}^{e_{вых\ max}} P_{потр}(e_{вых})\, de_{вых}}{2e_{вых\ max}}$$

is the average power, consumed by the output cascade from high-voltage sources. Since

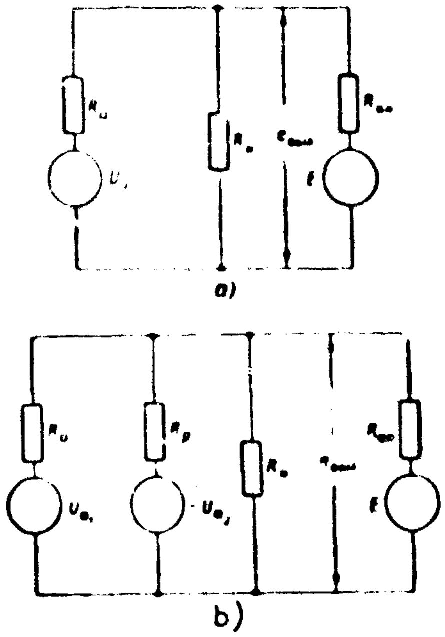
$$P_{потр} = I_1 U_{a_1} + I_2 U_{a_2},$$

$$I_1 = \frac{U_{a_1} - e_{вых}}{R_a} \text{ and } I_2 = I_1 - I_н = \frac{U_{a_1}}{R_a} - e_{вых}\left(\frac{1}{R_a} + \frac{1}{R_н}\right).$$

for the given case we will receive

$$\eta = \frac{e_{вых\ max}^2}{3\frac{R_н}{R_a} U_{a_1}(U_{a_1} + U_{a_2})}. \tag{4.10}$$

when $R_a = R_H = 10$ kilohms $U_{a_1} = 300$ v, $U_{a_2} = 190$ v and $e_{вых\ max} = 100$ v, $\eta \approx 2.27\%$.



a)

b)

Fig. 42. Equivalent diagrams of the output cascade for two limit cases.

Significant advantages from the viewpoint of economy of power consumption of power supplies are presented by the diagram of an output cascade shown in Fig. 43 (V. V. Gurov, B. Ya. Kogan, A. D. Talantsev, V. A. Trapeznikov [1]). In this diagram resistor R_a is replaced by electron tube $Л_2$. Selecting initial biases on tubes $Л_1$ and $Л_2$, it is possible to attain a state where at a zero input signal we receive equal currents through $Л_1$ and $Л_2$

and, consequently, zero load voltage. The magnitude of initial current can be here selected sufficiently low. So that with change of the input signal the circuit works correctly, it is necessary to ensure change of grid potentials of tubes $Л_1$ and $Л_2$ in anti-phase. This is attained by connection of the grid of tube $Л_2$ to the second cascade not directly, and through an additional inverting cascade (Fig. 44).* As compared with the above diagram of the amplifier here there is required a superfluous half envelope. However, if one were to put two amplifiers together on one chassis, then the total number of envelopes remains approximately the same as for the usual amplifier (3.5 per amplifier).

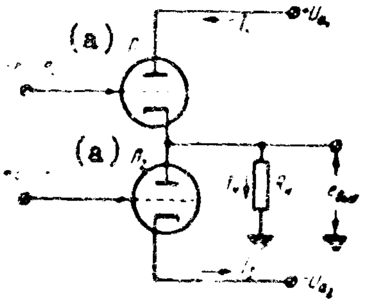


Fig. 43. Economic output cascade, e_c is the output voltage of the second cascade, e_2 — bias voltage. KEY: (a) Tube.

Since for the considered circuit

$$P_{вых} = \frac{e^2_{вых}}{R_н}, \text{ and } P_{потр} = \frac{U_a e_{вых}}{R_н},$$

then the average efficiency of the output cascade for a zero value of the resting current will be

$$\eta = \frac{P_{вых\ ср}}{P_{потр\ ср}} = \frac{2}{3}\frac{e_{вых\ max}}{U_a}. \qquad (4.11)$$

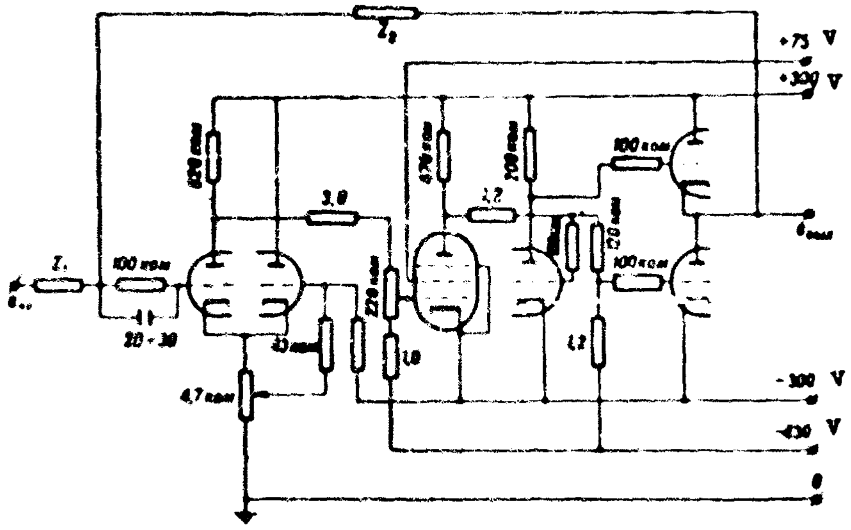


Fig. 44. Diagram of amplifier with economic output cascade.

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*Modification of the considered diagram, not requiring an additional inverting cascade, is shown in Fig. 59.

When [illegible] 100 v and U_{a1} = 300 v, [illegible] ≈ 20%.

Besides a higher efficiency, such an output cascade also possesses that advantage that it allows us to lower approximately 4 times the power of sources of plate supply.

Capacitors C_2, C_3 and C_4 in the amplifier circuit, depicted in Fig. 37, serve for correction of the gain-phase response for the purpose of eliminating the possibility of self-excitation of the amplifier. With careful fulfillment of such amplifiers and feeding from stabilised sources with accuracy of maintenance of constancy of voltage of the order of ± 0.05% we can reduce zero drift to 1 to 2 millivolts (voltage of drift is brought to input).*

As example of a circuit with compensation of zero drift by a series couples triode let us consider the diagram of an operational amplifier C. A. Meneley and C. D. Morrill [1](, presented in Fig. 45. The operational amplifier here is made of four cascades.

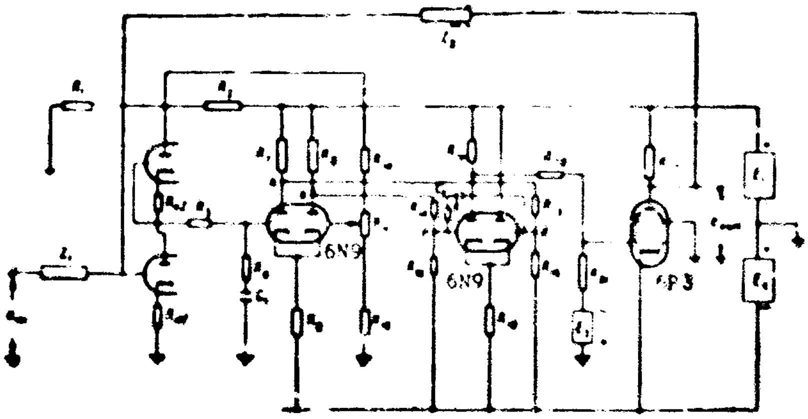

Fig. 45. Operational amplifier with use of a series-balancing input cascade.

The first cascade is assembled by a series-balancing circuit, the second cascade — by in a parallel-balancing circuit, the third — by a "subtractor" circuit and the fourth — by a circuit, analogous to the circuit of the output cascade, of the above considered (Fig. 37) amplifier. The upper tube of the series-balancing cascade it is possible to consider as a plate load with equivalent resistance

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*Amplifiers of this type (UPT-4 and UPT-12) enter into a number of electronic analog installations. See I. M. Vittenberg [1].

$$R_{eq} = R_{i_2} + R_{k_2}(1 + \mu) \qquad (4.12)$$

where R_{i_2} is the internal resistance of tube, R_{k_2} — the resistance, connected in the cathode circuit, μ — the amplification factor. Here the amplification factor of the cascade $K_1 = \frac{U_{out}}{U_{in}}$ can be found from consideration of the equivalent diagram when $R_{i_1} = R_{i_2}$ and $R_{k_1} = R_{k_2}$:

$$K_1 \approx \frac{\mu}{2}.$$

With application of tube 6N8 one can obtain $K_1 \approx 10$.

If the characteristic of both tubes are identical, then with a constant input voltage increase of filament voltage evokes identical increase of current in the upper and lower tubes, and consequently, identical change of grid voltage. Therefore, the fixed distribution of voltage on the tubes will not change. The circuit of the first cascade can be considered a bridge, two arms of which are formed by tubes $Л_1$ and $Л_2$ and two other — by resistances R_{10}, R_{11} and R_{12}. In connection with the fact that during change of incandescence resistances of tubes $Л_1$ and $Л_2$ change the same magnitude, the voltage, removed from the diagonal, remains here constant. Likewise this voltage will not change during change of the magnitude of feeding voltage.

If characteristics of both halves of the tubes somewhat differ in incadescence, then it is possible to bring them together by means of change of resistance R_3 in circuits of the cathode of the upper tube. Changes of input voltage on tube $Л_1$ are minute due to the presence of negative feedback and a large amplification factor of the amplifier. Therefore, the bridge is practically always in balanced state.

The second cascade of the amplifier is made according to the circuit of an asymmetric parallel-balancing cascade. In the absence of a signal both grids are under identical potentials with respect to the ground. If characteristic of the tubes and load resistances R_7 and R_8 are identical, the plate currents in the tubes will be equal and the output voltage, obtained as the difference of potentials between points a — b and e — d, will equal zero. Change of potential of the grid of the left half of the tube leads to a change of plate current of this half of the tube, and consequently, to change of the voltage drop in resistance R_9. This evokes change of plate current of the right half of the tube, opposite in sign to the change of plate current of the left half of the tube. As a result between points a — b and e — d there appears a difference of potentials, whose sign depends on the polarity of the signal applied to the grid of the left half of the tube. Potentials at points a — b will change symmetrically only in the case of large values of resistance R_9.

The amplification factor of the cascade can be found from calculation of the equivalent circuit. When the tubes have identical characteristics and parameters:

$$R_{e9} = \frac{R_{\cdot}(R_{1} + R_{\cdot})}{R_{7} + R_{11} + R_{1\cdot}} \quad \frac{R_{6}(R_{\cdot\cdot} + R_{8})}{R_{6} + R_{\cdot\cdot} + R_{6}} \quad \frac{R_{\cdot 6}}{R_{\cdot\cdot} + R_{6}} \quad \frac{R_{1\cdot}}{R_{13} + R_{\cdot 3}} \qquad (4.13)$$

the expression for the amplification factor can be obtained in the form

$$K_2 = \frac{\mu R_{11}}{R_i + R_{11}} \frac{R_{\cdot 8}}{R_{\cdot\cdot} + R_{\cdot 8}} \qquad (4.14)$$

Thus, the amplification factor of this cascade is calculated just as for the triode. With application of tube 6N9 we can obtain $K_2 = -5$.

The third cascade, besides amplifying, should provide reverse transition to the asymmetric circuit. It is made by the known diagram (A. A. Sokolov [1], [2]) of a differential amplifier, called a "subtractor" circuit. Output voltage of the cascade, taken from resistance R_{21} onto the grid of the fourth cascade, should be the amplified difference of potentials $e_e - e_d$.

The amplification factor of cascade K_3 can be approximately found from the expression

$$K_3 \approx \frac{\mu}{1+\frac{2R_i}{R_{e3}}} \cdot \frac{R_{21}}{R_{20}+R_{21}}. \qquad (4.15)$$

Here

$$R_{e3} = \frac{R_{19}(R_{20}+R_{21})}{R_{19}+R_{20}+R_{21}}$$

The total amplification factor of the cascade, obtained with use of the tube 6N9, constitutes

$$K_3 = 19.$$

The fourth cascade, made of a beam tetrode, is built similarly to the output cascade of the operational amplifier considered earlier. During use of tube 6P3 and R_{22} = 10 kilohms we can obtain

$$K_4 = 17.$$

Thus, the total amplification factor of the amplifier constitutes

$$K_y = K_1 \cdot K_2 \cdot K_3 \cdot K_4 \approx 10 \cdot 25 \cdot 19 \cdot 17 \approx 80\,000.$$

With well-selected tubes, according to the data of C. A. Meneley and C. D. Morrill [1], the zero drift brought to the input is less than 200 microvolt hour: noises on the output do not exceed 0.3 millivolts.

The frequency characteristic does not introduce phase and frequency distortions in signals up to 1000 cycles (with a transmission factor of 10).*

3. Diagrams of Operational Amplifiers with Automatic Stabilization of Zero Level

Operational amplifiers with automatic stabilization of zero level are based on the known property of systems with negative feedback to decrease the effect of external disturbances, influencing elements covered by feedback. Indeed, from expression (3.39) it follows that the output magnitude consists as if of two components: one, determined by the input magnitude and the given law of its conversion, and the other, error caused by disturbances to the system. The nearer to the input of the system the disturbance is applied, the greater the error it will cause in the output. Decrease of effect of application of the disturbance is directly proportional to the amplification factor of the section from input to the place of application of the disturbance.

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*The authors do not indicate the magnitude of the amplitude of the input signal.

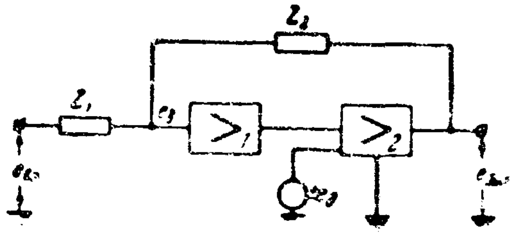

Fig. 46. Skeleton diagram of an operational amplifier with series coupling of an auxiliary amplifier. 1 -- auxiliary amplifier, not possessing zero drift, 2 — main d-c amplifier.

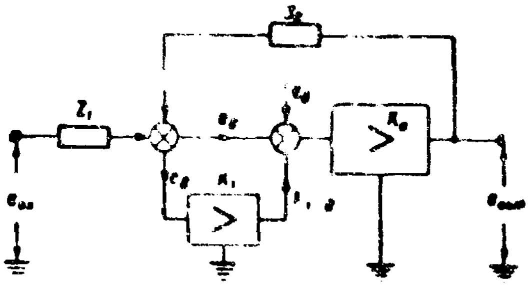

Fig. 47. Skeleton diagram of an operational amplifier with parallel connection of the auxiliary amplifier. K_1 — auxiliary amplifier, K_0 -- the main d-c amplifier.

Considering zero drift as a certain equivalent disturbance, brought to the input of the operational amplifier, it is possible to show that the connection between the integrating point and the main d-c amplifier (Fig. 46) of an additional amplifier, to which in principle there is not inherent zero drift, should lead to decrease of the drift voltage by K_1 times, where K_1 — is the amplification factor of the additional amplifier.

The additional amplifier can be built on the principle of modulation of input voltage, subsequent amplification by an a-c amplifier and demodulation. It is most rational here as modulator and demodulator to use an electromechanical vibrator, since all electronic circuits and circuits with the application of dry-disc rectifiers possess significant zero drift. However, series coupling of an additional amplifier by the diagram of Fig. 46 turns out to be not too rational, since simultaneously with decrease of zero drift it leads to sharp narrowing of the passband of the amplifier due to the equation of necessity of operating at a comparatively low carrier frequency, determined by the possibilities of electromechanical vibrators. More rational is the circuit diagram of the additional amplifier, shown in Fig. 47. In considering this diagram one should keep in mind the circumstance that the disturbance, equivalent to zero drift, changes comparatively slowly and by force of this the narrow passband of the additional amplifier is not an obstacle to limitation of zero drift. Indeed, at low frequencies of the input signal it is possible to disregard parallel coupling for by signal e and, thus, the circuit is equivalent to that brough in Fig. 46. At signal frequencies, lying beyond the limits of the passband of the additional amplifier, it is as if the circuit formed by this amplifier is opened, and there is accomplished transition to the ordinary circuit. This eliminates the deficiency of the circuit of Fig. 46 and provides the possibility of work of the operational amplifier in a comparatively wide range of input signal frequencies.

We will derive the relationship between the output voltage of such an operational amplifier and input* for a given conductance of the input circuit and feedback circuit. Following the method, stated above, we will receive

the equation of the mismatch indicator

$$\bar{e}_4 = f_{11}(p)\bar{e}_{\text{вх}1} + f_2(p)\bar{e}_{\text{вых}}, \tag{4.16}$$

where, as before,

$$f_{11}(p) = \frac{Y_{11}(p)}{Y_{11}(p) + Y_2(p) + Y_3(p)}$$

is the transfer function of the input circuit, a

$$f_2(p) = \frac{Y_2(p)}{Y_{11}(p) + Y_2(p) + Y_3(p)}$$

is the transfer function of the feedback circuit;

---

*For simplification of computations we consider that to the input there is fed only one variable.

the main channel of the system

$$\bar{e}_{\text{вых}} = -K_0 f(p)\{[1 + K_1 f_1(p)]\bar{e}_1 \pm \bar{e}_2\}. \tag{4.17}$$

where K_0 is the static amplification factor of the main d-c amplifier, f(p) is its transfer function, K_1 — the static amplification factor of the additional d-c amplifier, $f_1(p)$ — its transfer function.

Solving equations (4.16) and (4.17) for $\bar{e}_{\text{вых}}$, we will receive

$$\bar{e}_{\text{вых}} = -f_{11}(p)\frac{K_0 f(p)[1 + K_1 f_1(p)]\bar{e}_{\text{вх}}}{1\; K_0 f(p)[1 + K_1 f_1(p)] f_2(p)} \mp \\ \mp \frac{K_0 f(p)\bar{e}_2}{1 + K_0 f(p)[1 + K_1 f_1(p)] f_2(p)}. \tag{4.18}$$

If the amplification factor K_0K_1 is selected so large that for the operating range of frequencies in the denominator of expression (4.18) it is possible to disregard unity as compared with the modulo

$$|K_0 f(p)[1 + K_1 f_1(p)] f_2(p)|,$$

then equation (4.18), taking into account expression for transfer functions f_{11} and f_2 can be presented in the form

$$\bar{e}_{\text{вых}} = -\frac{Y_{11}(p)}{Y_2(p)}\bar{e}_{\text{вх}} \mp \frac{1}{[1 + K_1 f_1(p)]\dfrac{Y_2(p)}{Y_{11}(p) + Y_2(p) + Y_3(p)}}\bar{e}_2. \tag{4.19}$$

From the resulting expression it follows that error, introduced by zero drift, for such an operational amplifier will be $[1 + K_1 f_1(p)]$ times less as compared with an operational amplifier, not supplied with an additional amplifier ($K_1 = 0$).

Furthermore, at low frequencies the total amplification factor of the main channel sharply increases.

Indeed,

$$K_{\text{общ}} = \frac{e_{\text{вых}}}{e_1} = (1 + K_1)K_0.$$

when $K_1 \gg 1$

$$K_{\text{общ}} = K_1 K_0.$$

If I_1 = 1000, and K_0 = 50,000, we obtain $K_{\text{общ}} = 50 \cdot 10^6$. The great total amplification factor of the main channel leads to decrease of error (especially during work of the operational amplifier as an adder), caused by the limited value of the amplification factor.

One typical diagram (E. A. Goldberg [1]) of such an operational amplifiers is shown in Fig. 48.*

As can be seen from the figure, as the main amplifier there is used a somewhat modified d-c amplifier, shown earlier in Fig. 37. In this amplifier the diagram of the first cascade was subjected to a small change and there were introduced other correcting circuits (C_1, R_4; C_2, R_7; C_4, R_{11}; C_3, R_{10}).

The input of the additional amplifier is connected to the integrating point Σ through filter (R_{12}, R_{13}, C_5, C_6, R_{14}) and connecting capacitance C_7, and the output after demodulation (right half of the vibrator) is connected through smoothing filter C_{12}, R_{23} to the grid of the cathode follower of the first cascade, utilized in the given circuit as the adder. The filter at the input of the auxiliary amplifier serves to prevent penetration into the amplifier of alternating current of higher harmonics of voltage e_{Σ}, multiples of the commutation frequency of the vibrator, and the filter at the output — for smoothing of the pulsating voltage, received after half wave rectification by the vibrator. The comparatively low commutation frequency (from 50 to 400 c) and parameters of smoothing filter limit the passband of the auxiliary amplifier by approximately a frequency of 5 c. To avoid self-excitation of the a-c amplifier and decrease of inductions there is required thorough the shielding of the vibrator leads feeding its contacts, and also such adjustment of the span of contacts and amplitude of vibration of the armature, with which is ensured work with "coverage" of contacts.

During work as a scale amplifier with a transmission factor K = 1 ($Z_1 = Z_2 = R$)

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*Amplifiers UPT-10 made on a similar diagram, enter into the composition of electronic analog MPT-9.

such an operational amplifier according to source material (E. A. Goldberg [1]) has a passband up to a 100 kc. Zero drift brough to the input here does not exceed 50 microvolts.

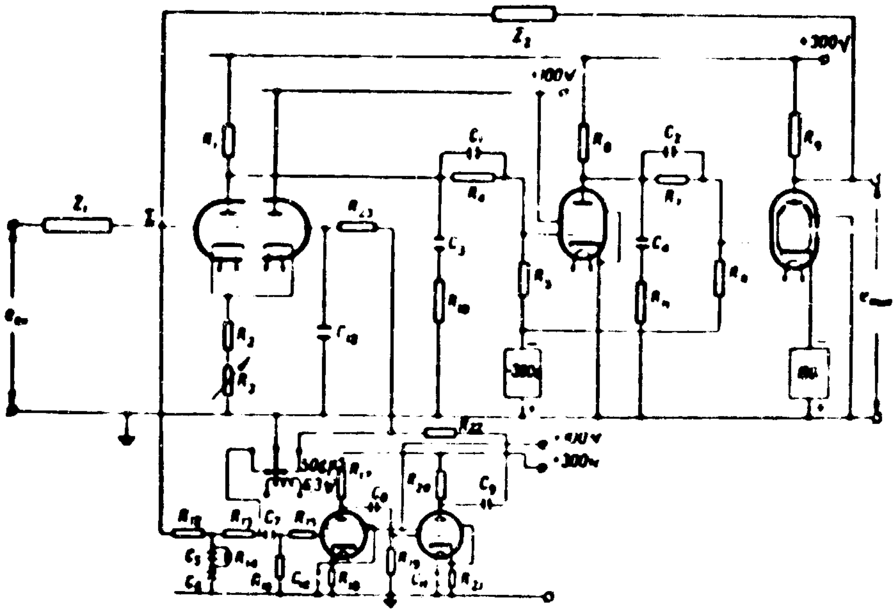

Fig. 48. Fundamental circuit of an operational amplifier with automatic control of sero level.

$R_1 = 1$ мгом; $R_2 = 1$ ком, $R_3 = 2$ ком, $R_4 = 570$ ком, $R_5 = 4.7$ мгом; $R_6 = 100$ ком; $R_7 = 10$ мгом, $R_8 = 4.7$ мгом, $R_9 = 33$ ком, $R_{10} = 2.4$ ком; $R_{11} = 4.7$ ком, $R_{12} = R_{13} = 1$ мгом; $R_{14} = 300$ ком, $R_{15} = 1$ ком; $R_{16} = 2$ мгом $R_{17} = 170$ ком, $R_{18} = 6.8$ ком, $R_{19} = 1$ мгом, $R_{20} = 470$ ком; $R_{21} = 6.8$ ком, $R_{22} = 1$ мгом, $R_{23} = 10$ мгом, $C_1 = 10$ пф, $C_2 = 10$ пф; $C_3 = 0.5$ мкф, $C_4 = 100$ пф; $C_5 = C_6 = 0.1$ мкф, $C_7 = 0.05$ мкф; $C_8 = C_9 = 0.1$ мкф, $C_{10} = C_{11} = 20$ мкф, $C_{12} = 1$ мкф.*

*Editor's note: мгом = Mohm; ком = Kohm; пф = pf; мкф = μf.

In projecting the auxiliary amplifier one should consider the necessity of providing negative feedback for low frequency signals. Therefore, in the presence of the auxiliary amplifier it is necessary to be sure that this inequality is observed

$$K_1 K_0 < 0.$$

Since $K_0 < 0$, then, consequently, K_1 should be greater than zero. The sign of the amplification factor of the auxiliary amplifier, as follows from analysis of the diagram of Fig. 48, is determined by three factors: the number of cascades of the a-c amplifier, which change sign, the method of connection of the vibrator and the character of summation of signals by the main amplifier.

Number of amplifier stages can be even or odd. With an odd number of cascades output voltage of the amplifier has a reverse phase, and consequently, voltage after the demodulator has a reverse sign.

Depending upon the construction and circuit diagram of the vibrator it is possible to carry out commutation of input circuits (modulation) and output circuits (demodulation) in the same a-c half period (commutation in phase) or with shift by one half period (commutation in anti-phase). In the first case the sign of output voltage does not change, in the second case — it changes to the reverse. Finally, summation can be carried out in cathode or plate circuits of the first cascade of the main amplifier. During summation in the cathode there is carried out summation of signals of main and auxiliary amplifiers with opposite signs; during summation in plate circuits — with the same signs.

In the considered diagram of Fig. 46 is used an even number of amplifier stages, commutation in anti-phase and summation in the cathode circuit which ensures an even number of changes of sign in the M-DM amplifier (modulation — demodulation) and, consequently, $K_1 > 0$.

It is possible to carry out other variants of circuits (A. D. Talantsev [11]), in which the total amplification factor of the auxiliary amplifier remains larger than zero. For example, it is possible to make an a-c amplifier with an odd number of cascades, connect the vibrator to the circuit of anti-phase commutation, and to carry out summation in the plate circuits of the first cascade of the main amplifier or with an odd number of cascades of the a-c amplifier carry out commutation in phase and summation in the cathode circuit.

With an even number of cascades it is possible to have also a variant with commutation in phase and summation in the plate circuits of the first cascade of the main amplifier. This variant is not of practical interest.

In Fig. 49a and b are brought fundamental circuits, showing how new variants of circuits are realised.

Use of an odd number of cascades removes the inclination of an a-c amplifier (with an even number of cascades) to self-excitation due to spurious coupling between input and output and, thereby, allows one to facilitate work of the vibrator, since the necessity of adjustment with "covering" of contact drops and large amplitudes of oscillation of the armature connected with it. During summation of signals in the plate of the first cascade of the main amplifier it is possible to use a vibrator of ordinary construction with commutation in anti-phase. Summation in the cathode circuit requires application either of two vibrators, or one special one with a split insolated armature and two pairs of contacts (Fig. 50). The diagram of an operational amplifier with such a vibrator is shown in Fig. 51.* As vibrator in the circuits of Figs. 48 and 49 there can be used both special electromagnetic interrupters and ordinary polarised relays.

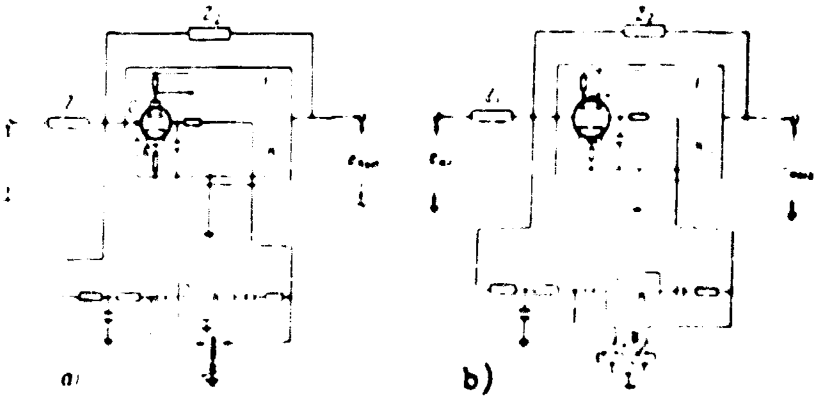

Fig. 49. Variants of circuits of operational amplifiers with automatic control of zero level. 1 — main d-c amplifiers, 2 — auxiliary amplifiers.

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Fig. 50. Electromagnetic interrupter.

Application of electromagnetic interrupters has the advantage that they give a doubled or quadrupled commutation frequency of the current feeding their winding, (Fig. 49a and Fig. 49b accordingly) and thereby decrease interferences with the frequency of the network. At the same time electromagnetic interrupters require more thorough shielding of the contact system from the coil and its lead wires, since currents, which it must pass for creation of electromagnetic forces on the armature, generate a noticeable field of scattering.

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*It is applied in the analog installation of the Academy of Sciences of the USSR, type EMU-4.

Polarized relays in this respect have the advantage that the magnitude of variable magnetic flux in their magnetic system is significantly less. Comparison of electromagnetic vibrators and polarized relays shows that the level of interferences in the M-DM are approximately identical, if in both cases we use for feeding of coils source of non-standard frequency (for example, 120 c for the relays). The fundamental circuit of an operational amplifier with polarized relays as the vibrator of the M-DM amplifier and economic output cascade is shown in Fig. 52.*

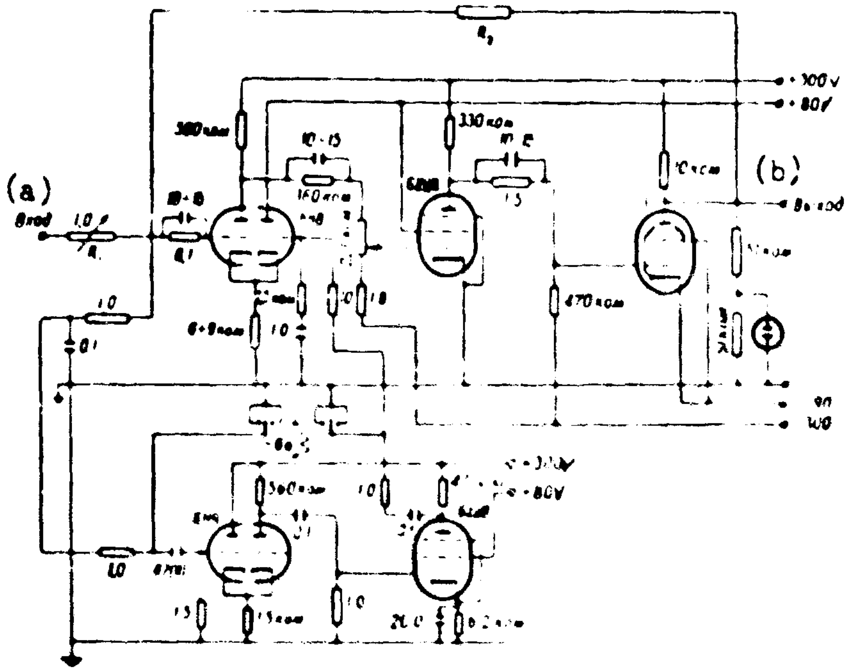

Fig. 51. Diagram of operational amplifier with electromagnetic interrupter (Fig. 50).
KEY: (a) Input; (b) Output

An electromagnetic vibrator without adjustment of contacts for "covering" can also be used in the circuit of Fig. 48, but on the condition of thorough shielding of contacts and the armature so that leak between input and output of the a-c amplifier is reduced to leaks between the input and the ground and between the output and the ground. (N. N. Lenov (1)).

*By this diagram were made the operational amplifiers of electronic analog EMY-5 of the Academy of Sciences of the USSR (see V. V. Gurov, B. Ya. Kogan, A. D. Talantsev, V. A. Trapeznikov [1]).

This method of combatting leakage between the input and output of the a-c amplifier is useful to use also in circuits with an odd number of cascades for removal of spurious negative feedback, lowering in this case the amplification factor K_1 of the a-c amplifier.

Connection of 1000 pf capacitors between the fixed contacts of the vibrator (relay) and the common point also helps to remove the above-indicated spurious couplings.

The amplification factor of the auxiliary amplifier at zero frequency it is possible to present in the form

$$K_1 = k_{\text{м}} \cdot \bar{k} \cdot k_{\phi\text{и}} \cdot k_{\text{м}1}. \tag{4.20}$$

where $k_{\text{м}}$ is the transmission factor of the modulator taking into account the input filter, $\bar{k}$ — the natural amplification factor of the a-c amplifier at the carrier frequency, $k_{\phi\text{и}}$ — the transmission factor of phasing circuit at the a-c amplifier output, $k_{1\text{м}}$ — the transmission factor of the demodulator taking into account the output filter.

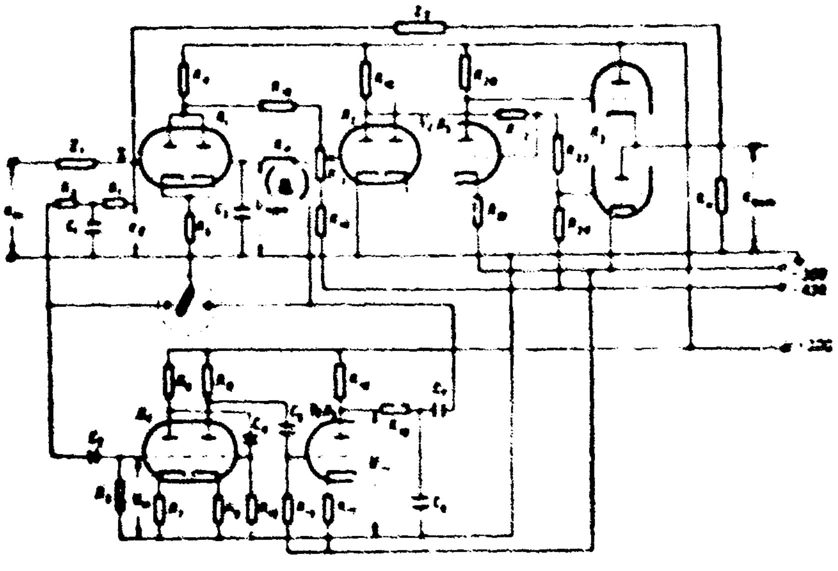

Fig. 52. Fundamental circuit of operational amplifier with a polarized relay as vibrator of the M-DM amplifier and economic output cascade.

R_1 = 510 ком, R_2 = 1,0 мгом, R_3 = 2,0 мгом, R_4 = 2[illegible] ком, R = 2,0 ком, R_6 = 100 ком; R_7 = 6,2 ком; R_8 = 500 ком, R_9 = 6,2 ком, [illegible] мгом, [illegible] мгом, R = 1,0 мгом, R_{13} = 270 ком; R_{14} = 2,2 мгом, R_{15} = 1,0 мгом, [illegible] мгом, R = 12 ком; R_{18} = 330 ком; R = 1,0 мгом, R_{20} 1,0 мгом, R = 2,0 ком, R = 1,3 мгом, R_{23} = 16 ком; R_{24} = 420 ком; C_1 = 0,25 мкф, C_2 1000 пф, C_3 = 1,0 мкф, C_4 = 1000 пф, C_5 = 2200 пф, C_6 = 1000 пф, C_7 = 0,[illegible] мкф*

KEY: (a) Modulator-demodulator voltage.

•Editor's note: мгом = MΩ; ком = kΩ; пф = μμf; мкф = μf.

Analysis of the work of the modulator together with the input filter shows that the form of voltage on the grid of the input cascade of the a-c amplifier in a steady-state regime will have the form, shown in Fig. 53.

Ordinate of pulses one can determine from the obvious relationships:

$$\begin{aligned} U_{g_1} &= U_{g_4} e^{-\frac{\tau}{2T_1}}, \\ U_{g_3} &= U_{g_2} e^{-\frac{\tau}{2T_2}}, \\ U_{g_2} &= (e_1 - U_{g_1}) \frac{R_3}{R_2 + R_3}, \\ U_{g_4} &= (e_1 - U_{g_3}) \frac{R_3}{R_2 + R_3}. \end{aligned} \qquad (4.21)$$

where τ is the period of work of the vibrator, $T_1 = C_2(R_2 + R_3)$ — the time constant of the circuit with opened contact of the vibrator, $T_2 = R_3C_2$ — the time constant of the circuit with closed contact of the vibrator. The remaining designations are shown in Fig. 53. Solution of the system of equations (4.21) gives:

$$\left.\begin{aligned} U_{g_1} &= e_1 \frac{e^{-\frac{\tau}{2T_1}} - 1}{e^{-\frac{\tau}{2}\left(\frac{1}{T_1} + \frac{1}{T_2}\right)} - 1}, \\ U_{g_3} &= e_1 \frac{e^{-\frac{\tau}{2T_1}} - 1}{e^{-\frac{\tau}{2}\left(\frac{1}{T_1} + \frac{1}{T_2}\right)} - 1} e^{-\frac{\tau}{2T_2}}, \\ U_{g_2} &= e_1 \frac{e^{-\frac{\tau}{2T_1}} - 1}{e^{-\frac{\tau}{2}\left(\frac{1}{T_1} + \frac{1}{T_2}\right)} - 1} e^{-\frac{\tau}{2T_1}} \frac{R_3}{R_2 + R_3}, \\ U_{g_4} &= e_1 \frac{e^{-\frac{\tau}{2T_1}} - 1}{e^{-\frac{\tau}{2}\left(\frac{1}{T_1} + \frac{1}{T_2}\right)} - 1} \frac{R_3}{R_2 + R_3}. \end{aligned}\right\} \qquad (4.22)$$

Usually parameters of the input circuit are selected in such a way that:

$$R_2 = 0.5R_3, \qquad T_1 = 4\frac{\tau}{2}, \qquad T_1 = T_2 \frac{R_2 + R_3}{R_3}.$$

Here for approximation of the transmission factor of the modulator it is possible to consider that at the amplifier input there will be formed a pulse of rectilinear form, whose amplitude is $0.4e_1$.*

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*More accurate derivation of relationships for calculation of transfer function of input circuit of M-DM amplifier see D. E. Polonnikov, Input elements of electronic amplifiers of autocompensators. Cand. dissertation, IAT Academy of Sciences of the USSR (1956); I. C. Hutcheon, A. M. I. Mech E., Properties of some D. C. — A. C. Chopper circuits. Proc. IEE, N 6, Sept. (1957).

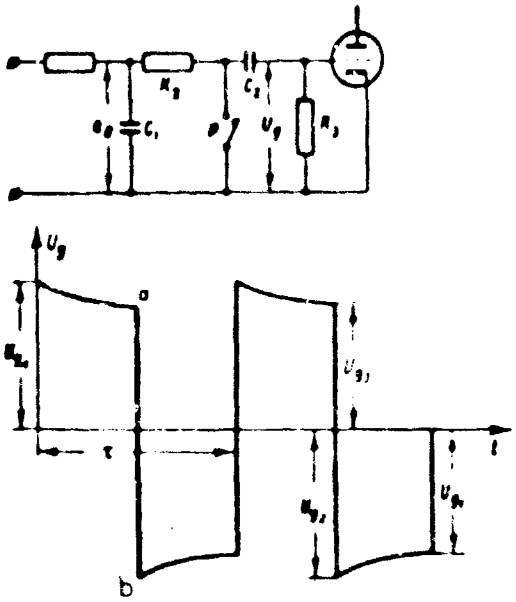

Fig. 53. Form of the voltage curve at the a-c amplifier input; a — contact P is open, b — contact P is closed.

Thus, the initial transmission factor of the modulator $k_м = 0.4$. The transmission factor of the output phasing circuit ($R_{19}C_6$) (Fig. 52) at the frequency of modulation $\omega_м$ will be

$$k_{ф1} = \frac{1}{\sqrt{(R_э C_6 \omega_м)^2 + 1}}, \tag{4.23}$$

where

$$R_э = \frac{R_{18}[R_i + (\mu + 1)R_{17}]}{R_{18} + R_i + (\mu + 1)R_{17}} + R_{19}.$$

The value of $R_э C_6$ is chosen from the condition

$$R_э C_6 \omega_м = \tan\varphi,$$

where φ is the angle of phase shift, introduced by the a-c amplifier at the frequency of commutation $\omega_м = 2\pi f$.

The more the phase of output voltage is shifted by the amplifier, the greater should be the time constant in the output correcting circuit and the greater the lowering of the transmission factor.

When $R_э C_6$ = 0.004 sec and $\omega_м = 2\pi f = 240\pi$

$$k_{ф1} = 0.314$$

Transmission factor of demodulator it is possible to consider equal to one. Thus, $k_{дм} = 1$

For effective lowering of zero drift it is sufficient to have a total transmission factor of M-DM amplifier of order of $K_1 = 1000$. It is obvious that

$$\tilde{k} = \frac{K_1}{k_{ф.к.} k_м k_{дм}} = \frac{1000}{0.4 \cdot 1 \cdot 0.314} \approx 8000$$

In reality the required amplification factor of the a-c amplifier should be higher (15,000 to 20,000). This is possible to explain by the fact that in output signal of modulator, besides voltage with carrier frequency, there are higher harmonics, for which transmission factor of phasing contour and amplification of amplifier is significantly below described values.

Use of auxiliary amplifier does not completely remove zero drift. Residual drift is caused by change of contact potentials on vibrator, induction from excitation circuit of vibrator and grid currents of first cascade of d-c amplifier. Drift can also appear due to unsuccessful selection of point of grounding of common lead of amplifiers, thanks to which currents, flowing through the common lead, create additional signal at input of amplifier in the form of voltage drop in this lead. Therefore, all leads to the common terminal, have to be connected to it in one place and here be reliably grounded with small transitional resistance of grounding.

As analysis of errors shows (see Chapter III), presence of grid current of first cascade causes error in output voltage, determined by expression

$$\delta e_{вых.г} = \mp \frac{I_c}{C e_{вых\ max}} t. \qquad (4.24)$$

For series receiving-amplifying tubes grid current is of the order 10^{-8} a and, therefore, error $\delta e_{вых.г}$ already at $t_{раб} = 100$ sec can become very perceptible. Therefore, question of lowering or removal of influence of grid currents is very urgent also for amplifiers with automatic stabilisation of zero level. Decrease of grid current can be attained, for example, by introduction of corresponding compensating voltage in grid circuit of first cascade of main amplifier, by selection of tubes with small grid current for work in this cascade, use of special circuits for consturction of first cascade, ensuring decrease of grid currents (for example, circuit of cathode follower with lowered tension of plate supply (G. Korn and T. Korn [1]), and also introduction of connection of grid of first cascade of main amplifier with integrating point through capacitor (see RC circuit, delineated by dotted line in Fig. 49a). In last case appears danger of accumulation of charges on this capacitor in period of overloading of amplifier and slow return of it to a normal regime.

Zero drift, caused by presence of grid currents, it is possible also to lower considerably, if one were to use additional feedback circuit for low frequency channel (H. Hamer [1]). By this component of drift voltage due to grid currents of first cascade of main amplifier is removed from input of the M-DM amplifier. Indeed, if one were to use designations from Fig. 54, then for considered case there can be found connection between output and input voltages taking into account grid current in the form:

$$\bar{e}_{\text{вых}} = -K_0(\bar{e}_1 + K_1\bar{e}_2). \tag{4.25}$$

$$\bar{e}_1 = \bar{e}_{\text{вх}}\frac{1}{RCp+1} + \bar{e}_{\text{вых}}\frac{RCp}{RCp+1} \pm \frac{I_g R}{RCp+1}. \tag{4.26}$$

$$\bar{e}_2 = \bar{e}_{\text{вх}}\frac{1}{RCp+1} + \bar{e}_{\text{вых}}\frac{RCp}{RCp+1}. \tag{4.27}$$

Solving equations (4.25), (4.26), (4.27) for $\bar{e}_{\text{вых}}$ we receive

$$\bar{e}_{\text{вых}} = -\frac{\bar{e}_{\text{вх}}}{\frac{RCp+1}{K_0(1+K_1)} + RCp} \mp \frac{I_g}{\frac{RCp+1}{RK_0} + Cp(1+K_1)}. \tag{4.28}$$

For very large K_0 equation (4.28) reduces to

$$\bar{e}_{\text{вых}} = -\frac{\bar{e}_{\text{вх}}}{RCp} \mp \frac{I_g}{Cp(1+K_1)}. \tag{4.29}$$

From equation (4.29) it follows that separation of channels of low and high frequency with the help of additional feedback circuit gives decrease of error due to grid currents by $(1 + K_1)$ times. Here is required doubled quantity of styroflex capacitors and accurate trimming of time constant of both feedback circuits.

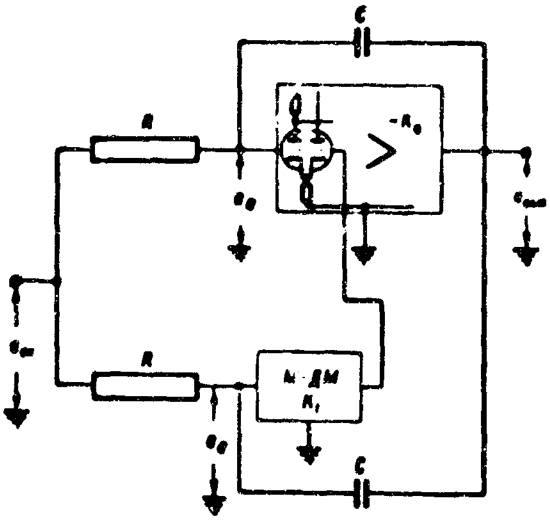

Fig. 54. Diagram of integrating operational amplifier.

4. Amplifiers with Parallel Amplification Channels.*

The principle of action of amplifiers with parallel amplification channels is that channels of amplification are divided into two: high-frequency and low-frequency (assembled by M-MD circuit) — with subsequent summation in broad-band amplifier with relatively low amplification factor (see Fig. 55).

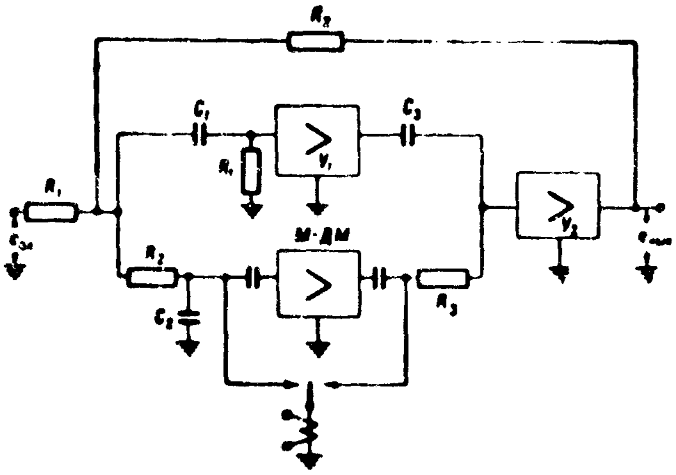

Fig. 55. Skeleton diagram of T-shaped operational amplifier.

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*Idea of parallel channels of amplification in reference to amplifiers of self-recording insturments was expressed in 1952 in work of Buckerfield [1]. Apparently in reference to operational amplifiers this idea was not used due to a number of difficulties.

The passband of separate channels is selected in such a way that during joining there does not occur essential troughs in the gain-frequency response and there is ensured stability during closing of feedback both at high, and low frequencies.

The high-frequency channel, as a rule, is executed by the diagram of the a-c amplifier which lowers requirement on stability of sources of supply. Absence of resistance coupling with the integrating point almost completely removes influence of grid currents.

Assuming for simplification that amplifiers Y_1 and amplifiers M-DM are inertialess, we obtain expression of transfer function of parallel channels in the form

$$W(p)=\frac{k_1T_1T_2T_3p^3+k_1T_1T_3p^2+k_{\text{м.дм}}T_1p+k_{\text{м.дм}}}{T_1T_2T_3p^3+(T_1T_3+T_2T_3+T_1T_2)p^2+(T_1+T_2+T_3)p+1}\qquad(4.30)$$

where $T_1 = R_1C_1$, $T_2 = R_2C_2$, $T_3 = R_3C_3$, k_1 — amplification factor of amplifier Y_1, $k_{\text{м.дм}}$ — amplification factor of d-c M-DM amplifier.

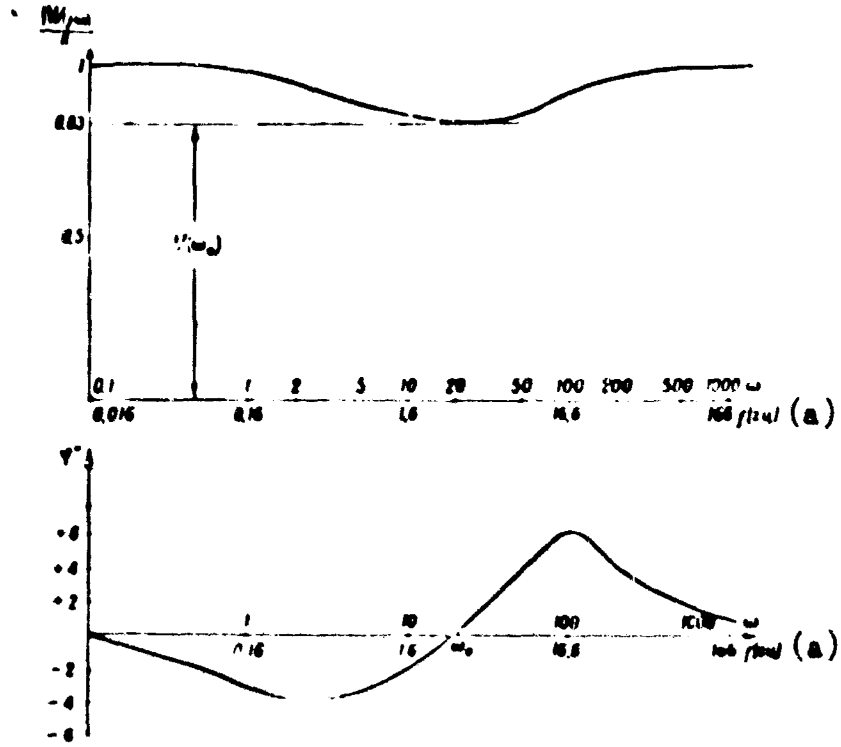

Fig. 56. Gain-phase responses of amplifying channel: W(jω) — complex amplification factor of parallel channels, k — static amplification factor of circuit of parallel channels.
KEY: (a) Cycles.

Frequency response is found, as we know, from (4.30) by replacement $p = j\omega$ in the form

$$W(j\omega) = \frac{(k_{\text{М.ЭМ}} - T_1 T_3 k_1 \omega^2) + j\omega(k_{\text{М.ЭМ}} T_1 - k_1 T_1 T_2 T_3 \omega^2)}{[1 - (T_1T_2 + T_2T_3 + T_1T_3)\omega^2] + j\omega[T_1 + T_2 + T_3 - T_1T_2T_3\omega^2]}. \qquad (4.31)$$

Consequently, when $\omega \to 0$ $W(j\omega) \to k_{\text{М.ЭМ}}$ and when $\omega \to \infty$ $W(j\omega) \to k_1$.

We will estimate character of change of phase response

$$\varphi = \arctan \varphi_1 - \arctan \varphi_2,$$

where

$$\tan \varphi_1 = \frac{\omega(T_1 k_{\text{М.ЭМ}} - k_1 T_1 T_2 T_3 \omega^2)}{k_{\text{М.ЭМ}} - T_1 T_3 k_1 \omega^2},$$

$$\tan \varphi_2 = \frac{\omega(T_1 + T_2 + T_3 - T_1 T_2 T_3 \omega^2)}{[1 - (T_1T_2 + T_2T_3 + T_1T_3)\omega^2]}.$$

When $\omega \to 0$ $\tan \varphi_1 \to \omega T_1$, and $\tan \varphi_2 \to \omega(T_1 + T_2 + T_3)$. Hence we conclude that at low frequencies $\varphi_1 < \varphi_2$ and, consequently, parallel channels give a lag ofoutput signal in reference to the input signal.

When $\omega \to \infty$ $\tan \varphi_1 \to \omega T_2$, and $\tan \varphi_2 \to \omega T_2 \frac{1}{1 + \frac{T_1}{T_3} + \frac{T_3}{T_1}}$, whence it follows that $\varphi_1 > \varphi_2$ and, consequently, we get lead of output signal in reference to input.

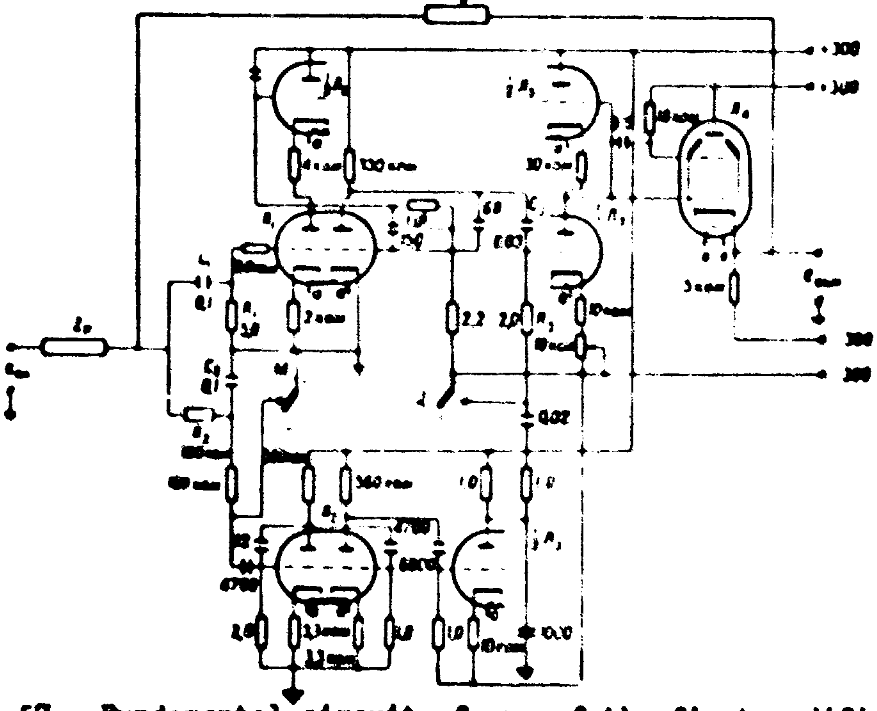

Fig. 57. Fundamental circuit of one of the first modifications of operational amplifier with parallel channels of amplification of installation of type EMU-8.

Finally, phase response passes through zero at certain frequency $\omega = \omega_0$, when $\varphi_1 = \varphi_2$ or $\tan\varphi_1 = \tan\varphi_2$. This leads to relationship

$$\frac{T_1 k_{\text{м дм}} - k_1 T_1 T_2 T_3 \omega_0^2}{k_{\text{м дм}} - T_1 T_3 k_1 \omega_0^2} = \frac{T_1 + T_2 + T_3 - T_1 T_2 T_3 \omega_0^2}{1 - (T_1 T_3 + T_2 T_3 + T_1 T_2)\omega_0^2} = T. \tag{4.32}$$

Frequency $\omega = \omega_0$ at which $\varphi_1 = \varphi_2$ is also called frequency of union of passbands of parallel channels of amplification.

In Fig. 56 are brought gain and phase frequency responses of one of the first modifications of operational amplifiers with parallel channels of amplification (see Fig. 57), developed in the Academy of Sciences of USSR (V. M Syreyev [1]) in reference to analogs. Parameters of the circuit were selected in such a way that at the frequency of union ω_0 slump of the gain-frequency response does not exceed 30%.

If it is accepted for simplification that $k_1 = k_{\text{м дм}} = k$, then

$$\left|\frac{W(j\omega)}{k}\right| = \frac{1 - T_1 T_3 \omega^2}{1 - (T_1 T_2 + T_1 T_3 + T_2 T_3)\omega^2} \times \sqrt{\frac{1 + \dfrac{\omega^2 [T_1 - T_1 T_2 T_3 \omega^2]^2}{[1 - T_1 T_3 \omega^2]^2}}{1 + \dfrac{\omega^2 [T_1 + T_2 + T_3 - T_1 T_2 T_3 \omega^2]^2}{[1 - (T_1 T_2 + T_2 T_3 + T_1 T_3)\omega^2]^2}}}.$$

Considering that at frequency $\omega = \omega_0$ the radical equals one, we receive finally

$$\left|\frac{W(j\omega_0)}{k}\right| = \frac{1 - \dfrac{T_1}{T_2}(T_3\omega_0)^2}{1 - (T_3\omega_0)^2\left(\dfrac{T_1}{T_3} + \dfrac{T_2}{T_3} + \dfrac{T_1}{T_3}\cdot\dfrac{T_2}{T_3}\right)}. \tag{4.33}$$

Considering $\left|\dfrac{W(j\omega_0)}{k}\right| = 0.7$, $\omega_0 T_3 = 1$ and $\dfrac{T_1}{T_2} = \dfrac{T_2}{T_3} = a$, we find

$$-0.3a^2 + a + 0.7 = 0,$$

whence

$$a = 4.$$

Time constant T_3 can be selected from permissible value of voltages of pulsations at output of operational amplifier. Assuming that pulsations of voltage at output of operational amplifier are caused only by voltage of higher harmonics after the demodulator $(e_{\text{п}})_{\text{М-ДМ}}$, we receive

$$(\Delta e_{\text{вых}})_{\text{п}} = -k_3\left[(e_{\text{п}})_{\text{М-ДМ}} + k_1 e^{\sim}_{\text{п}}\right].$$

where $e^{\sim}_{\text{п}} = (\Delta e_{\text{вых}})_{\text{п}} \dfrac{Y_2}{Y_2+Y_1}$ is variable component of voltage at integrating point;

$(e_{\text{п}})_{\text{М-ДМ}} = \dfrac{(e' k_{\text{м}} + e^{\sim}) k^{\sim} k_{\text{дм}}}{\omega_{\text{м}} T_3}$ — voltage of pulsations at output of M-DM amplifier;

$e^{\sim}$ — voltage of interferences, brought to input of a-c amplifier of M-DM channel;

k_3 — amplification factor of integrating and output cascades;

$e' = -\dfrac{(e_{\text{вых}})_{\text{т}}}{k_3 k_{\text{М-ДМ}}}$ — constant component of voltage at integrating point;

$\omega_{\text{м}}$ — frequency of modulation.

Considering these expressions, we receive finally

$$|(\Delta e_{\text{вых}})_{\text{п}}| = \frac{(e_{\text{вых}})_{\max} + k_3 k^{\sim} k_{\text{дм}} e^{\sim}}{\omega_{\text{м}} T_3\left[1 - k_1 k_3 \dfrac{Y_2}{Y_1+Y_2}\right]}.$$

whence

$$T_3 = \frac{k_3 k^{\sim} k_{\text{дм}} \dfrac{e^{\sim}}{(e_{\text{вых}})_{\max}} + 1}{|(\Delta e_{\text{вых}})_{\text{п}}|\,\omega_{\text{м}} k_1 k_3 \dfrac{Y_2}{Y_1+Y_2}} \qquad (4.34)$$

The less the required quantity T_3, the wider the passband of M-DM amplifier and the higher the frequency, with which the upper channel of amplification should enter and, consequently, the less the time constant T_1. The last very considerable affects reduction of return of time amplifier to normal conditions after overloading. As follows from (4.34), to decrease T_3 it is expedient to increase frequency $\omega_{\text{м}}$.

For the circuit of Goldberg $k_1 = 1$ and T_3 other conditions equal, will be considerably larger. Indeed*

$$T_{\text{Г}} = \frac{k_3 k_1^{\sim} k_{\text{дм}} \dfrac{e^{\sim}}{(e_{\text{вых}})_{\max}} + 1}{|(\Delta e_{\text{вых}})_{\text{п}}|\,\omega_{\text{м}} k_3 \dfrac{Y_2}{Y_1+Y_2}}.$$

---

*Subscript Г in subsequent formulas indicates Goldberg circuit.

The ratio of time constant T_3 for the circuit of parallel channels of amplification and Goldberg's circuit will be

$$\frac{T_3}{T_{3\Gamma}} = \frac{k^-}{k_\Gamma} \cdot \frac{1}{k_1}.$$

Thus, for example, for operational amplifier of an installation of type EMU-8 (Fig. 58; B. Ya. Kogan, A. A. Maslov, D. Ye. Polonnikov [1]) we have $\frac{k^-}{k_\Gamma} = 20$, and $k_1 = 1000$, then

$$\frac{T_3}{T_{3\Gamma}} = 0.02$$

In real circuits this ration attains a magnitude 0.001, since by considerations of stability of the Goldberg circuit value $T_{3\Gamma}$. found from the given conditions, must considerably increase.

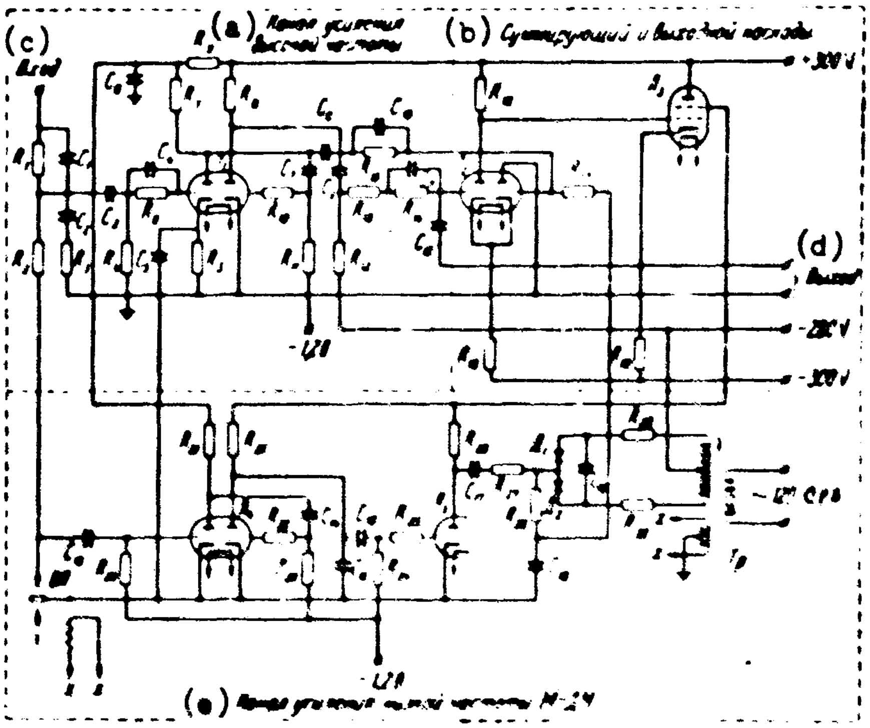

Fig. 58. Fundamental circuit of operational amplifier with parallel channels of amplification of installation of type EMU-8.
KEY: (a) Channel of amplification of high frequency; (b) Integrating and output cascades; (c) Input; (d) Output; (e) Channel of amplification of low frequency M-DM.

Further development of circuit of parallel channels of amplification forced us already in circuit of modified amplifier EMU-8 (Fig. 58) to depart from initial structural diagram of Fig. 57. To guarantee stability of low frequency with a very large amplification (10^6) it was necessary to remove filter R_2C_2, and for increase of stability reserve to apply a third channel (C_8 — R_{15}), connecting output of first cascade with input of integrating cascade.

Application as demodulator of semiconductor diodes (D_1 — D_2) allowed us to lower requirement of accuracy of adjustment of the contact vibrapack-modulator (VP). In modified amplifier EMU-8 we managed significantly to lower influence of change of feeding voltage by decrease of time constant $R_{11}C_7$, $R_{12}C_9$ and application in feed circuits of first cascades of decoupling filter R_9 — C_6 with large time constant (2 sec).

Use of differential integrating cascade, grounding of median point of incandescence, additional shielding, and also correction at high frequencies promoted essential decrease of level of interferences and expansion of the passband.

Main characteristics of such an amplifier: the passband when e_{out} = 100 v (peak) and attenuation is not more than 3 db constitutes 15 kc; level of pulsations when k = 1 constitutes 10 mv (peak); amplification factor at zero frequency of the order of $1.2 \cdot 10^6$; zero shift with change of net voltage by ± 10% is equal to 20 mv. In spite of application for feeding the amplifier of two unstabilized power supplies, drift of the zero level, brought to the input in scale amplifier regime, does not exceed 40 mv, and in integrator regime 2 mv for 100 sec.

Amplifiers of this type, besides the installation EMU-8, are used also in the set of nonlinear blocks NMB (see table XIV Appendix II).

In last years were offered circuits of amplifiers with parallel channels of amplification (D. Ye. Polonnikov [1]), for which cascades of high-frequency channel were as if disconnected in series with increase of frequency. Such a principle allows us considerably to expand the passband of amplifier. It can be considered a result of development of the method, known from theory of automatic control, of supplying of signal in advance, first applied for broad-band inverter by C. S. Deering [1].

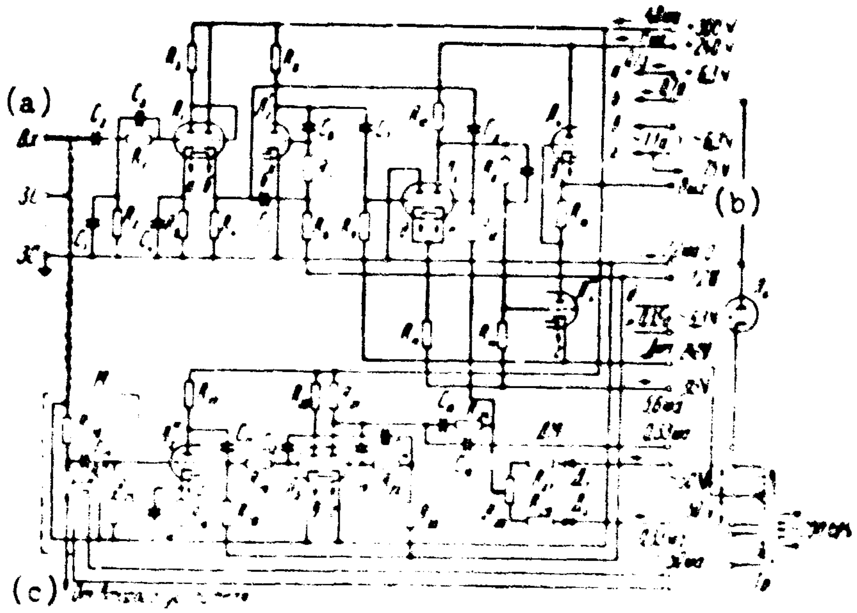

Fig. 59. Fundamental circuit of operational amplifier (TU-10) with parallel channels of amplification of installation of type EMU-10.
KEY: (a) Input; (b) Output; (c) From second amplifier.

In Fig. 59 is brought diagram of amplifier of TU-10, developed in Academy of Sciences of USSR for the EMU-10. In this diagram the principle of series eliminating cascades is used partially. In upper channel the signal from integrating point proceeds only to first cascade (Л7_1) possessing wide passband. From output of first cascade signal is branched to second cascade (Л7_2) and through C_8 to integrating cascade (Л7). Thus, with increase of frequency the whole signal moves to integrating cascade, passing (as if eliminating) the second cascade. Time constant of first cascade due to application of cathode follower is lowered to $8 \cdot 10^{-8}$ sec, and time constant of integrating cascade is of order of $3 \cdot 10^{-7}$ sec. Comparatively small magnitude of time constants of these cascades, and also application of correcting circuits ensure sufficient reserve of stability with a wide passband.

Separating capacitors C_2, C_5, C_7, C_8 and resistances R_2, R_6, R_9, R_{12} are selected in such a way as to ensure stability at low frequency and correct union of passbands of the channels. Output cascade is made by the circuit of the economic cascade without a signinvertor. Amplifier M-DM and upper channel are fed from stabilized voltages, the remaining cascades from an unestabilized source.

Main technical characteristics of such amplifier:

1. Drift of zero level in scale amplifier regime, brought to input (when k = 10), is 30 microvolt in 8 hours.

2. Drift of zero level in integrator regime when RC = 1 constitutes 15 millivolts for 1000 sec (1 millivolt per 100 sec).

3. Interferences (background) at output with transmission factor k = 1 constitute 5 millivolt (peak vlaue).

4. Linearity in output voltage of ± 145 v when load is 10 kilohm.

5. Amplification factor of open d-c amplifier (1 to 2) $\cdot 10^6$, at frequency 200 c $8 \cdot 10^4$ and at frequency 10 kc 600.

6. Passband (slope of gain-frequency response at 30%) when [illegible] = 100 v, load 10 kilohm and shunting capacitance 600 pf constitutes 65 kilocycles.

7. Maximum permissible capacitances at output of amplifier are not less than 0.1 microfarad, at integrating point not less than 0.1 microfarad (with output capacitance 1000 pf).

CHAPTER V

DIODE FUNCTIONAL GENERATORS

1. General Information.

Nonlinear computing elements or functional generators are devices, reproducing given nonlinear functions of one, two or many arguments.

A particular case of a functional generator of two arguments are devices, intended to execute factoring and dividing operations. In view of the great importance of these devices they are usually separated into a special group and considered separately. Equally subject to separate consideration are devices for reproduction of discontinuous nonlinear functions, to which consideration of typical nonlinear characteristics leads CAP (Automatic Control System).

Functional generators it is possible to divide into general-purpose and specialised devices. General-purpose functional generators allow us with the help of one device mechanism, by rebuilding, to reproduce various functional dependences. Specialised generators are adapted (by principle of action) for reproduction of only one specific dependence. Examples of specialized functional generators are devices, using the quadratic nature of dependence of plate current (with small plate currents) on grid voltage of a three-electrode tube (A. Ya. Breytbart [1]) or the logarithmic dependence of grid voltage on grid current (M. J. Tucker [1]). In practice of construction of electronic integrators and analogs, and also separate computers both types of devices found application. Specialized devices have advantage over general-purpose only when, other conditions equal, they are simpler in construction, are cheap in manufacture and are reliable in operation.

Depending upon the character of passage of signals through the functional generator we distinguish devices of open type, for which signals pass directly from input to output, and devices of compensational type, reproducing the given function only with coverage by negative feedback. Advantage of compensational devices consists in lowering the influence of external and internal disturbances and oscillations of parameters on accuracy of operation.

Depending upon method of physical realization we distinguish devices made with electron tubes, electron-beam tubes, electromechanical servo systems with functional potentiometers (G. M. Zhdanov [1]) or specially contoured cams (N. Ye. Kobrinskiy [1]), etc. Electronic functional generators as compared with electromechanical ones possess this advantage, that with equal accuracy they have a significantly wide passband and are less labor-consuming in manufacture and setting up.

Use of functional generators in combination with linear computing elements allows us with the help of electronic analogs to conduct investigation also of nonlinear CAP. In last years very simple functional generators became widely used also in equipment of automatic control. Here they used for improvement of dynamic properties of the CAP (A. Ya. Lerner [1], A. A. Fel'dbaum [2]).

Main requirements, presented to functional generators of electronic analogs, it is possible to reduce to the following:

1. Functional generation should be executed for input magnitudes, given in the form of d-c voltages, with a total range of change ± 100 v.

2. Input impedance of functional generator should not be lower than 10 to 50 kilohms, output no more than 10 to 20 ohms.

Functional generator should possess sufficient output power for convenience of union with other elements.

3. Reproduction of given function should be carried out with accuracy of at least 1 — 2% of full scale.

4. Noise content in output voltage (direct current) should not be greater than

5 to 10 millivolts (peak value).

5. Functional generators must reproduce unambiguous and multivalue nonlinear dependences, and also nonlinear dependences, reduced to elementary functions and obtained from experiment.

6. It is necessary to ensure possibility of reproduction of nonlinear dependences with low and very large values of first derivative, and also non-monotonic functions with a large number of extrema.

It is natural that one construction of a functional generators can not satisfy all the enumerated requirements simultaneously.

During construction of functional generator is essential selection of the most suitable method of presentation of the function given for reproduction. Here can be used approximate methods, for example, assignment of function in the form of a series, by a totality of poinet on foundaries of discrete intervals of change of the argument with corresponding interpolation between these points (for example, piecewise-linear approximation). Last, the function can be given in the form of a pattern or curve, drawn on paper.

In recent years from the large variety of functional generators, functional generators built on diodes and electron-beam tubes received the biggest application. Therefore, in the future account main attention is allotted to these types of functional generators.

2. Diode General-Purpose Functional Generators.

Diode functional generators are constructed in the form of general-purpose or specialized devices. They represent in most cases parametric devices, using piecewise-linear approximation of the given function.

Let y = f(x) be an unambiguous continuous function everywhere on the considered interval, with the possible exception of a finite number of points of discontinuity of type 1.

Then this function it is possible approximately to present by expression

$$y = y_0 + a_0 x + \sum_{i=1}^{n} b_i (x - x_{\text{нач}\, i}). \tag{5.1}$$

where

$$b_i = \begin{cases} 0 & \text{when } x < x_{\text{нач}\, i}, \\ B_i = \text{const} & \text{when } x > x_{\text{нач}\, i}, \end{cases}$$

and $x_{\text{нач}\, i}$ is the value of x at beginning of every segment of the decomposition of the argument.

When initial variables in an electronic analog are in the form of d-c voltage, after transformation of variables $y = M_y e_{\text{вых}},\ y_0 = M_{y_0} e_0,\ x = M_x e_{\text{вх}},\ x_{\text{нач}\, i} = M_x e^0_{\text{вх}\, i}$, we receive

$$e_{\text{вых}} = \frac{M_{y_0}}{M_y} e_0 + \frac{M_x}{M_y} a_0 e_{\text{вх}} + \sum_{1}^{n} b_i \frac{M_x}{M_y} (e_{\text{вх}} - e^0_{\text{вх}\, i}); \tag{5.2}$$

if $\frac{M_{y_0}}{M_y} = \frac{M_x}{M_y} = 1$, then

$$e_{\text{вых}} = e_0 + a_0 e_{\text{вх}} + \sum_{1}^{n} b_i (e_{\text{вх}} - e^0_{\text{вх}\, i}). \tag{5.3}$$

Summation of voltages in expression (5.3) is most simply executed on resistors, proceeding to summation of currents proportional to each component separately. Indeed, the first two currents can be obtained with the help of ordinary circuits, consisting of ohmic resistances, and the last with the help of circuits which are a combination of ohmic resistances and diodes.

In Fig. 60 are presented four diagrams of diode elements, ensuring required current characteristics with positive tangent of angle of inclination. For obtaining current characteristics with negative slope these diagrams must be supplemented with sign-inverting amplifier at the input of the considered diode elements. Here current characteristics will be a mirror image of characteristics of Fig. 60 with respect to the axis of ordinates.

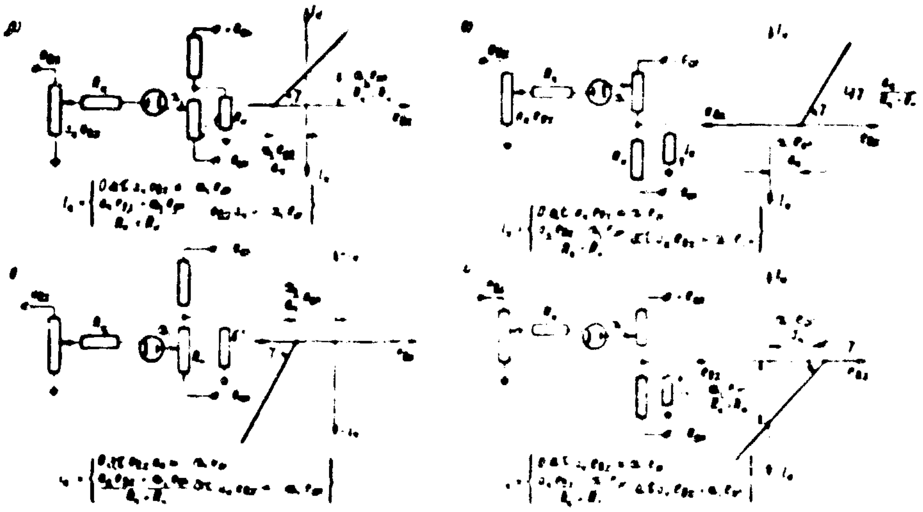

Fig. 60. Diode elements and their current characteristics.

Synthesizing such diode elements in the diagram, for example, depicted in Fig. 61, we can obtain total current in output resistor, equal to

$$I_\Sigma = I_0 + I_1 + \sum_{i=1}^{n} I_{1i}$$

If we disregard internal resistance of the diode, then

$$I_0 = (e_0 - e_{вых})A_0, \quad I_1 = (e_{вх} - e_{вых})A_1,$$

$$I_{1i} = \begin{cases} 0 & \text{when } a_{1i}e_{вх} \le \alpha_i e_{оп} + e_{вых} \\ A_{1i}(a_{1i}e_{вх} - \alpha_i e_{оп} - e_{вых}) & \text{when } a_{1i}e_{вх} > \alpha_i e_{оп} + e_{вых} \end{cases}$$

where $e_{оп}$ is the reference voltage. Output voltage of circuit here will be

$$e_{вых} = \frac{e_0 A_0 + e_{вх} A_1 + \sum_{1}^{n} A_{1i}(a_{1i} e_{вх} - \alpha_i e_{оп})}{A_н + A_0 + A_1 + \sum_{1}^{n} A_{1i}}, \tag{5.4}$$

where

$$A_н = \frac{1}{R_н}, \qquad A_0 = \frac{1}{R_0}, \qquad A_1 = \frac{1}{R_1}, \qquad A_{1i} = \frac{1}{R_{1i}}$$

As follows from comparison of expressions (5.4) and (5.3), with the help of the considered circuit we can not correctly reproduce the given relationship, since with change of the number of switched on diode elements $\sum_{1}^{n} A_{1i}$ in the last addend of denominator also changes.

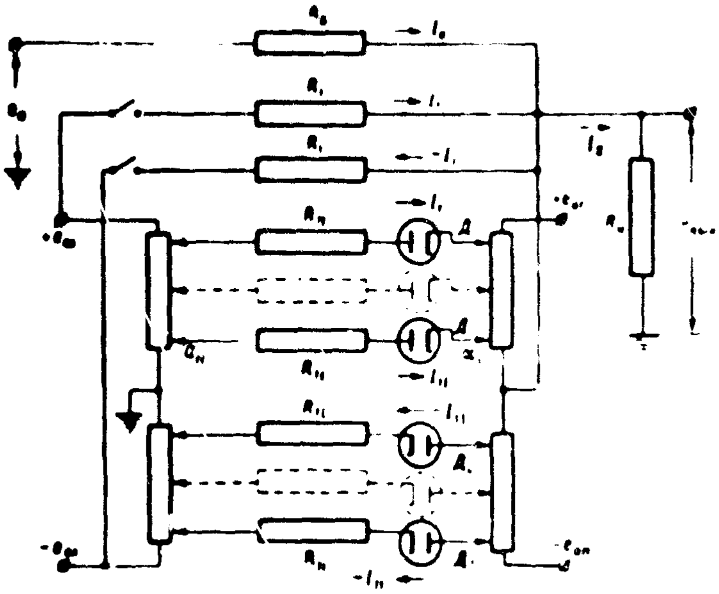

Fig. 61. Diagrams of summation of currents of diode elements.

Obviously, in the given case, just as for linear computing elements with parametric compensation, it would have been possible by means of addition of amplifier and positive feedback to compensate error, introduced by considered circuit. In Fig. 62 is brought a diagram of such a functional generator, offered and developed in the laboratory of L. I. Gutenmakher (L. G. Kogan [1]).

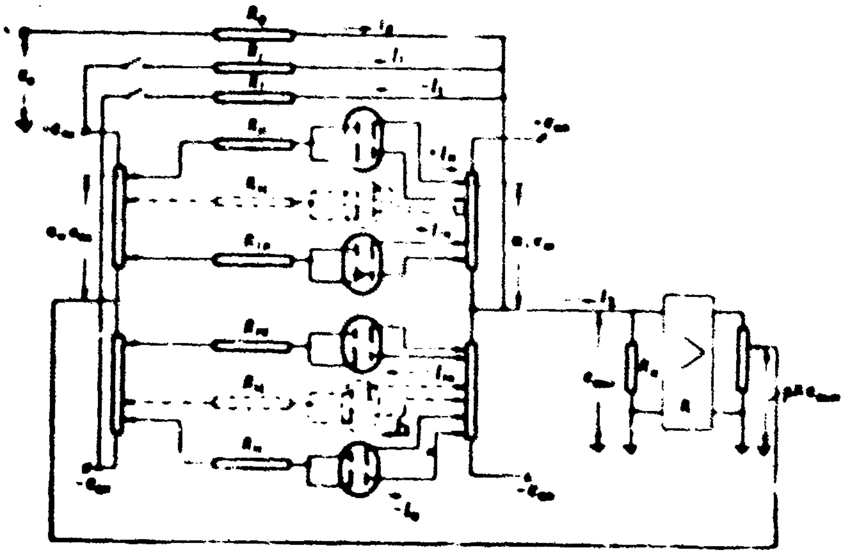

Fig. 62. Diagram of functional generator with parametric compensation of error of summation.

In this circuit feedback voltage is passed in such a way that it affects currents of all diode elements.

As before,

$$I_\Sigma = I_0 + I_1 + \sum_1^n I_{1i}. \qquad (5.5)$$

and

$$I_0 = (e_0 - e_{\text{вых}})A_0, \qquad I_1 = (e_{\text{вх}} - e_{\text{вых}})A_1,$$

$$I_{1i} = \begin{cases} 0 \text{ when } a_{1i}e_{\text{вх}} \leq \alpha_i e_{\text{оп}} + e_{\text{вых}}(1 - \beta K_i), \\ A_{1i}(a_{1i}e_{\text{вх}} - \alpha_i e_{\text{оп}} - e_{\text{вых}} + \beta K_i e_{\text{вых}}) \text{ when} \\ \qquad a_{1i}e_{\text{вх}} > \alpha_i e_{\text{оп}} + e_{\text{вых}}(1 - \beta K_i). \end{cases}$$

After substituting these values of currents in equation (5.5) we receive

$$e_{\text{вых}} = \frac{e_0 A_0 + e_{\text{вх}} A_1 + \sum_1^n A_{1i}(a_{1i}e_{\text{вх}} - \alpha_i e_{\text{оп}})}{A_\Sigma + A_0 + A_1 + \sum_1^n A_{1i}(1 - \beta K_i)} \qquad (5.6)$$

If one were to select β in such a manner that $\beta K_i = 1$, the denominator of expression (5.6) no longer will depend on the number of coupled diode elements and then output voltage $e_{\text{вых}}$ will with accuracy up to a constant scale factor reproduce the given relationship (5.3).

However, such a functional generator possesses a number of peculiarities, preventing its direct use in d-c electronic analogs. Among these peculiarities should be mentioned:

1. Necessity of additional stabilized power source (reference voltage $e_{\text{оп}}$), for which no terminal is grounded.

2. Necessity of stabilization of amplification factor of integrating amplifier.

3. Impossibility of direct connection with operational amplifiers with a common grounded pole.

In Fig. 63 is presented a diagram of a functional generator (A. A. Fel'dbaum and L. N. Fitsner [1]), free from the indicated deficiencies and possessing additional possibility of obtaining of two nonlinear functions from one argument. The basis of the circuit are diode elements, from whose output is removed not current, as in the preceding example, but voltage. Voltages of diode elements are summed by operational amplifiers 1, 2, 3 and 4. All diode elements are identical (Fig. 64) and there is assumed identical coupling of the diode in the circut. The characteristic of such a diode element taking into account resistance of diode can be received from the expression for current I_{11} *

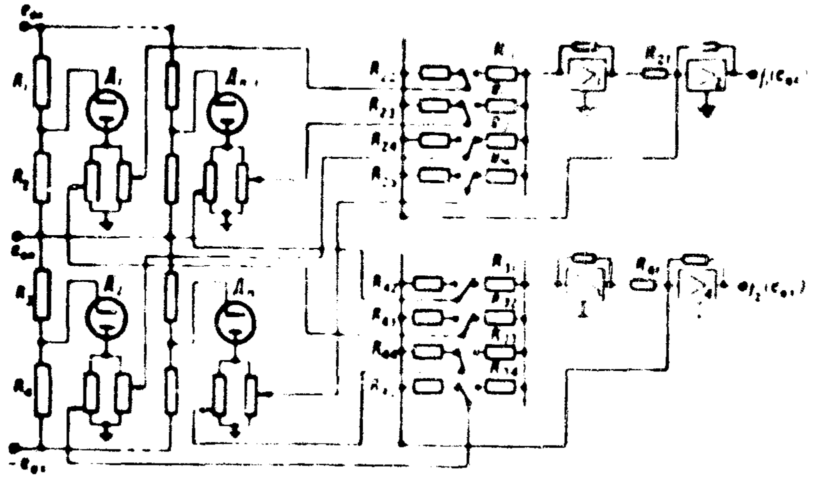

Fig. 63. Diagram of general-purpose diode functional generator with use of integrating d-c amplifiers.

Indeed,

$$I_{11} = \frac{e_{01}R_2 + e_{0n}R_1}{R_1R_2 + (r_d + R_n)(R_1 + R_2)}.$$

where for a vacuum diode

$$r_d \approx \begin{cases} 500 \text{ ohms} & \text{when } e_{01} \quad -e_{0n}\frac{R_1}{R_2} \\ 50 \text{ megohms} & \text{when } e_{01} \quad -e_{0n}\frac{R_1}{R_2} \end{cases}$$

Using these expressions, we find

$$e'_{out} = I_{11}R_{n3} = \begin{cases} 0 \text{ when } e_{01} > -e_{0n}\frac{R_1}{R_2}. \\ \frac{(e_{01}R_2 + e_{0n}R_1)R_{n3}}{R_1R_2 + (r_d + R_n)(R_1 + R_2)} \text{ when } e_{01} \quad -e_{0n}\frac{R_1}{R_2} \end{cases} \tag{5.7}$$

Graphic presentation of this dependence $e_{out} = f(e_{01})$ is shown in Fig. 64. As

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*In this expression and further e_0 must be placed with its sign.

follows from these relationships, input voltage, at which the diode opens, is determined by relationship $e^{н}_{вх i} = e_{оп}\frac{R_1}{R_2}$ and the slope of the resulting characteristic to the axis of abscissas — by expression

$$\tan\gamma = \frac{R_1}{R_1R_2 + (r_д + R_н)(R_1 + R_2)}\,R_3 \qquad (5.8)$$

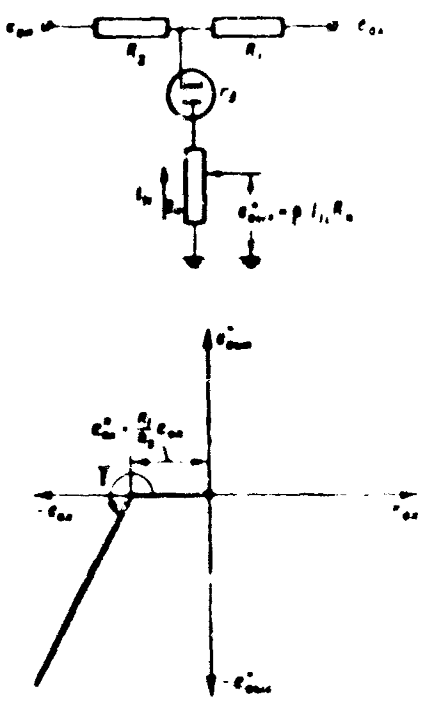

Fig. 64. Diode element and its current characteristic.

The whole characteristic is located only in the third quadrant. In order to obtain a characteristic in all four quadrants with the help of diode elements of only this type, it is necessary to resort to connecting at the output of sign-inverting amplifiers (Fig. 65).

As follows from expression (5.8) and Figure 65, the steepness of the characteristic of such a diode element

$$\tan\gamma = \frac{R_1R_2\alpha K_1K_2}{R_1R_2 + (r_д + R_н)(R_1 + R_2)}$$

depends both on resistances R_1, R_2 and $R_н$, and on setting of the divider at the output and transmission factors K_1, K_2 of output integrating amplifiers.

Increase of steepness due to increase of K_1 and K_2 runs into difficulties, connected with the resulting lowering of the transmission factor of the diode element and increase of zero drift of integrating amplifiers.

Resistances R_1 and R_2 are selected from condition of decomposition of the argument on the given intervals (proceeding from the accepted accuracy of approximation). In general-purpose functional converters to decrease the number of adjustments they often make this decomposition uniform. Then for n segments of decomposition, of magnitude $\Delta e_{вх}$ each, we receive

$$n\Delta e_{вх} = e_{оп}\frac{R_1}{R_{2n}}. \qquad (5.9)$$

where i is the number of the considered segment of the decomposition of the argument.

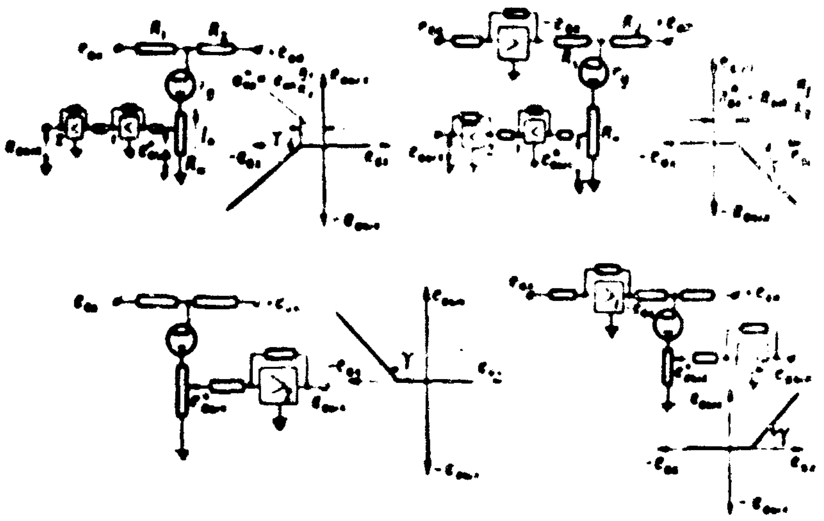

Fig. 65. Current characteristics of diode elements, used in the circuit of Fig. 63.

Considering $R_{1_i} + R_{2_i} = R$, we receive $R_{2_i} = R - R_{1_i}$, and then on the basis of (5.9) we will have

$$R_{1i} = R\frac{1}{1+\frac{e_{\text{оп}}}{i\,\Delta e_{\text{вх}}}} \qquad (5.10)$$

when

$$\Delta e_{\text{вх}} = 2.5\ \text{в}, \qquad e_{\text{оп}} = 25\ \text{в}$$

$$R_{1i} = R\frac{i}{i+10}$$

For various intervals in order of increase of number we receive:

$$R_{11} = \frac{1}{11}R, \qquad R_{12} = \frac{2}{12}R, \ \ldots, \qquad R_{1n} = \frac{n}{n+10}R.$$

To lower the current, flowing through the diode with voltage on it, near zero, in the circuit is provided lowering of filament voltage to 5 v.

The general appearance of a general-purpose functional generator, made by the diagram of Fig. 63, is shown in Fig. 66. The face panel of the instrument is divided into two parts symmetric relative to the middle. On the panel are located handles of potentiometers, a switch and voltmeter, by which nonlinear functions $f_1(e_{\text{вх}})$ and $f_2(e_{\text{вх}})$ are set up. During setting up the whole scale of input voltage, proportional to variable $e_{\text{вх}}$ is divided into 40 equal intervals by the number of diode elements in the circuit. At input terminals + x and — x is established voltage, corresponding to boundaries of intervals, and with the help of potentiometers of the given interval we establish by voltmeter the required value of the function.

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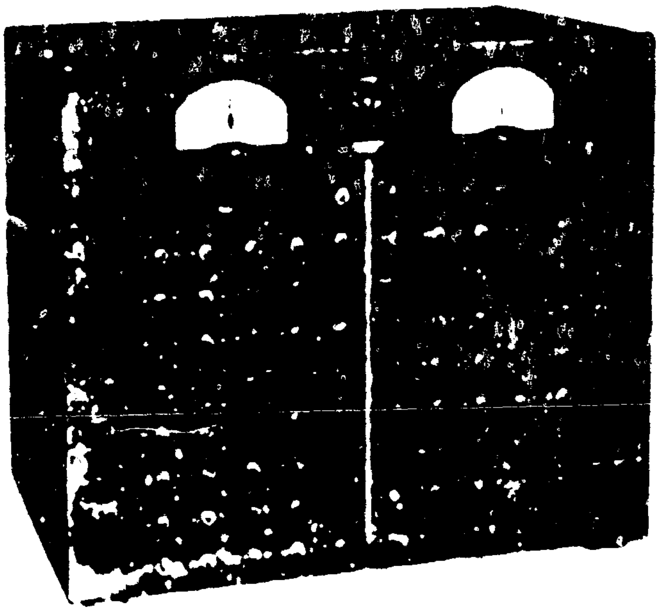

Fig. 66. General appearance of general-purpose diode functional generator made by the diagram of Fig. 63.

In Fig. 67 is brought a diagram of a general-purpose diode generator (I. M. Vittenberg [2]) made with the described diode elements, differing by the fact that in it are provided possibilities of changing the sign of output voltage and additional adjustment of the relationship between R_1 and R_2 to obtain nonuniform breakdown of the argument. As compared with the circuit considered above besides the quantity of operational amplifiers it requires two reference-voltage sources, doubled quantity of potentiometers and large quantity of switching equipment (toggle switches and change-over switches).

Total number of diode elements is lowered here to 12 which during supplying of input signal through resistance and with a constant bias ensures 14 segments decomposition of the total scale of change of the argument. The diagram is composed in operation, since set-up of the given function requires a large number of switching operations, including the necessity of selecting the place of installation of diodes, and, as the preceding one, does not allow one to reproduce a function with great steepness.

Of great interest is the general-purpose diode-triode functional generator (see A. A. Fel'dbaum, L. N. Fitsner [1]), providing reproduction of curves with steepness up to 30 v/v. One modification of the circuit of such a functional generator is shown in Fig. 68. In this circuit is used a two-cascade d-c amplifier Y - 1 controlled by divider $R_{1_1}, R_{1_2}, \ldots, R_{1_n}, R_{1_{n+1}}$ by output triode cascades $Л_1, Л_2, \ldots, Л_n$ which with increase of input signal in turn open, and close the feedback circuit of amplifier Y - 1 through resistances R_{21}, R_{22}, ..., R_{2n}. Output voltages of these triode cascades are limited by diodes $Л_1, Л_2, \ldots, Л_n$

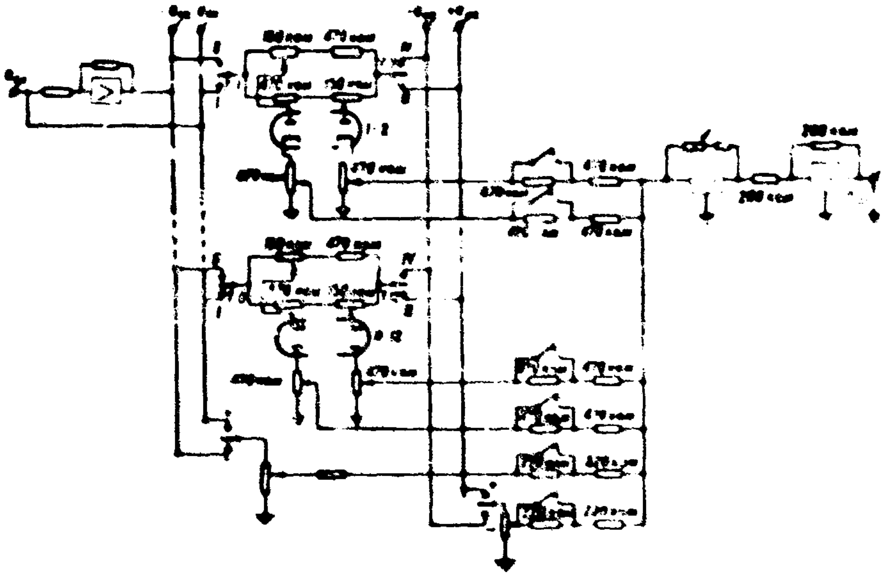

Fig. 67. Diagram of general-purpose diode functional generator (industrial model).

Setting of the steepness of the characteristic of every diode-triode element is carried out by selection of corresponding magnitude $\frac{R_{2i}}{R_0}$ and can be lowered by divider $П_1$. Moment of beginning of limitation is established by assignment of negative bias on divider $R_{1_1}, R_{1_2}, \ldots, R_{1_n}, R_{1_{n+1}}$. Analogous circuits of functional generators are used in electronic analog, MN-1 and MN-8.

The considered circuit can work only with positive input signals. When $e_{вх} < 0$ at the input of amplifier Y - 1 there is connected through R_0 a voltage of "support" + 100 v (see Fig. 68). Use of the principle of feedback allows one to increase accuracy and steepness of the reproduced nonlinear dependences.

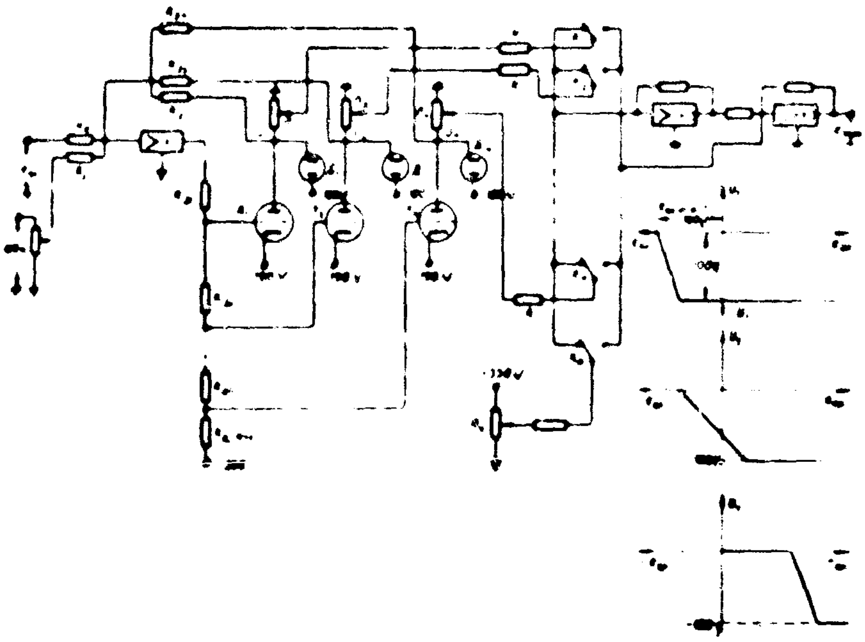

Fig. 68. Diagram of a general-purpose functional generator for reproduction of functions with great slope.

Following the method of synthesis of diode circuits, presented in Chapter VI, it is possible to carry out reproduction of nonlinear dependences with the same steepness, but with use of diode elements alone.

3. Diode Specialized Functional Generators.

The considered diode general-purpose functional devices along equally with positive qualities (wide passband, comparatively low error of the order of 0.5-1%) have also essential deficiencies, consisting in the first place of the fact that they are excessively complicated in construction, require fulfillment of a large number of switching and regulating operations during adjustment on a given nonlinerity, introduce limitation in the permissible steepness of the reproduced function and difficulty during reproduction of nonmonotonic functions, especially in the case of several extrema.

During solution of technical problems, creation of various kinds of computers and especially during investigation of systems of automatic control it is very often necessary to deal with nonlinearities, expressed by elementary functions (for example, $y = \sin x$, $y = \cos x$, $y = x^2$, $y = x^3$, etc.). Such nonlinear dependences, apparently, are better reproduced with the help of specialized devices, beforehand designed for the given nonlinear dependence and not requiring further adjustment during operation. Summation, multiplication and division of output signals of such nonlinear generators allows us to receive new nonlinear dependences, to expand the class of functions reproduced by these devices.

Method of construction of specialized devices should ensure simplicity of calculation, minimum elements, small cost and large operating stability and reliability.

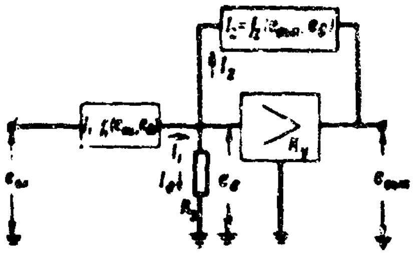

Fig. 69. Functional diagram of a diode specialized functional device.

During projection and development of such devices it turned out to be more expedient to use another approach or, more correct, another treatment of the principle of work of the functional device, consisting of the fact that it is considered as an operational amplifier, furnished with nonlinear converters of output and input voltages in corresponding currents. Indeed, for the diagram of Fig. 69 in a steady-state regime these equalities are correct:

$$\left.\begin{aligned} I_1 &= f_1(e_{\text{вх}}, e_c). \\ I_2 &= f_2(e_{\text{вых}}, e_c). \\ I_1 &= I_c + I_2 \end{aligned}\right\} \qquad (5.11)$$

and

$$\left.\begin{aligned} e_c &= R_c I_c. \\ e_{\text{вых}} &= K_y e_c \end{aligned}\right\} \qquad (5.12)$$

With very large K_y the system of equations (5.11), (5.12) reduces itself to equation

$$f_2(e_{\text{вых}}) = -f_1(e_{\text{вх}}). \qquad (5.13)$$

Function $f_2(e_{\text{вых}})$ we call the current characteristic of the feedback circuit, and function $f_1(e_{\text{вх}})$ the current characteristic of the input circuit of the operational amplifier. Relationship (5.13) is a more general equation of the considered functional device.

To satisfy conditions of static stability (presence of negative feedback) of such an operational amplifier it is necessary that

$$\frac{de_{\text{вых}}}{de_{\text{вх}}}\,\frac{df_2(e_{\text{вых}})}{de_{\text{вых}}} < 0.$$

Since

$$\frac{de_{\text{вых}}}{de_{\text{вх}}} = -K_y < 0,$$

then this condition is reduced to

$$\frac{df_2(e_{\text{вых}})}{de_{\text{вых}}} > 0.$$

If one were to have in mind, henceforth, consideration of diode converters, then it turns out to be to be expedient to translate the derived general relationship (5.13) into the language of conductance of the input circuit and feedback circuit.

Conductance of the circuit determines steepness of the reverse volt-ampere characteristic of this circuit. In the presence of a nonlinear volt-ampere characteristic, as takes place in the considered case, the concept of steepness as a ratio of current to voltage loses its meaning and should be replaced by the concept of steepness at a given point of a nonlinear curve.

Let us arbitrarily call $\frac{dI}{de} = Y(e)$ the differential conductance of a nonlinear circuit. Then after differentiation of the fundamental equation (5.13) for $e_{\text{вх}}$ we receive

$$\frac{df_2}{de_{\text{вых}}}\,\frac{de_{\text{вых}}}{de_{\text{вх}}} = -\frac{df_1}{de_{\text{вх}}}.$$

Since by definition

$$\frac{df_2}{de_{\text{вых}}} = Y_2(e_{\text{вых}}). \text{ and } \frac{df_1}{de_{\text{вх}}} = Y_1(e_{\text{вх}}).$$

that

$$de_{\text{вых}} = -\frac{Y_1(e_{\text{вх}})}{Y_2(e_{\text{вых}})}\, de_{\text{вх}}. \qquad (5.14)$$

Expression (5.14) is the equation of a nonlinear operational amplifier in differential form.

During piecewise-linear approximation for the i-th segment of the decomposition, differential relationship (5.14) should be replaced by a difference relationship:

$$\Delta e_{\text{вых}\, i} = -\frac{\Delta Y_{1i}}{\Delta Y_{2i}}\, \Delta e_{\text{вх}\, i},$$

where

$$\begin{aligned}
\Delta e_{\text{вх}\, i} &= (e_{\text{вх}} - e^{n}_{\text{вх}\, i}),\\
\Delta e_{\text{вых}\, i} &= (e_{\text{вых}} - e^{n}_{\text{вых}\, i}),\\
\Delta Y_{1i} &= \begin{cases} 0 & \text{when } e_{\text{вх}} < e^{n}_{\text{вх}\, i},\\ \Delta Y_{1i} = \text{const} & \text{when } e_{\text{вх}} > e^{n}_{\text{вх}\, i},\end{cases}\\
\Delta Y_{2i} &= \begin{cases} 0 & \text{when } e_{\text{вых}} < e^{n}_{\text{вых}\, i},\\ \Delta Y_{2i} = \text{const} & \text{when } e_{\text{вых}} > e^{n}_{\text{вых}\, i}.\end{cases}
\end{aligned}$$

With n segments of the decomposition we receive

$$\sum_{1}^{n} \Delta Y_{2i}\, \Delta e_{\text{вых}\, i} = -\sum_{1}^{n} \Delta Y_{1i}\, \Delta e_{\text{вх}\, i}. \qquad (5.15)$$

Expression (5.15) shows that nonlinear dependences thus will have only negative slope (located in the 2nd and 4th quadrants) and with their help it will be possible to reproduce only monotonic functions.

To reproduce nonlinear functions one should introduce negative conductance either at the input, or in the feedback circuit. As is known, negative conductance can be created only with the introduction of additional sources of emf. Therefore, for reproduction of nonlinear functions or monotonic functions with positive slope one must connect at the input or output of the main integrating amplifier sign-inverting amplifiers (Fig. 70a and Fig 70b). The circuit diagram of the sign-inverter at the input has this advantage, that here only one operational amplifier works as an adder.

When the given nonlinear dependence is such that the current characteristic of the input circuit or feedback circuit contains sections with constant value of current on a definite interval of change $e_{вых}$ or $e_{вх}$, equation (5.15) should be written in the form

$$\sum_{1}^{n}\Delta Y_{oi}\,\Delta e_{вых\,i} = -\sum_{1}^{n}(\Delta Y_{1i}\,\Delta e_{вх\,i} + Y_{0i}e_{0n}). \qquad (5.16)$$

where Y_{0i} is the conductance of the auxiliary circuit, taking two values: either A = const, or A = 0 depending upon values $e_{вх}$ or $e_{вых}$.

Every component of the right and left part is reproduced by a diode element, an elementary circuit, made of a conductance, a gate and a reference-voltage source.

Types of diode elements. For synthesizing, specialised functional devices the above-considered diode elements (Fig. 65 and Fig. 67) turn out to be of little use, since they require excessively an large number of resistors. practically do not yield to design and hamper reproduction of functions with comparatively great steepness.

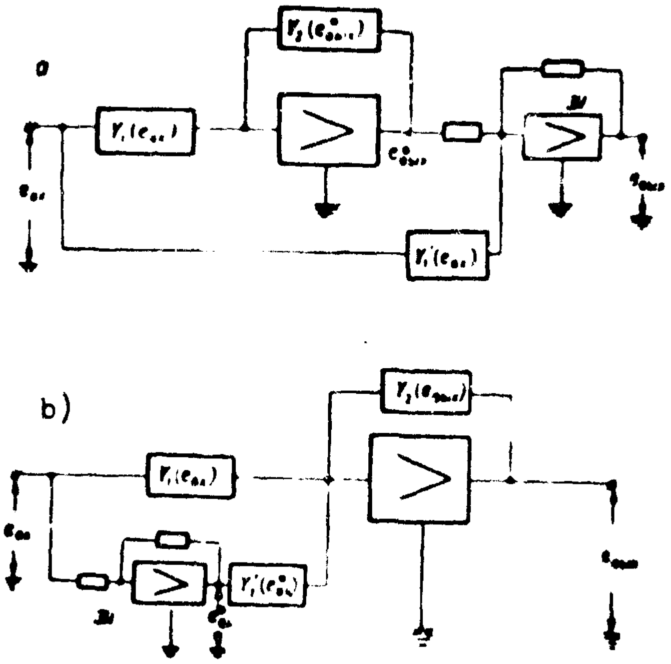

Fig. 70. Circuit diagrams of a sign-inverting amplifier for obtaining negative conductance. ZI — sign-inverting amplifier.

Lately for specialised devices there was offered use of the following types of diode elements:

1) diode elements with potentially grounded diodes (A. D. Talantsev [2]);

2) diode elements, made like limiters (V. V. Gurov, B. Ya. Kogan, A. A. Maslov, V. A. Trapeznikov [1]);

3) combined diode elements (see the same).

A diagram of a diode element with potentially grounded diodes is shown in Fig. 71a. If we disregard resistance of the diode in the conducting state as compared with resistances R_1 and r_1 and designate input voltage, with which the diode changes conductance (there is switching of conductance to another step), by $e''_{\text{вх}\,i}$, then for the current characteristic we will receive the expression

$$I_{1i} = \begin{cases} \dfrac{e_{\text{вх}}}{R_{1i}} - \dfrac{e_{\text{оп}}}{r_{1i}} & \text{when } e_{\text{вх}} > e^{\text{п}}_{\text{вх}\,i}, \\ 0 & \text{when } e_{\text{вх}} \quad e''_{\text{вх}\,i}. \end{cases} \tag{5.17}$$

Hence when $e_{\text{вх}} > e^{\text{п}}_{\text{вх}\,i}$

$$\frac{dI_{1i}}{de_{\text{вх}}} = \Delta Y_{1i} = \frac{1}{R_{1i}}. \tag{5.18}$$

and relationship (5.17) takes the form

$$I_{1i} = \begin{cases} e_{\text{вх}}\Delta Y_{1i} - e_{\text{оп}}\Delta y_i & \text{when } e_{\text{вх}} > e^{\text{п}}_{\text{вх}\,i}, \\ 0 & \text{when } e_{\text{вх}} \quad e''_{\text{вх}\,i}. \end{cases}$$

where $\Delta y_i = \dfrac{1}{r_i}$.

Magnitude $e^{\text{п}}_{\text{вх}\,i}$ is determined from the condition of equality of the potential of point Σ_1 to the potential of point Σ, taken with accuracy sufficient for practice for zero, when summation of currents of diode elements is carried out by an integrating operational amplifier.

Therefore

$$e^{\text{п}}_{\text{вх}\,i} = \frac{R_i}{r_i}e_{\text{оп}} = \frac{\Delta y_i}{\Delta Y_{1i}}e_{\text{оп}}. \tag{5.19}$$

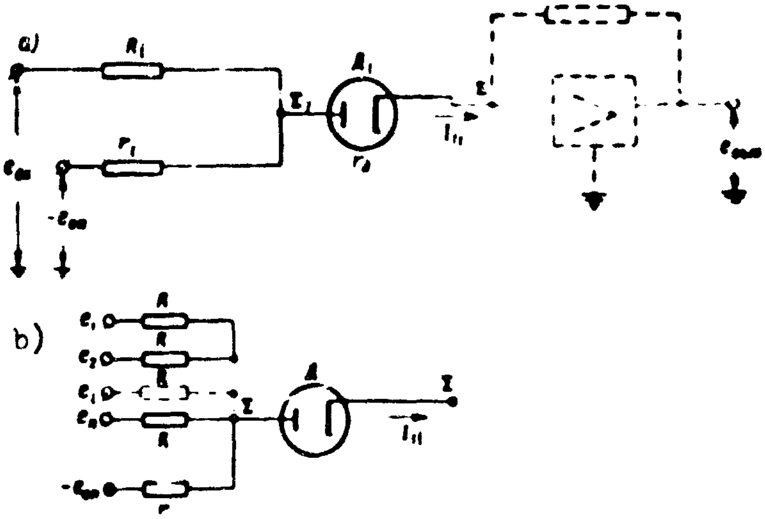

Fig. 71. Diagram of diode element with potentially grounded diodes.

From expression (5.19) it follows that $e^{п}_{вх\,i}$ can be equal to, greater and less than $e_{оп}$.

When $e_{вх} < e^{п}_{вх\,i}$, voltage at integrating point Σ_1 is determined by the expression

$$e_{\Sigma_1} = e_{вх}\frac{\Delta Y_{11}}{\Delta Y_{11}+\Delta y_i} - e_{оп}\frac{\Delta y_i}{\Delta Y_{11}+\Delta y_i}. \qquad (5.20)$$

Voltage e_{Σ_1} with increase of $e_{вх}$ will decrease. When $e_{\Sigma_1} = e_{\Sigma}$ the diode will open and potential of point Σ_1 will differ from the potential of point Σ by the magnitude of the voltage drop on the diode. This difference of potentials will constitute $\Delta U = I_{11}r_d$. In the worst case, when I_{11} = 0.2 ma and r_d = 500 ohms, ΔU = 0.1 v. Therefore, during coupling of diode the integrating point Σ_1 approaches potentially to point Σ and consequently, is as if potentially grounded.

With the help of such a diode element it is possible also to effect summation of several voltages, if to integrating point Σ_1 we join several voltages through identical conductances $\Delta Y = \frac{1}{R}$. Indeed, the current characteristic for the circuit of Fig. 68b upon the former assumptions will be

$$I_{u} = \begin{cases} 0 & \text{when } e_{\Sigma_1} < 0, \\ \left(\Delta Y \sum_{1}^{n} e_i - e_{\text{оп}} \Delta y\right) & \text{when } e_{\Sigma_1} > 0. \end{cases} \tag{5.21}$$

$$e_{\Sigma_1} = \frac{\Delta Y \sum_{1}^{n} e_i - e_{\text{оп}} \Delta y}{n \Delta Y + \Delta y}, \text{ where } \Delta y = \frac{1}{r}. \tag{5.22}$$

Combination of several such diode cells allows us to effect functional generation of the sum of input voltages (especially valuable during creation of functional devices of two or more argument):

$$e_{\text{вых}} = f[(e_1 + e_2 + \ldots + e_n)]. \tag{5.23}$$

In Fig. 72 are brought possible methods of connecting of diode elements with potentially grounded diodes, ensuring obtaining of current characteristics, located in all four quadrants. Change of character of current characteristic here is attained by change of sign of input signal (auxiliary amplifier), sign of support voltage and method of coupling of diode in the circuit. Circuits of diode elements thus obtained are divided into two groups (Fig. 72a and 72b): diode elements, working on switching on, and diode elements, working on switching off. Diode elements, working on switching on, are those for which with increase of input signal by an absolute value the differential conductance with respect to the input signal changes from zero to a certain constant value ΔY_i. For diode elements, working on switching off, with increase $e_{\text{вх}}$ conductance decreases a certain constant value ΔY_i.

Calculation of parameters of the circuit of the considered diode elements with potentially grounded diodes does not cause difficulties. Indeed, after piecewise-linear approximation of the given nonlinear dependence (see Sect. 1 Chapter VI) we already know the values $e_{\text{вх}\,i}$ and ΔY_i, and, therefore, for a selected value of

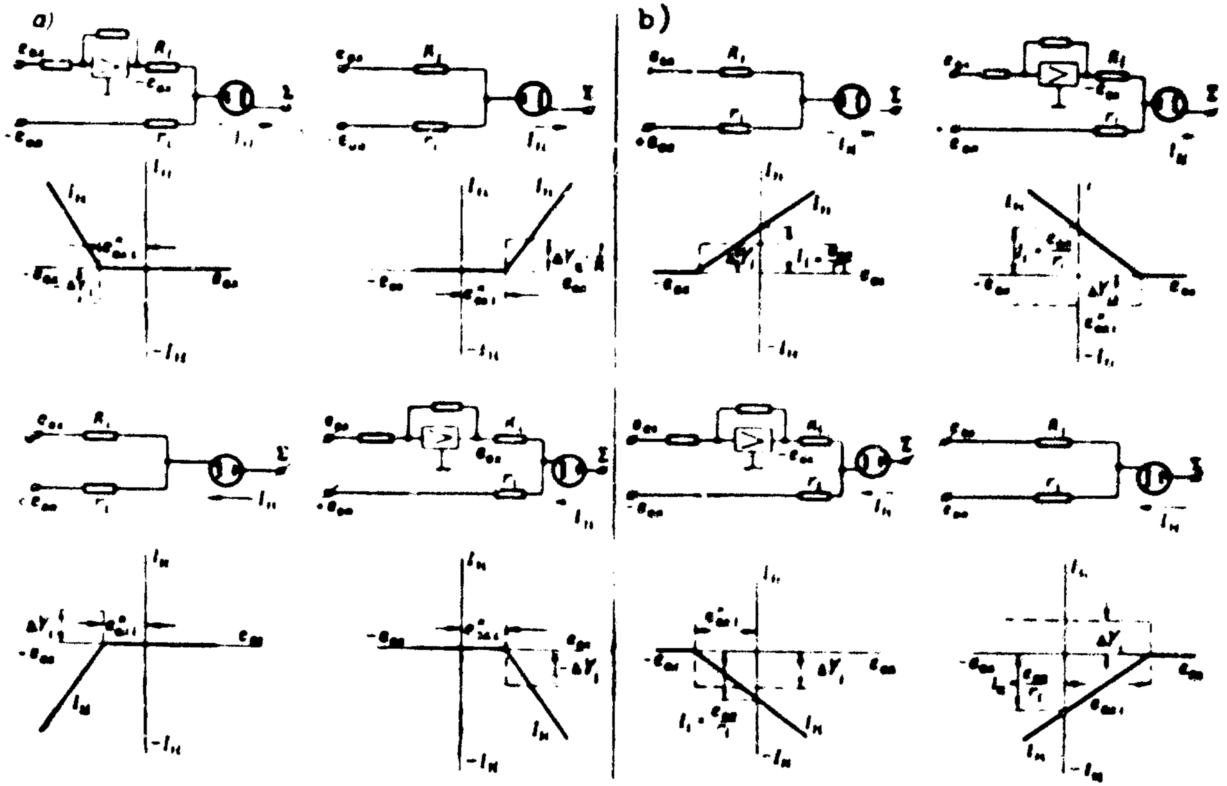

Fig. 72. Possible methods of coupling diode elements with potentially grounded diodes.

calculation reduces itself to determination of Δy_i by relationship (5.19).

$$\Delta y_i = -\frac{\Delta Y_i e^{п}_{\text{вх}\,i}}{e_{\text{оп}}}. \tag{5.24}$$

where i is the number of the diode element.

The current characteristic of the considered diode element taking into account resistance of diode can be presented in the form

$$I_i = \begin{cases} 0 & \text{when } e_{\text{вх}} \quad e^{п}_{\text{вх}\,i}, \\ \dfrac{e_{\text{вх}} - e_{\text{оп}}\dfrac{R_i}{r_i}}{R_i + r_{\text{д}}\left(1 + \dfrac{R_i}{r_i}\right)} & \text{when } e_{\text{вх}} > e^{п}_{\text{вх}\,i}. \end{cases}$$

If one were to compare this expression for the current characteristic with equation (5.17), then one can be convinced that calculation of resistance of diode can be presented as a simultaneous change of R_i and r_i. Designating $R_i + r_{\text{д}}\left(1 + \frac{R_i}{r_i}\right) = R_i^*$ and $r_i + r_{\text{д}}\left(1 + \frac{r_i}{R_i}\right) = r_i^*$, it is possible to write the equation of current characteristic taking into account resistance of diode in the form

$$I_i = \begin{cases} 0 & \text{when } e_{\text{вх}} \quad e^{п}_{\text{вх}\,i}, \\ \dfrac{e_{\text{вх}}}{R_i^*} - \dfrac{e_{\text{оп}}}{r_i^*} & \text{when } e_{\text{вх}} > e^{п}_{\text{вх}\,i}. \end{cases} \tag{5.24a}$$

A peculiarity of the considered diode elements is the fact that at zero slope of their current characteristic $(\Delta Y_i = 0)$ current through the diode element also is equal to zero $(I_i = 0)$

Therefore, for reproduction of current characteristics, possessing sections with zero increment of current when $I_i \neq 0$, it is necessary to resort to switching on of an additional diode element with a current characteristic, compensating increase of current from all switched on diode elements of the circuit. This circumstance increases the total consumption of current in the circuit, the number of simultaneously switch on diode elements and error of reproduction of the given function. In similar cases large advantages can result from application of diode elements, built on the principle of a limiter. In Figs. 73a and 73b are brought two typical circuits of such diode elements.

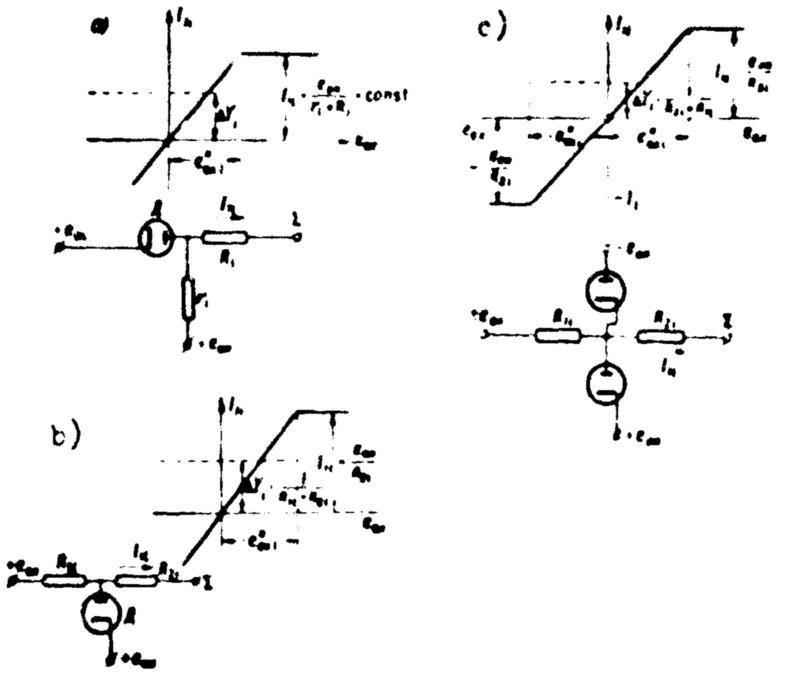

Fig. 73. Diagrams of diode elements, made like a limiter, and their current characteristics.

For the diagram of Fig. 73c with the accepted designations the equation of the current characteristic will be

$$I_u = \begin{cases} \dfrac{\Delta Y_1 \left(e_{вх} + \dfrac{\Delta y_1}{y_д} e_{оп} \right)}{\dfrac{\Delta y_1}{y_д} + 1 + \dfrac{\Delta Y_1}{y_д}} & \text{when } e_{вх} < e''_{вх1}, \\ e_{оп} \dfrac{\Delta y_1 \Delta Y_1}{\Delta Y_1 + \Delta y_1} = \text{const} & \text{when } e_{вх} > e''_{вх1}, \end{cases} \tag{5.25}$$

where

$$\Delta y_1 = \frac{1}{r_1}, \quad \Delta Y_1 = \frac{1}{R_1}, \quad e''_{вх1} = e_{оп} \frac{\Delta y_1}{\Delta Y_1 + \Delta y_1}. \tag{5.26}$$

If we disregard resistance of diode $r_д = \frac{1}{y_д}$, then expression for the current characteristic can be reduced to the form

$$I_u = \begin{cases} \Delta Y_1 e_{вх} & \text{when } e_{вх} < e''_{вх1}, \\ e_{оп} \dfrac{\Delta y_1 \Delta Y_1}{\Delta y_1 + \Delta Y_1} = \text{const} & \text{when } e_{вх} \quad e''_{вх1}. \end{cases} \tag{5.27}$$

As follows from these relationships, slope of current characteristic of this diode element depends only on the resistance, determined by ΔY_1. Therefore, its calculation does not differ at all from calculation of diode element with potentially grounded diode. However, here there is an essential limitation, since $e^{*}_{\text{вк}\,i}$ must be less than $e_{\text{оп}}$. Equation of current characteristic for the diode element, depicted in Fig. 73b will be

$$I_{1i} = \begin{cases} e_{\text{вх}}\,\Delta Y_i^{\circ} & \text{when } e_{\text{вх}} \quad e^{*}_{\text{вк}\,i}, \\ e_{\text{оп}} Y_{2i} & \text{when } e_{\text{вх}} > e^{*}_{\text{вк}\,i}, \end{cases} \tag{5.28}$$

where

$$e^{*}_{\text{вк}\,i} = \frac{Y_{1i} + Y_{2i}}{Y_{2i}}\, e_{\text{оп}}, \tag{5.29}$$

$$\Delta Y_i^{\circ} = \frac{1}{R_{2i} + R_{1i}}, \quad Y_{2i} = \frac{1}{R_{2i}}, \quad Y_{1i} = \frac{1}{R_{1i}}. \tag{5.30}$$

From expression (5.29) it follows that voltage of switching for such diode element always should be larger than $e_{\text{оп}}$. Using relationships (5.29) and (5.30), it is possible always to determine required values of R_{1i} and R_{2i} by the given values of $e^{*}_{\text{вк}\,i}$ and ΔY_i°. Diode element of this type allows one to decrease number of resistances in circuit with reproduction of odd characteristics. Indeed, if in the circuit of Fig. 73b we were to switch in still another diode with reference voltage $e_{\text{оп}}$ as shown in Fig. 73c, then we can obtain a limiter with bilateral limitation, and such a doubled diode element will work during both signs of input signal, not requiring additional diodes for commutation of input signal in sign and using the same resistances R_{1i} and R_{2i}.

Diode elements shown in Figs. 73a, 73b and 73c are elements, working on switching off. Other possible methods of coupling of diode elements of this type are shown in Fig. 74. As follows from the figure, diode elements of the limiter type, working on switching on, always up to moment of switching on give at the integrating point a constant current component. Diode elements, assembled by diagram I, can be used with switching voltages from 0 to $e_{\text{оп}}$ and by diagram II — for switching voltages, varying from $e_{\text{оп}}$ to $e_{\text{вх}\,\max}$.

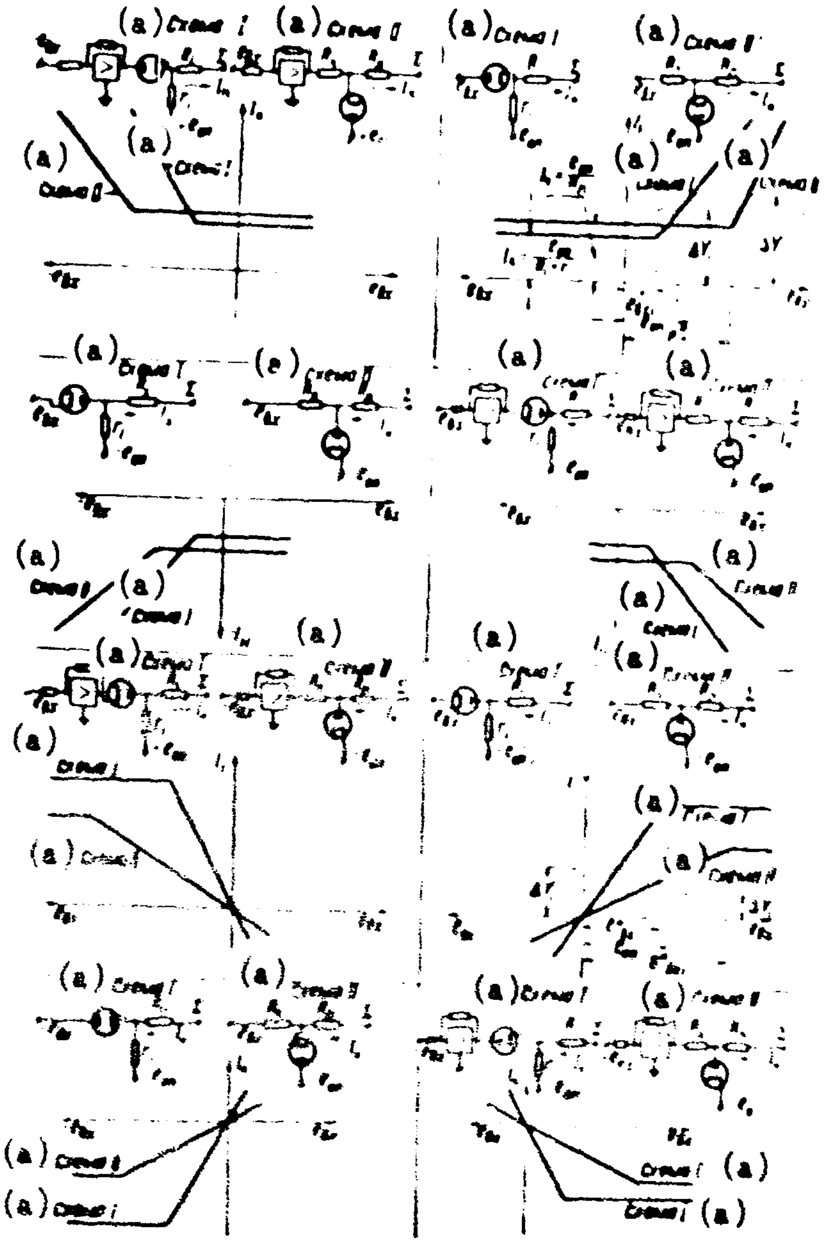

Fig. 74. Possible methods of coupling diode elements of the limiter type.
KEY: (a) Diagram.

Combined diode elements are the totality of various types of diode elements. Actually they are not really an element, but a diode circuit, reproducing a nonlinear characteristic of simplest form. In Figs. 75a and 75b are brought two examples of combined diode elements and their resultant current characteristics. Diode element, depicted in Fig. 75a, is a combination of a diode limiter, made by diagram I ($Д_1$, r, R_1 and R_2), with diode limiter, made by diagram II ($Д_2$, R_1 and R_2). Diode element, presented in Fig. 75b, consists in turn of a diode element with potentially grounded diode ($Д_1$, r, R_1 and R_2) and a diode limiter, made by diagram II (R_1, R_2 and $Д_2$).

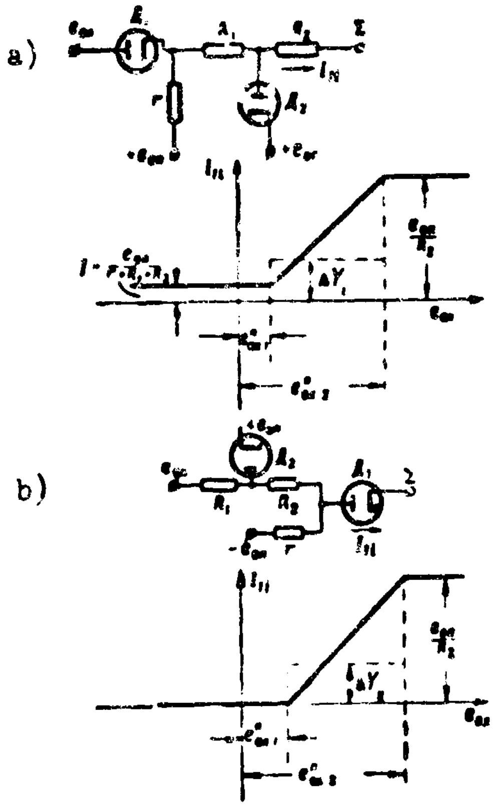

Fig. 75. Doubled diode elements.

Comparison of current characteristics of both types of doubled diode elements shows that their distinction consists of the fact that in current characteristic of first circuit there is initial current, different from zero. This current can be compensated by supplying additional current at the integrating point from a reference voltage through a corresponding resistance. During use of the second circuit it is possible in the same way to displace the characteristic along the axis of ordinates.

Circuits of diode generators. Usually during construction of the circuit of a nonlinear generator separate diode elements are connected in parallel. This means that current at the integrating point of the operational amplifier at a given voltage of the argument, is equal to the sum of currents, set to this point by every diode element under the influence of the voltage of argument and the reference voltage. Thus, for example, during construction

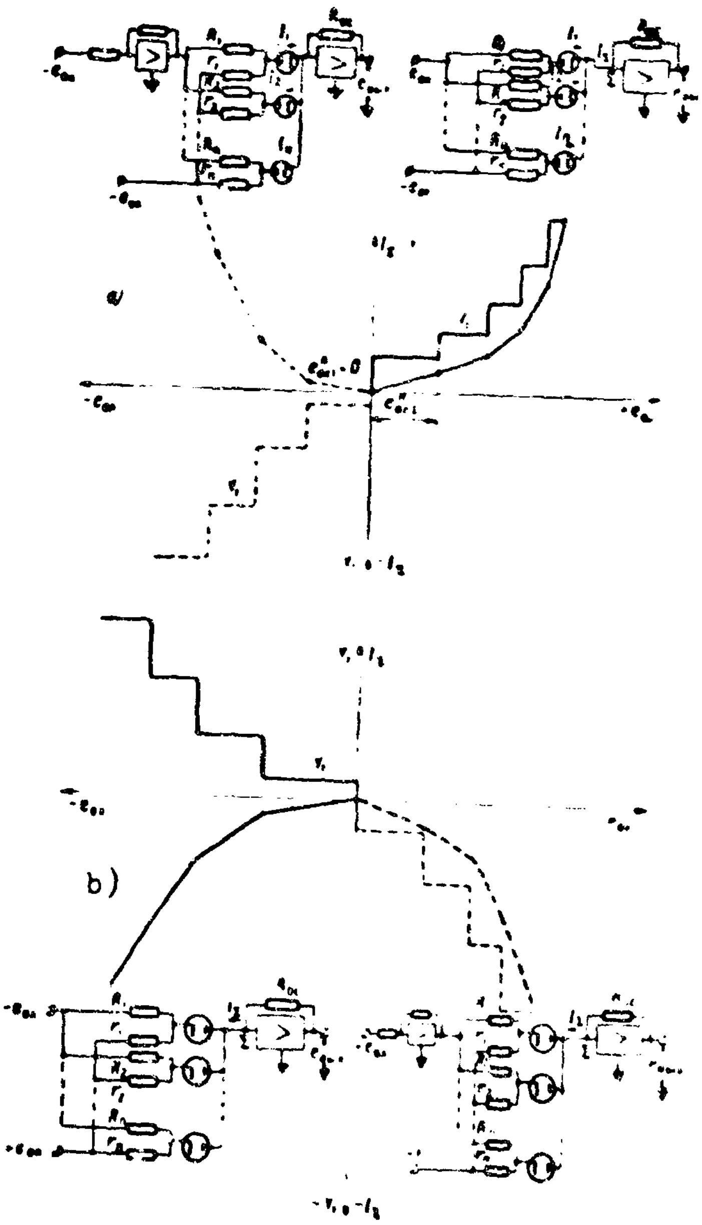

Fig. 76. Diagrams of connection of diode elements (with potentially grounded diodes), working on switching on.

of a circuit only on diode elements with potentially grounded diodes, working on switching on, we have

$$I_\Sigma = \sum_{1}^{k} \pm \Delta Y_i \left(e_{вх} - e^{н}_{вх\,i}\right). \tag{5.31}$$

where

$$\Delta Y_i = \begin{cases} A_i & \text{when } e_{вх} > e^{н}_{вх\,i}, \\ 0 & \text{when } e_{вх} < e^{н}_{вх\,i}, \end{cases}$$

and k is the number of simultaneously switched-on diode elements.

When $e_{вх} = e_{вх.\max}$

$$I_\Sigma = I_{\Sigma\max} = \sum_{1}^{n} \pm \Delta Y_i \, \Delta e_{вх\,i}, \tag{5.32}$$

$$\Delta e_{вх\,i} = e_{вх.\max} - e^{н}_{вх\,i},$$

where n is the total number of diode elements. When $e_{вх} = 0$, $I_\Sigma = 0$.

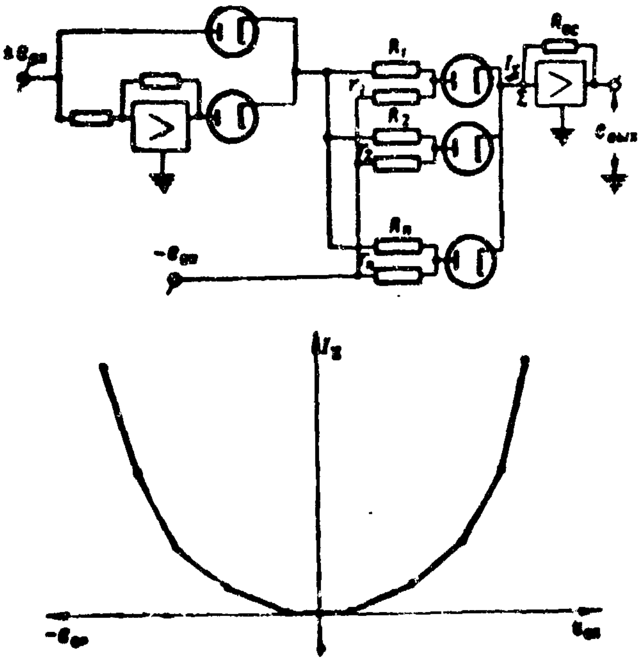

Fig. 77. Circuit for reproduction of even functions.

Current characteristics, which can be obtained with the help of such elements, are presented in Fig. 76a and Fig. 76b. By change of magnitude r_1 in first diode element characteristics can be displaced along the axis of abscissas a required magnitude. In particular, characteristics start from the origin of coordinates, as this is shown on these figures, when $r_1 \to \infty$, i.e., with disconnecting of reference voltage from first diode element. Shift of current characteristics along the axis of ordinates parallel to themselves is attained by addition of a constant current component by means of connection of a source of constant (reference) voltage through constant resistance to integrating point. Considered circuits give build-up of differential conductance with increase of input voltage. Therefore, they are directly useful for reproduction of monotonic functions with positive value of second derivative (with respect to absolute value of change of argument).

During reproduction of even functions it is possible for two quadrants to use one diode circuit, supplying it additionally with commutating diodes, connecting to the input of the circuit always a voltage of the same sign (Fig. 77).

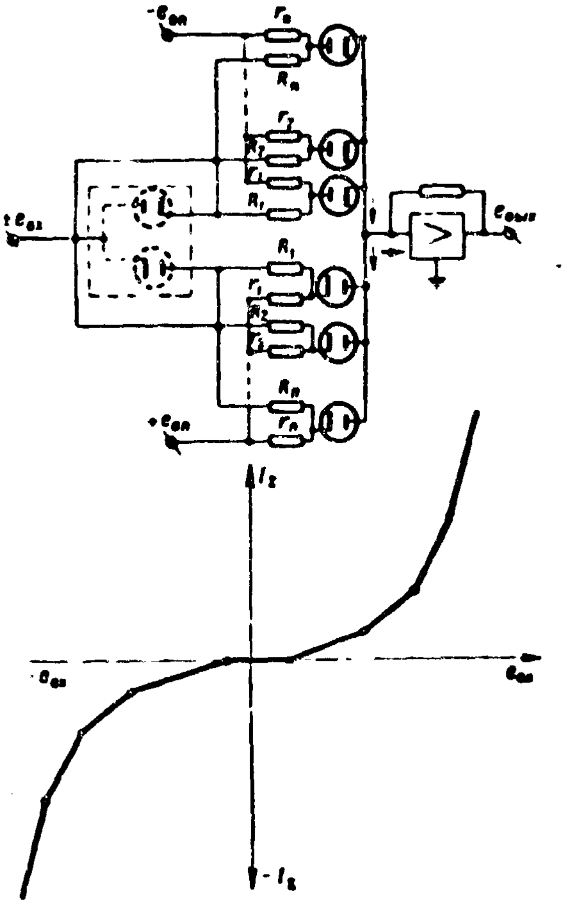

Fig. 78. Circuit for reproduction of odd functions.

During reproduction of odd functions it is necessary to use in every quadrant its own diode circuit. Transition from one circuit to another is carried out here automatically (Fig. 78). If on first interval of change of argument it is necessary to have current characteristic, whose slope is different from zero, and if with this aim first diode elements are replaced by resistances, then it is necessary to connect to the input of the circuit a commutating diode cell as shown in Fig. 78 by the dotted line.

During use of diode elements of the same type, but working on switching off, we receive

$$I_{\Sigma}=\sum_{1}^{n}(\pm\Delta y_i e_{\text{on}}\mp\Delta Y_i e_{\text{вх}}). \qquad (5.33)$$

and

$$\Delta y_i=\begin{cases}\frac{1}{r_i}\text{ when } e_{\text{вх}}<e^{n}_{\text{вх}\,i}.\\ 0\text{ when } e_{\text{вх}}>e^{n}_{\text{вх}\,i}.\end{cases}$$

$$\Delta Y_i=\begin{cases}\frac{1}{R_i}\text{ when } e_{\text{вх}}<e^{n}_{\text{вх}\,i}.\\ 0\text{ when } e_{\text{вх}}>e^{n}_{\text{вх}\,i}.\end{cases}$$

when

$$e_{\text{вх}}=e_{\text{вх.max}}.\qquad I_{\Sigma}=0.$$

and when

$$e_{\text{вх}}=0.\qquad I_{\Sigma}=I_{\Sigma\,\max}=\sum_{1}^{n}\pm\Delta y_i e_{\text{on}}.$$

Current characteristics, corresponding to expression (5.33), are shown in Fig. 79 for all four quadrants with various parameters of the circuit. They give decrease of differential conductance with increase of absolute value of input signal and therefore, are directly useful for reproduction of nonlinear dependences with negative values of second derivative. As in preceding case, by addition of direct current at integrating point it is possible to shift considered current characteristics along the axis of ordinates up or down. As example, let us consider reproduction of nonlinear dependence

$$e_{\text{вых}} = -10\sqrt{e_{\text{вх}}}.$$

Current characteristics of input circuit and feedback circuit will be:

$$I_1 = 10Y_2\sqrt{e_{\text{вх}}},$$
$$I_2 = Y_2 e_{\text{вых}}.$$

Character of change of differential conductance is determined by relationship

$$Y_1 = \frac{dI_1}{de_{\text{вх}}} = \frac{5Y_2}{\sqrt{e_{\text{вх}}}}.$$

Magnitude Y_1 decreases with increase of $e_{\text{вх}}$. Consequently, for reproduction of given nonlinear dependence one should use diode elements, working on switching on, ensuring positive value of differential conductance. However, in given case when $e_{\text{вх}} = 0$, $I_1 = 0$, and when $e_{\text{вх}\,\max} = 100$ v, $I_1 = 100Y_2$.

Although parallel connection of diode elements with potentially grounded diodes, working on switching off, ensures required character of change of conductance with change of input signal, it does not give coincidence of current characteristics, since when $e_{\text{вх}} = 0$, $I_1 = I_{1\,\max}$ and when $e_{\text{вх}} = e_{\text{вх}\,\max}$, $I_1 = 0$. In order to receive coincidence of given and obtained current characteristics, it is necessary to carry out shift of current characteristics by supplying at integrating point an additional constant current component, equal to value of I_1 and $e_{\text{вх}} = 0$ for a circuit of purely parallel connection of these diode elements.

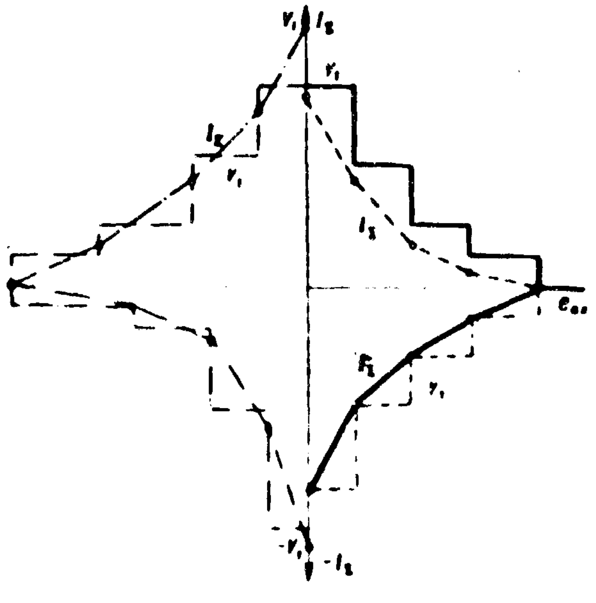

Fig. 79. Current characteristics of circuits, consisting of parallel conection of diode elements (with potentially grounded diodes), working on switching off.

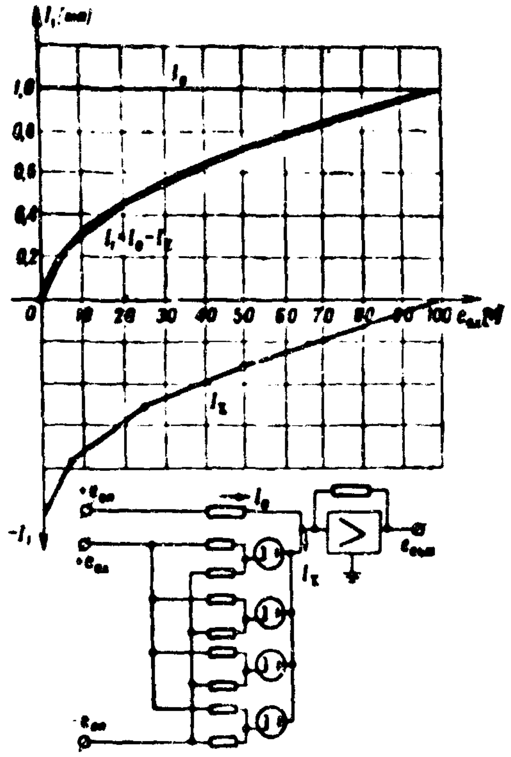

Fig. 80. Method of reproduction of dependence $I_\Sigma = 10Y_2\sqrt{e}$ diode elements (with potentially grounded diodes), working on switching off.

The circuit of a diode nonlinear generator thus obtained is shown in Fig. 80. Here for simplification is brought approximation by four diode elements. Required quantity of diode elements and initial data for calculation of their parameters are determined after piecewise-linear approximation (see Section 1, Chapter VI).

Current characteristic in Fig. 80, can be obtained also by diode elements of limiter type and working on switching off. Here there is not required additional shift of the characteristic and there do not appear accompanying increase of consumption of current by the circuit and growth of error for small input signal values. During construction from these diode elements of a circuit one must on the first two segments of the decomposition (for which $e^{н}_{вх\, i} < e_{0н}$) apply diode elements, made according to diagram I, and for the last two segments — ones made by diagram II (see Fig. 74b). During reproduction of the last segment of the decomposition it is possible to replace the diode element a resistor, connected to the integrating point and input signal. The circuit of such a diode converter for reproduction of nonlinear dependence of the above-considered example is shwon in Fig. 81.

Equation of the current characteristic of the circuit, composed of diode elements of limiter type, working on switching off, can be found on the basis of (5.27) and (5.28) from expression

$$I_\Sigma = \begin{cases} \sum\limits_{1}^{m} \Delta Y_i e_{вх} + \sum\limits_{m+1}^{n} \Delta Y'_i e_{вх} & \text{when } e_{вх} < e^{н}_{вх\, i}, \\ \sum\limits_{1}^{m} \Delta Y_i e^{н}_{вх\, i} + \sum\limits_{m+1}^{n} \Delta Y'_i e^{н}_{вх\, i} & \text{when } e_{вх} \geqslant e^{н}_{вх\, i}. \end{cases} \qquad (5.34)$$

where m is the number of diode elements, made by diagram I, and n is the total number of diode elements.

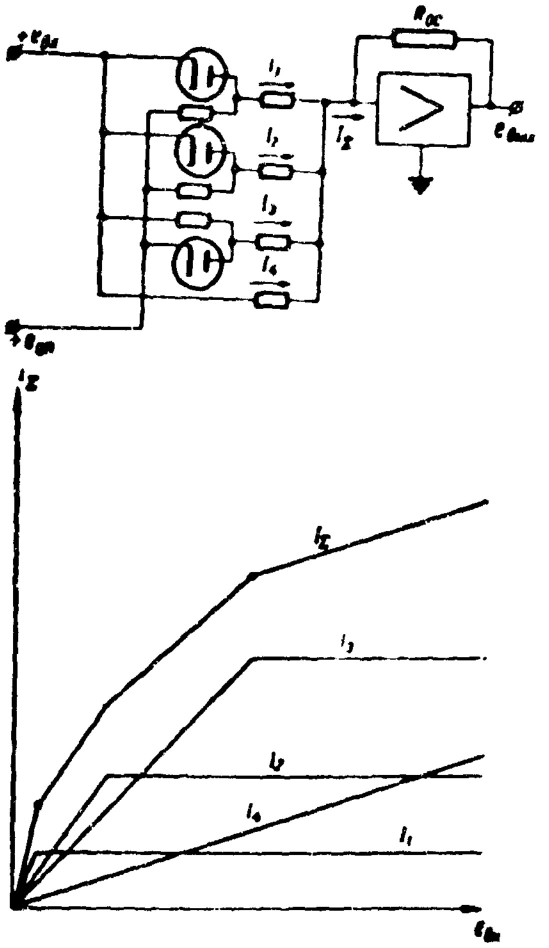

Fig. 81. Circuit of connection of diode elements of limiter type, working on switching off, for reproduction of dependence $I_\Sigma = 10 Y_\Sigma \sqrt{e_{вх}}$.

As follows from the expression, with growth of absolute value of input signal total differential conductance of the circuit falls and when $e_{вх} = e_{вх\,max}$ conductance $Y_\Sigma = 0$ and

$$I_\Sigma = \sum_{1}^{m} \Delta Y_i e^{n}_{вх\,i} + \sum_{m+1}^{n} \Delta Y_i^{*} e^{n}_{вх\,i}.$$

When $e_{вх} = 0$ the circuit gives:

$$Y_\Sigma = Y_{max} = \sum_{i=1}^{m} \Delta Y_i + \sum_{m+1}^{n} \Delta Y_i^{*}. \quad I_\Sigma = 0.$$

For a circuit, composed of diode elements of limiter type, but working on switching on, the expression for the resultant current characteristic can be written in the form

$$I_\Sigma = \begin{cases} \sum_{1}^{m} \frac{e_{оп}}{R_i + r_i} + \sum_{m+1}^{n} \frac{e_{оп}}{R_{2i}} & \text{when } e_{вх} \quad e^{n}_{вх\,i}, \\ \sum_{1}^{m} \Delta Y_i e_{вх} + \sum_{m+1}^{n} \Delta Y_i^{*} e_{вх} & \text{when } e_{вх} > e^{n}_{вх\,i}. \end{cases} \tag{5.35}$$

Here

$$\Delta Y_i^{*} = \frac{1}{R_{1i} + R_{2i}}, \quad \text{and } \Delta Y_i = \frac{1}{R_i}.$$

In considered case total differential conductance of circuit increases with increase of absolute value of input signal.

When

$$e_{вх} = 0, \quad Y_\Sigma = 0,$$

$$I_\Sigma = \sum_{1}^{m} \frac{e_{оп}}{R_i + r_i} + \sum_{m+1}^{n} \frac{e_{оп}}{R_{2i}}.$$

When $e_{\text{вх}} = e_{\text{вх max}}$.

$$Y_{\Sigma} = \sum_{1}^{m} \Delta Y_i + \sum_{m+1}^{n} \Delta Y_i^{*}.$$

As in earlier considered cases, it is possible by coupling to the integrating point of an additional constant current component to compensate initial displacement of the characterisitc.

The preceding shows that synthesizing of circuits of nonlinear generators from considered diode elements can be carried out, proceeding from given the character of change of differential conductance and the initial and finite values of current, sent by the circuit to the integrating point. The first condition allows us to explain what kind of circuit (based on switching on or switching off) of diode element of considered type one should use, and the second gives certain indications of the type of diodo element which one should apply or about character of shift of total current characteristic.

Final selection of type of diode element can be made proceeding from analysis of errors introduced by diode elements of various types, and taking into account necessity of obtaining parameters of circuit, not exceeding permissible limits. Thus, by conditions of work of operational amplifiers feeding the circuit of the nonlinear generator, total consumption of current must not exceed 10 ma. Therefore, minimum values of resistances r_1, R_1, R_{11} and R_{21} should be selected in such a manner that the equavalent input impedance of diode circuit is lower than 10 kilohms. Here quantities r_1, R_1, R_{11} and R_{21} will be greater than 10 kilohms and, consequently, error from final resistance of diodes in the straight direction will be negligible. The upper limit of these resistances is also limited by presence of leaks in the circuit, which for large resistances develop more sharply, and by technical difficulties of selection of required resistances. In units developed by the Academy of Sciences of the USSR, the upper limit for these resistances constitutes 5 megohms, and the lower 50 to 100 kilohms.

CHAPTER VI

PRINCIPLES OF THEORY OF DIODE FUNCTIONAL GENERATORS

1. Determining the Law of Decomposition of the Axis of an Argument and Values of Differential Conductance of a Circuit on Every Segment During Piecewise-Linear Approximation

Problem of piecewise-linear approximation of nonlinear function $e_{\text{вых}} = \nu f(\xi e_{\text{вх}})$* consists in finding a series of fixed values $e^{n}_{\text{вх}\,1}, e^{n}_{\text{вх}\,2}, \ldots, e^{n}_{\text{вх}\,n}$ which provides approximation of broken line with vertices $\nu f(\xi e^{n}_{\text{вх}\,1}), \nu f(\xi e^{n}_{\text{вх}\,2}), \ldots, \nu f(\xi e^{n}_{\text{вх}\,n})$ to curve $\nu f(\xi e_{\text{вх}})$ in such a manner that on the whole interval of change of $e_{\text{вх}}$ absolute error of approximation of the function does not exceed a given value $\Delta e_{\text{вых}}$.

Of function $\nu f(\xi e_{\text{вх}})$ it is assumed that it is unambigous and continuous in the given interval of change of $e_{\text{вх}}$ and has a continuous first derivative everywhere on it, with the exception of, perhaps, a finite number of points of discontinuity of the first kind.

In such a formulation the problem of piecewise-linear approximation present for every interval $e^{n}_{\text{вх}\,k+1} - e^{n}_{\text{вх}\,k}$ the problem of approximation of the given function $e_{\text{вых}} = \nu f(\xi e_{\text{вх}})$ by a ploynomial of the first degree. As is known, error of such approximation is determined by the remainder of the interpolating formula.

For Newton's interpolating formula during linear interpolation the remainder will have the form

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* ν is the scale of the function, ξ — the scale of the argument.

$$R(e_{\text{вх}}) = \frac{e''_{\text{вых}}(\xi)}{2!}(e_{\text{вх}} - e^{n}_{\text{вх}\,k})(e_{\text{вх}} - e^{n}_{\text{вх}\,k+1}). \qquad (6.1)$$

where $e''_{\text{вых}}(\xi)$ is the value of second derivative at a certain point inside interval $e^{n}_{\text{вх}\,k+1} - e^{n}_{\text{вх}\,k}$.

From analysis of formula (6.1) it follows that $R(e_{\text{вх}})$ will a value of maximum modulo when $e''_{\text{вых}}(\xi)$ and expression $[(e_{\text{вх}} - e^{n}_{\text{вх}\,k})(e_{\text{вх}} - e^{n}_{\text{вх}\,k+1})]$ simultaneously attain their maximum. For that worst case we obtain

$$|R(e_{\text{вх}})|_{\max} = \frac{|e''(\xi)|_{\max}}{2!}\left[\frac{|e^{n}_{\text{вх}\,k+1} - e^{n}_{\text{вх}\,k}|}{2}\right]^{2}. \qquad (6.2)$$

Considering $\Delta e_{\text{вых}} = \varepsilon = \max|R(e_{\text{вх}})|$, we obtain from (6.2) expression for determination of length of a segment of the decomposition in the form* $h_k = (e^{n}_{\text{вх}\,k+1} - e^{n}_{\text{вх}\,k})$

$$h_k = \sqrt{\frac{8\varepsilon}{|e''_{\text{вых}}(\xi)|_{\max}}}. \qquad (6.3)$$

during subdivision of the axis of the argument there is determined in the given range the dependence $e''_{\text{вых}} = \varphi(e_{\text{вх}})$. The whole range of approximation is divided into subranges, in which function $e''_{\text{вых}} = \varphi(e_{\text{вх}})$ changes monotonically. In every subrange subdivision is begun from the value of $e_{\text{вх}} = e^{n}_{\text{вх}\,0}$, with which $e''_{\text{вых}}$ has the greatest value.

Considering piecewise-linear approximation by diode elements, connected to the input of the circuit, it is expedient to cross from initial dependence $e_{\text{вых}} = \nu f(\zeta e_{\text{вх}})$ to current characteristics, for example $I_1 = \nu Y_2 f(\zeta e_{\text{вх}})$ and $I_2 = Y_2 e_{\text{вых}}$. In this case formula (6.3) takes the form

$$h_k = \sqrt{\frac{8(\Delta I_1)_{\text{доп}}}{|I''_1(\xi)|_{\max}}}. \qquad (6.4)$$

where

$$(\Delta I_1)_{\text{доп}} = \varepsilon Y_2.$$

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*For derivation of formula see book of N. Ye. Kobrinskiy [1].

Breaking down the current characteristic, it is possible to directly determine the value of conductance of diode circuit on every interval $e^n_{\text{вх}\,k+1} - e^n_{\text{вх}\,k}$, and then for every diode element.

Indeed,

$$\left.\begin{aligned} Y_k &= \frac{I(e^n_{\text{вх}\,k+1}) - I(e^n_{\text{вх}\,k})}{e^n_{\text{вх}\,k+1} - e^n_{\text{вх}\,k}}, \\ Y_{k-1} &= \frac{I(e^n_{\text{вх}\,k}) - I(e^n_{\text{вх}\,k-1})}{e^n_{\text{вх}\,k} - e^n_{\text{вх}\,k-1}}, \end{aligned}\right\} \qquad (6.5)$$

whence

$$\Delta Y_k = Y_k - Y_{k-1}.$$

Formula (6.4) is correct when vertices of the approximating polygon lie on the ideal curve. If approximation is conducted so that vertices of approximating polygon lie on lower or upper boundaries of field of allowance, formula (6.4) remains in force during replacement of the given function $\gamma f(\gamma e_{\text{вх}})$ by $[\gamma f(\gamma e_{\text{вх}}) \pm \varepsilon]$ and substitution in (6.4) in place of ε its doubled value. Here as the ideal curve is taken the upper or lower found of the field of allowance.

The presented method is not connected with specifics of the given appraisal of accuracy of the considered functional device. Thus, for example, if as criterion of accuracy is selected not $\delta = \frac{\Delta e_{\text{вых}}}{e_{\text{вых}\,\max}}$, but $\delta_1 = \frac{\Delta e_{\text{вых}}}{e_{\text{вых}}}$, then decomposition can be effected, using expression (6.3), in which in place of ε is put $2\delta_1 e_{\text{вых}} =$ $= 2\delta_1 \gamma f(\gamma \bar{e}_{\text{вх}})$, and the condition that the approximating straight line inside the considered interval $e^n_{\text{вх}\,k+1} - e^n_{\text{вх}\,k}$ is simultaneously tangent to the upper or lower boundary of the field of allowance (depending upon curvature of the given curve) at point $e_{\text{вх}} = \bar{e}_{\text{вх}}$.

On the basis of the above we obtain

$$\frac{e_{\text{вых}}(e^n_{\text{вх}\,k+1}) - e_{\text{вых}}(e^n_{\text{вх}\,k})}{e^n_{\text{вх}\,k+1} - e^n_{\text{вх}\,k}} = (1 \pm \delta_1)\gamma f'(\gamma \bar{e}_{\text{вх}}) \qquad (6.6)$$

and

$$(e^n_{\text{вх}\,k+1} - e^n_{\text{вх}\,k})^2 = \frac{16\delta_1 \gamma f(\gamma \bar{e}_{\text{вх}})}{|e''_{\text{вых}}(\xi)|_{\max}}. \qquad (6.7)$$

Thus, we obtain two equations of two unknowns, $e_{\text{вх}}$ and $e^{n}_{\text{вх}\,k}$, from which we determine the sought value of $e^{n}_{\text{вх}\,k}$.

Changing, thus, from one interval to another, we obtain not only the length of separate intervals of the decomposition of an argument and law of decomposition, but also, very important, the value of conductance of the circuit on every interval which allows us in a number of cases immediately to calculate resistance of separate diode elements, and consequently, to obtain complete calculation of the circuit of the functional device.

Let us consider application of the presented method to a concreate example.

Example. It is required to desing a diode functional generator for reproduction of function $e_{\text{вых}} = 0.01\, e^{2}_{\text{вх}}$ with error of approximation, not exceeding ± 0.25%.

Diode elements are connected only to the input.

Current characteristic of feedback circuit is

$$I_2 = Y_2 e_{\text{вых}} \tag{6.8}$$

Current characteristic of input circuit is

$$I_1 = 0.01\, Y_2 e^{2}_{\text{вх}}. \tag{6.9}$$

Characteristic of ideal change of conductance of input circuit here is linear:

$$Y_1 = \frac{dI_1}{de_{\text{вх}}} = 0.02\, Y_2 e_{\text{вх}}. \tag{6.10}$$

Second derivative of given function will be a constant:

$$\frac{d^2 I_1}{de^2_{\text{вх}}} = 0.02\, Y_2. \tag{6.11}$$

Therefore, all segments $e^{n}_{\text{вх}\,k+1} - e^{n}_{\text{вх}\,k}$ will be identical and equal to

$$h_k = e^{n}_{\text{вх}\,k+1} - e^{n}_{\text{вх}\,k} = 20\sqrt{\varepsilon}.$$

The total number of segments of the decomposition will be

$$n = \frac{e_{\text{вх}\,\max}}{20\sqrt{\varepsilon}}. \tag{6.12}$$

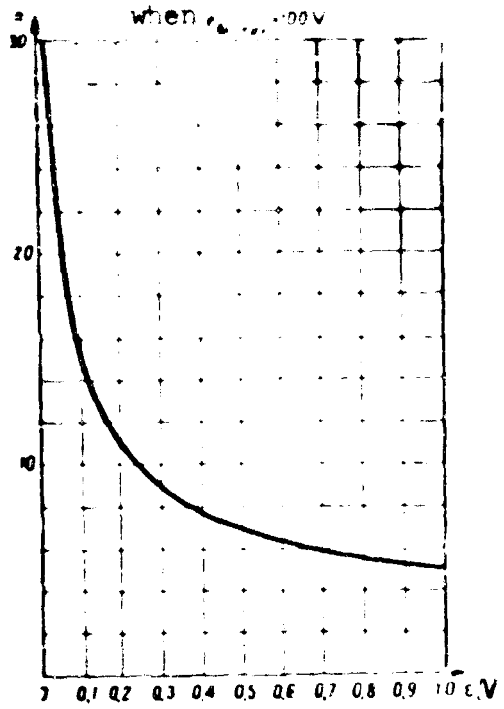

Fig. 82. Curve of dependence of number of segments of a decomposition on permissible error of output voltage.

Graph of dependence (6.12) when $e_{\text{вх max}} = 100$ v is shown in Fig. 82.

As follows from the graph, decrease of permissible error below 0.1 v (0.1%) leads to excessive increase of the number of required steps of decomposition and, apparently, does not have meaning, since due to error, introduced by diode elements themselves, total error here may not be lowered, and increases.

When $e_{\text{вх max}} = 100$ v and $\varepsilon = 25$ v (vertices of approximating polygon are located on the ideal curve)

$$n = 10, \quad h_k = 10 \text{ v}$$

On basis of (6.5) we obtain breakdown of conductances:

$$Y_i = [2e_{\text{вх max}} - (2i - 1)h]\,0.01Y_2,$$

where i = 1, 2,..., n — the number of the interval of the decomposition, starting from the section of the greatest steepness.

In the considered case

$$\Delta Y = Y_i - Y_{i-1} = 2h\,0.01Y_2 = 0.2Y_2.$$

Since when $e^{n}_{\text{вх 1}} = e_{\text{вх max}} = 100$ v, $Y_1 = 1.9Y_2$, increases of conductance will be identical on all steps, except the last, and are equal for the first 9 intervals to $\Delta Y_{1,\ldots,9} = 0.2Y_2$, and for the last $\Delta Y_{10} = 0.1Y_2$. Quantity Y_2 we select, limiting the maximum value of resistance of a diode element to 0.5 megohms.

Then $Y_2 = 2 \cdot 10^{-5}$ moh, and $\Delta Y_{10} = 2 \cdot 10^{-6}$ mho.

In Fig. 83 is brought the resulting decomposition of the parabola for that value of Y_2.

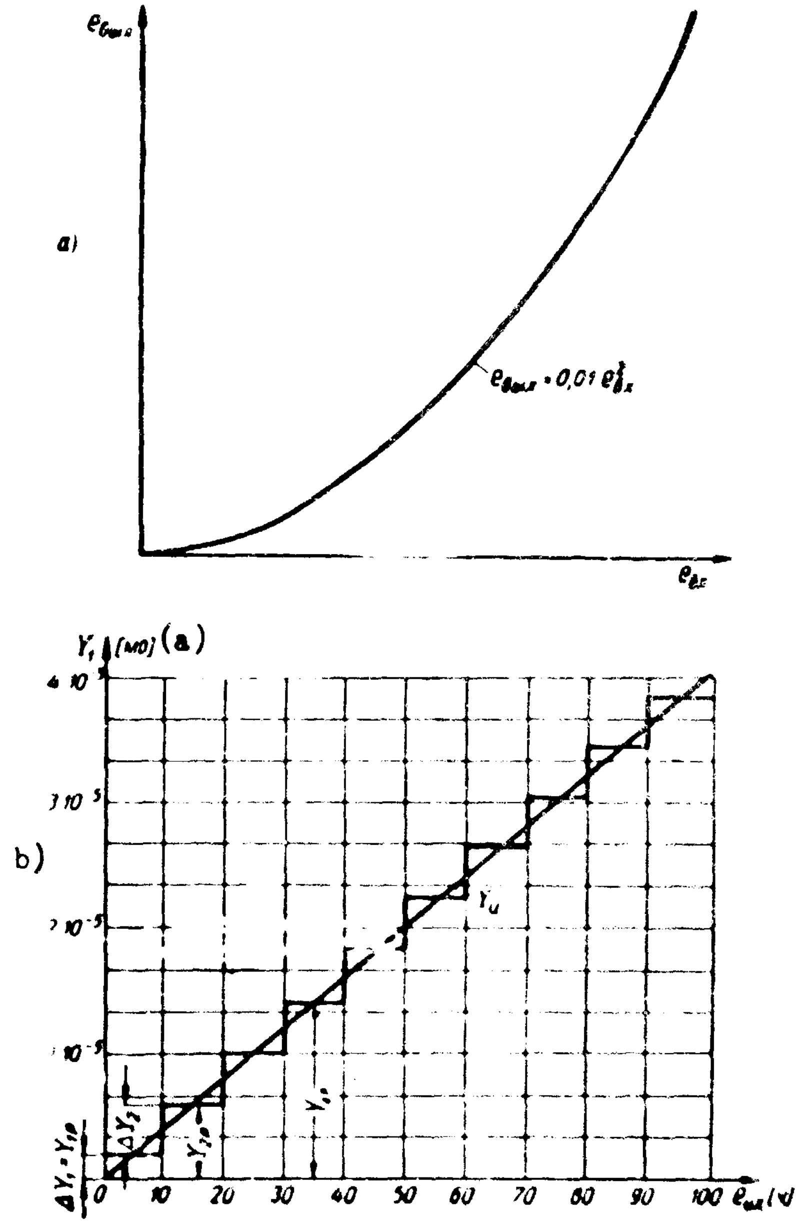

Fig. 83. Piecewise-linear approximation of quadratic dependence.
KEY: (a) Mho.

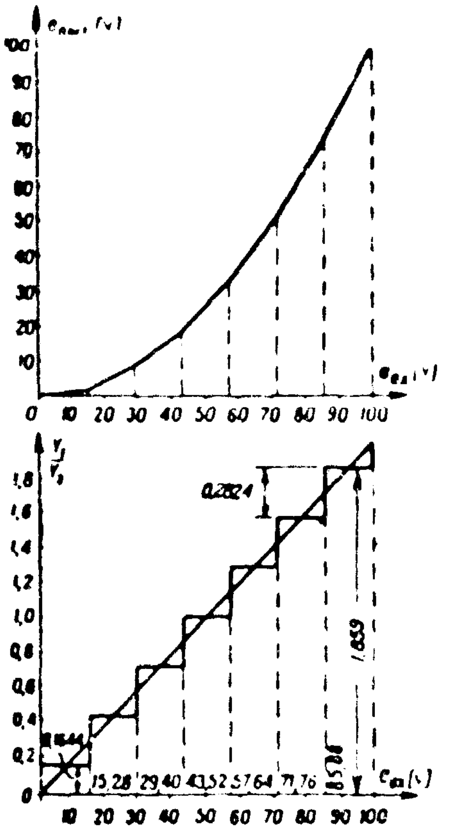

Fig. 84. Piecewise-linear approximation of quadratic dependence, when the points of switching are located on the lower boundary of the field of allowance.

When approximation is conducted by the lower boundary of the field of allowance, decomposition is carried out also by expression (6.4), but in place of ε we substitute its doubled value. Here

$$h_s = 40\sqrt{\frac{\varepsilon}{2}}.$$

The total number of segments of the decomposition will be

$$n = \frac{e_{in\ max}}{40\sqrt{\frac{\varepsilon}{2}}}.$$

When $e_{in\ max}$ = 100 v and ε = 0.25 v we obtain

$$n \approx 7,$$
$$h = 40\cdot0.353 = 14.12\ \text{v}.$$

Conductance on the first step when $e_{in\ max}$ = 100 v will be

$$Y_1 = 1.859\ Y_2$$

The increment of conductance will also be constant and equal

$$|\Delta Y_1|_1 = 2h\cdot0.01Y_2 = 0.2824Y_2$$

The increment of conductance on the last step will be

$$|\Delta Y_1|_n = [Y_1 - |\Delta Y_1|_1(n-1)] = 0.1644Y_2.$$

The resulting decomposition is placed on a graph (Fig. 84).

As these calculations show, if one were not to put additional requirements on limitation of the magnitude of the first derivative, such approximation allows one to create a more economical circuit, since it requires a smaller number of diode elements.

Use of presented method allows us comparatively simply to determine required value of resistances of separate diode elements and the order of their switching on.

Thus, for example, above-mentioned decomposition of a quadratic function (see page 171) shows that conductance of a circuit increases with increase of input signal, and, consequently, diode elements should work on principle of switching on.

Conductance of diode element, switched on first, should be equal to

$$(\Delta Y_1)_7 = 0.1644 Y_2,$$

and conductance of the remaining six elements are identical and are equal to

$$\Delta Y_i = 0.2824 Y_2.$$

When $Y_2 = 1 \cdot 10^{-5}$ mho, $(\Delta Y_1)_7 = 0.1644 \cdot 10^{-5}$ mho, $\Delta Y_{1i} = 0.2824 \times 10^{-5}$ mho, $R_7 = 6.1 \cdot 10^5$ ohm, $R_1 = R_2 = R_3 = R_4 = R_5 = R_6 = 3.54 \cdot 10^5$ ohm.

If one were to use in the circuit diode elements with potentially grounded diodes, then on the basis of (5.19) we obtain

$$\Delta y_i = \frac{1}{r_i} = \frac{\Delta Y_{1i} e^{n}_{\text{вх}\, i}}{e_{\text{оп}}}.$$

Substituting values of ΔY_{1i} and $e^{n}_{\text{вх}\, i}$ received earlier, we receive, when $e_{\text{оп}} = 30$ v:

$r_7 = \infty.$
$r_6 = 695$ kilohm
$r_5 = 361$ »
$r_4 = 244$ »
$r_3 = 184$ »
$r_2 = 148$ »
$r_1 = 123.5$ »

The circuit of a square-law function generator for selected parameters is presented in Fig. 85a.

In order to ensure work of square-law function generator for both signs of input voltage, one must at the input of the circuit connect an additional sign-inverting amplifier and two diodes, as shown in Fig. 85b. With such coupling of diodes, to the input of the circuit of a nonlinear generator there always moves voltage of the same sign. It is necessary, however, to consider here that on the input diodes there appears a voltage drop and, therefore, there may be required additional correction of calculated values of resistances of the circuit, especially on the last steps.

In certain cases it is expedient to use the same diode circuit for obtaining direct and inverse functions, switching it for this purpose from the input circuit to the feedback circuit of the operational amplifier. Here it is necessary to take into account error of approximation of the direct and inverse functions are connected by relationship

$\varepsilon_{op} \approx$ [illegible] (6.13)

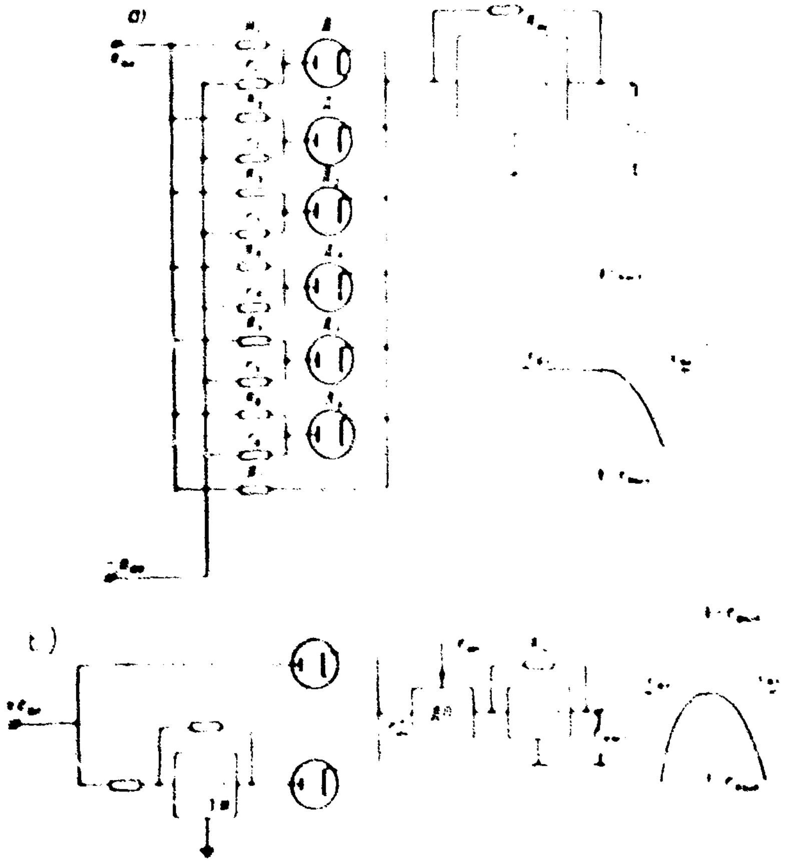

Fig. 85. Fundamental circuit of square-law function generators.

Therefore, if breakdown is carried out by a direct function and if the latter has sections with a steepness less than one, then such breakdown can lead to excessively great error of reproduction of the inverse function during coupling of the diode circuit into the feedback circuit. In these cases there is recommended breakdown segments of the direct function with a steepness larger than one, proceeding from the fact that $\varepsilon_{np} = \varepsilon$. and segments of the inverse function with steepness less than one, proceeding from the fact that $\varepsilon_{\Lambda p} = \varepsilon$. With such breakdown of the axis of an argument error of approximation will not excel a given value of with coupling of the diode circuit both to the input and to the feedback circuit.

If function $e_{\text{вых}} = f(e_{\text{вх}})$ is given graphically, then double graphic differentiation, necessary for construction of dependence

$$\frac{d^2 e_{\text{вых}}}{d e_{\text{вх}}^2} = \varphi(e_{\text{вх}}).$$

leads to large errors. In this case we usually resort to graphic construction of the approximating polygon. For this nonlinear dependence $e_{\text{вых}} = f(e_{\text{вх}})$ or the current characteristic equivalent to it $I_1 = Y_2 f(e_{\text{вх}})$ is depicted in the form of a graph on millimeter graph paper in coordinates $e_{\text{вых}}$ and $e_{\text{вх}}$ or I_1 and $e_{\text{вх}}$. So that graphic error is small compared with error of approximation, it is necessary to draft the given nonlinear dependence by sections in magnified scale. Then for every section there will be set a field of allowance (absolute error ε or εY_2) and in the resulting tube are inscribed straight lines in such a way that on every interval they do not go beyond the limits of the tube. The described method is illustrated in Fig. 86.

During approximation by polygon with vertices, located on the curve of the given nonlinear dependence (Fig. 87), there results a certain increase of number of segments of the decomposition as compared with the preceding case. True, here error in reproduction of slope of given characteristic somewhat decreases. Final choice among these methods of approximation can be made taking into account

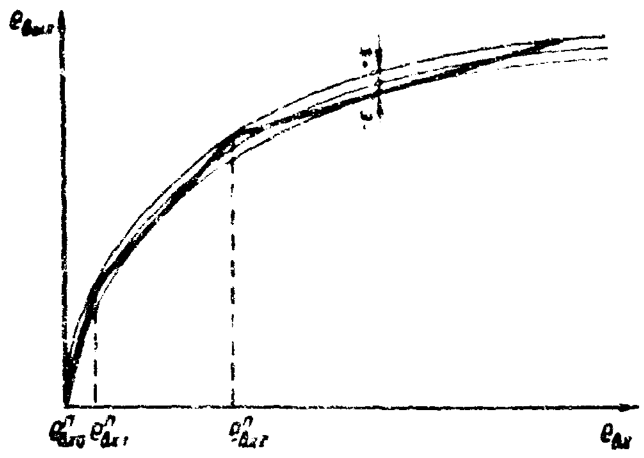

Fig. 86. Method of piecewise-linear approximation.

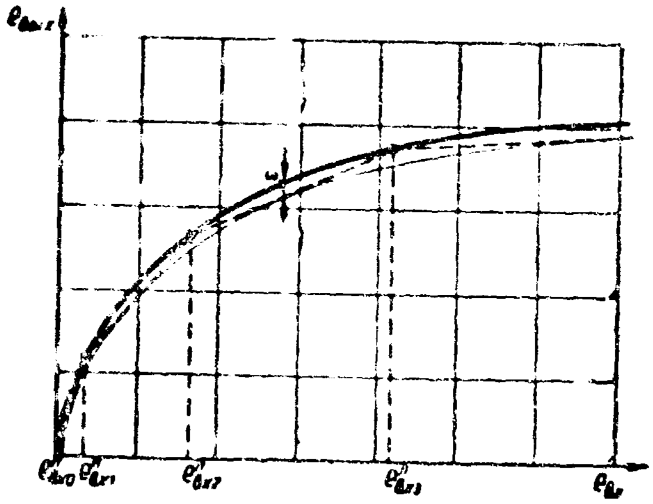

Fig. 87. An other method of piecewise-linear approximation.

2. Error of Diode Nonlinear Generators

As in study of linear computing elements, absolute error of nonlinear functional devices is determined as the difference of ideal and real values of output quantity, measured at the same moment of time during supplying to the device's input accordingly real and ideal values of the input signal:

$$\Delta e_{\text{вых}} = |e_{\text{вых}}(t)|_{р} - |e_{\text{вых}}(t)|_{и} \tag{6.14}$$

However the absolute error $\Delta e_{\text{вых}}$ obtained thus will depend on the value of $e_{\text{вых}}$ at which it is determined. Therefore, for appraisal of accuracy of a device here it is possible to use the maximum value of a particular practical criterion of accuracy:

$$[\delta e_{\text{вых}}]_{\max} = \frac{[\Delta e_{\text{out}}(e_{\text{вых}})]_{\max}}{e_{\text{вых}\,\max}}$$

or the integral practical criterion of accuracy (see Chapter III, page 85).

In connection with the fact that the circuit of diode converters clearly do not contain elements, able to store energy, and spurious capacitive and inductive resistances in the operating band strip of frequencies of these devices are negligible, further consideration is conducted exclusively in reference to error of the steady-state regime.

Depending upon the method of physical realization of the functional device this error can be caused by various factors. However, for most devices it can be divided into three main components:

1) error, caused by errors in input signal;

2) error, connected with the given method of representation of the given nonlinear function in the device (for example, error, introduced during piecewise-linear approximation in the case of diode generators);

3) error of reproduction of approximated nonlinear dependence.

Let us consider each component of error in reference to diode functional generators.

Error, caused by inaccuracy of input signal, can be estimated on the assumption that the generator ideally reproduces the given nonlinear dependence (no errors of approximation and reproduction of piecewise-linear curve).

With such consideration there are rejected errors of the second and higher orders of smallness and results turn out to be correct for any functional device. In fact, if there is given: $e_{\text{вых}} = v f(e_{\text{вх}})$, then

$$\Delta e_{\text{out}} = v \frac{df(e_{\text{вх}})}{de_{\text{вх}}} \Delta e_{\text{вх}}. \tag{6.15}$$

and

$$\delta e_{\text{вых}} = \frac{\Delta e_{\text{вых}}}{e_{\text{вых}\ \max}} = \delta e_{\text{вх}} \frac{d\left[\frac{f(e_{\text{вх}})}{f(e_{\text{вх}\ \max})}\right]}{d\left(\frac{e_{\text{вх}}}{e_{\text{вх}\ \max}}\right)}$$

or

$$\delta e_{\text{вых}} = S_e \delta e_{\text{вх}}. \tag{6.16}$$

where S_e is the steepness of the given function.

Thus, relative error, caused by inaccuracy of input signal, will be greater, the greater the steepness of the nonlinear dependence to be reproduced.

In case of use of a diode functional converter, for which a given nonlinear dependence is realized simultaneously by connecting a nonlinear resistor to the input and feedback circuit, we receive

$$\Delta e_{\text{вых}} = -\frac{\frac{dI_1}{de_{\text{вх}}}}{\frac{dI_2}{de_{\text{вых}}}} \Delta e_{\text{вх}}; \tag{6.17}$$

relative error is

$$\delta e_{\text{вых}} = -\frac{S_{I_1}}{S_{I_2}} \delta e_{\text{вх}}. \tag{6.18}$$

where $S_{I_1} = \frac{d\left(\frac{I_1}{I_{1\max}}\right)}{d\left(\frac{e_{\text{вх}}}{e_{\text{вх}\ \max}}\right)}$ — is the given steepness of the current characteristic of input circuit, $S_{I_1} = \frac{d\frac{I_2}{I_{2\max}}}{d\left(\frac{e_{\text{вых}}}{e_{\text{вых}\ \max}}\right)}$ — the given steepness of current characteristic of feedback circuit.

As follows from expression (6.18), relative error in output signal is greater, the greater the steepness of current characteristic of the input circuit and the less the steepness of the current characteristic of the feedback circuit. Since in feedback circuits there is realised a function, the reverse of the given one (and usually realisable at the input), then by simple transfer of the diode circuit into the feedback circuit we can not lower the error from inaccuracy in input signal.

During reproduction of nonlinear functions with value of steepness higher than 10 one should present rigid requirements on accuracy of the input signal, and consequently, to lowering of error of other computing elements of the installation. Thus, for example, when $S_\varepsilon = 10$ and $\delta e_{\text{вх}} = 1\%$ we obtain $\delta e_{\text{вых}} = 10\%$, which in a number of cases can be impermissible. Therefore, electronic analogs, intended for solution of nonlinear problems, should be, as a rule, supplied with operational amplifiers with automatic stabilization of zero level and in them should be thorough "ground insulation" (see Chapter IX, page 289), and provided other measures of increasing accuracy. Let us turn now to consideration of error of piecewise-linear approximation. Selection of the quantity of permissible error during piecewise-linear approximation is an independent and fairly complicated question. Here we will limit ourselves to only several remarks.

Practice of construction of specialized functional converters showed that for many conversions error of approximation $\varepsilon = 0.25$ v or $\delta\varepsilon = 0.25\%$ gives total error of the order of 0.75 — 1% during reproduction of a given function with steepness, not exceeding 10. Decrease of the quantity lower than the shown meaning leads to sharp increase of the required number of segments of the decomposition of the argument, and consequently, to increase of the number of diode elements and loss of total accuracy.

During reproduction of functions with steepness, exceeding 10, error due to inaccuracy of input signal becomes so great that it is possible without sacrifice ε of total accuracy of the device to increase the permissible value of ε and to receive thereby economy in the number of diode elements. It is rational to select by the relationship,

$$\delta\varepsilon = \frac{1}{4}\,\delta e_{\text{вх}}\,|S_\varepsilon|. \qquad (6.19)$$

Then when $\delta e_{\text{вх}} = 0.1\%$ and $S_\varepsilon = 10$ we will have $\delta\varepsilon = 0.25\%$.

Error of reproduction of approximated nonlinear dependence during coupling of circuits of diode elements to the input and in feedback circuits of an operational amplifier will be determined by error in value of currents, sent by these circuits to the integrating point of amplifier. Indeed, if given nonlinear dependence $e_{\text{вых}} = f(e_{\text{вх}})$ is reproduced by current characteristics $I_1 = f_1(e_{\text{вх}})$ and $I_2 = f_2(e_{\text{вых}})$, then in the worst case total error in output voltage will be

$$\Delta e_{\text{вых}} = \frac{de_{\text{вых}}}{dI_1}\Delta I_1 + \frac{de_{\text{вых}}}{dI_2}\Delta I_2. \qquad (6.20)$$

Here $\frac{de_{\text{вых}}}{dI_1}$ and $\frac{de_{\text{вых}}}{dI_2}$ have ideal meaning, given by piecewise-linear approximation. Including the error in these quantities would not change the result practically, since here there would be considered only error of the second order of smallness.

Relative error here will be

$$\delta e_{\text{вых}} = \frac{1}{S_{I_2}}(\delta I_1 + \delta I_2). \qquad (6.21)$$

since

$$S_e = \frac{de_{\text{вых}}}{de_{\text{вх}}} = \frac{S_{I_1}}{S_{I_2}}.$$

Thus, to explain total error of a functional generator from inaccurate reproduction of the approximated characteristic, it is necessary to know, besides steepness of current characteristic S_{I_2}, also relative errors in output currents of diode circuits, coupled to the input and feedback circuits. Since determination of error of diode circuits with respect to output current does not depend on the place of coupling of the diode circuit, then in the future it is conducted for circuits, switched into the input circuit. For circuits, switched into the feedback, it is possible to use results received, accordingly replacing the index for current and voltage.

Besides error, introduced by diode circuits, an integrating operational amplifier also introduces error from zero drift. During use of amplifiers with automatic stabilization of zero level this error as compared with others can be disregarded.

The main part of error in output current of diode circuits consists of systematic errors, caused by inaccuracy of selection of resistances of separate diode elements, change of current of incadescence, influence of internal resistance of diodes and their resting current,*), inaccuracy of setting of the quantity of reference voltage, influence of change of temperature.

Error of the circuit of a diode converter can be found, knowing errors introduced by separate diode elements, from which the circuit is composed. Therefore, we will conduct an analysis of error for every considered type of diode element separately.

As was shown earlier, current of a diode element with potentially grounded diode can be found from expression

$$I_{u}=\frac{e_{\text{вх}}-e_{\text{оп}}\frac{R_i}{r_i}}{R_i},$$

whence absolute error in current, created by one diode element will be

$$\Delta I_{u}=\left(\frac{\partial I_{u}}{\partial R_i}\right)_{R_i=(R_i)_{\text{н}}}\Delta R_i+\left(\frac{\partial I_{u}}{\partial r_i}\right)_{r_i=(r_i)_{\text{н}}}\Delta r_i+ \\ +\left(\frac{\partial I_{u}}{\partial e_{\text{оп}}}\right)_{e_{\text{оп}}=(e_{\text{оп}})_{\text{н}}}\Delta e_{\text{оп}}+I_{\text{н}}(e^{\text{н}}_{\text{вх}\,i}). \tag{6.22}$$

where $I_{\text{н}}(e^{\text{н}}_{\text{вх}\,i})$ is the resting current of the diode; ΔR_i, Δr_i, $\Delta e_{\text{оп}}$ — change of resistances R_i, r_i and reference voltage $e_{\text{оп}}$; $(R_i)_{\text{н}}$, $(r_i)_{\text{н}}$ — values of resistances, received from calculation of piecewise-linear approximation.

Proceeding to relative errors and expanding the value of expressions $\frac{\partial I_{u}}{\partial R_i}$, $\frac{\partial I_{u}}{\partial r_i}$ and $\frac{\partial I_{u}}{\partial e_{\text{оп}}}$, we receive

$$\delta I_{u}=-\frac{(e_{\text{вх}})_{\text{н}}}{(R_i)_{\text{н}}I_{i\,\max}}\delta R_i+\frac{(e_{\text{оп}})_{\text{н}}}{(r_i)_{\text{н}}I_{i\,\max}}\delta r_i-\frac{(e_{\text{оп}})_{\text{н}}}{(r_i)_{\text{н}}I_{i\,\max}}\delta e_{\text{оп}}+\frac{I_{\text{н}}}{I_{i\,\max}}. \tag{6.23}$$

---

*Resting current is the current, flowing through diode during zero difference of potentials between plate and cathode.

Relative error of resistances R_i and r_i in turn can be presented as consisting of three components: one, determined by inaccuracy of selection of resistances $(\delta R_{in}, \delta r_{in})$, the second, caused by change of resistance of diode during change of current flowing through it and oscillations of current of incadescence $(\delta R_{i\text{д}}, \delta r_{i\text{д}})$, and, finally, third, caused by change of resistances under the influence of change o' ambient temperature $(\delta R_{i\theta}, \delta r_{i\theta})$.

Therefore,

$$\delta R_i = \delta R_{in} + \delta R_{i\text{д}} + \delta R_{i\theta}, \quad \delta r_i = \delta r_{in} + \delta r_{i\text{д}} + \delta r_{i\theta}.$$

Quantities δR_{in} and δr_{in} are determined by allowance for selection of resistances. During use of resistors VS and MLT, having factory allowance ± 10%, usually selection is carried out with accuracy of 0.25%, applying for decrease of error compound resistors (two). Resistors R_i and r_i are best select with allowance of one sign. In accordance with expression (6.23) this should give certain decrease of total error.

Error due to change of ambient temperature can be determined from expression

$$\delta R_{i\theta} = \delta r_{i\theta} = \alpha(\theta - 20^\circ). \tag{6.24}$$

where α is the temperature coefficient, equal for VS and MLT to $\sim 2 \cdot 10^{-3} \frac{1}{\ }$ in the interval θ from + 20 to + 100°C.

In spite of comparatively large α influence of change of temperature from 20 to 60° C is small, since changes of $\delta R_{i\theta}$ are compensated partially by change of $\delta r_{i\theta}$ and, furthermore, resistance in feedback circuits changes in the same relation, so that with changing input current current of the feedback circuit, also changes, and voltage at output changes little.

The most essential component of the considered error is caused by change of resistance of the diods during operation. This component of error can be found from relationship, brought on page 154, in the form

$$\delta R_{i\text{д}} = \delta r_{i\text{д}} = \Delta r_\text{д}\left(\frac{1}{r_{in}} + \frac{1}{R_{in}}\right).$$

where $\Delta r_{\text{д}}$ is the maximum deflection of resistance of diode from the value of $r_{\text{д}} = 600$ ом taken for calculations.

Considering that

$$r_{i\text{н}} = R_{i\text{н}} \frac{e_{\text{оп}}}{e^{n}_{\text{вх}\,i}},$$

we receive

$$\delta R_{i\text{д}} = \delta r_{i\text{д}} = \frac{\Delta r_{\text{д}}}{R_{i\text{н}}} \left(\frac{e^{n}_{\text{вх}\,i}}{e_{\text{оп}}} + 1 \right). \tag{6.25}$$

In Fig. 88 are brought the curves of

$$\delta R_{i\text{д}} = \delta r_{i\text{д}} = f(R_{i\text{н}})$$

where $\Delta r_{\text{д}}$ = 200 ohm and various relationships $\frac{e^{n}_{\text{вх}\,i}}{e_{\text{оп}}}$.

From the curves it follows that for considered diode elements error $\delta R_{i\text{д}}$ and $\delta r_{i\text{д}}$ grows with increase of $e^{n}_{\text{вх}\,i}$ and when $R_{i\text{н}}$ = 200 kilohm it can attain magnitudes of the order of 0.4 — 0.5%.

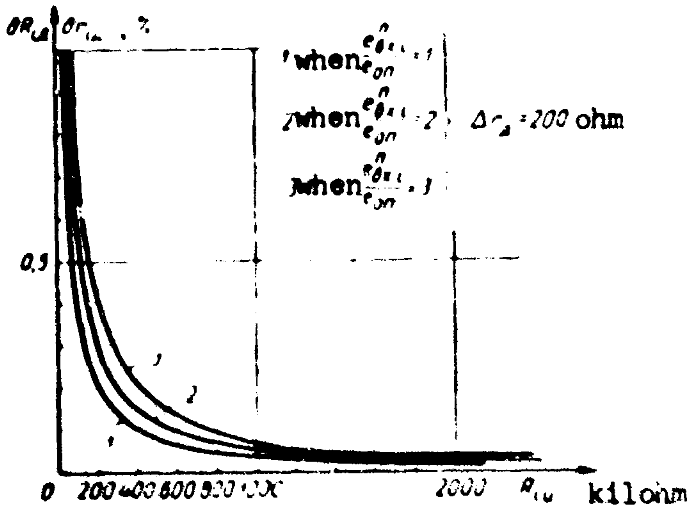

Fig. 88. Curves of change of errors $R_{i\text{д}}$ and $r_{i\text{д}}$ depending upon $R_{i\text{н}}$.

Error due to change of magnitude of reference voltage is proportional to relative changes of this voltage. Therefore, accuracy of operation of a stabilised rectifier for reference voltage should be not lower than 0.1 — 0.05%.

Resting current is developed at the moment of unlocking of the diode element and usually leads to lowering of jumps of the first derivative at the moment of switching.

It was experimentally fixed that during change of R_i and r_i from 50 kilohm to 1 megohm and filament voltage from 4 to 6 the resting current for diodes of type 6D6 oscillates from 1 to 8 microamperes.

Taking into account the remarks made and disregarding the resting current for the worst case, where signs of δR_i and $\delta e_{\text{оп}}$ are opposite to sign δr_i we receive for maximum relative error of the diode element the final expression

$$\delta I_i = \frac{e_{\text{вх max}}}{I_{1\,\text{max}} R_{i\text{н}}} \left[|\delta R_{i\text{н}}| + |\delta R_{i\text{в}}| \quad \frac{|\Delta r_2|}{R_{i\text{н}}} \left(\frac{e^{\text{н}}_{\text{вх}\,i}}{e_{\text{оп}}} + 1 \right) \right] + \\ + \frac{e^{\text{н}}_{\text{вх}\,i}}{I_{1\,\text{max}} R_{i\text{н}}} \left[|\delta r_{i\text{н}}| + |\delta r_{i\text{в}}| + \frac{|\Delta r_2|}{R_{i\text{н}}} \left(\frac{e^{\text{н}}_{\text{вх}\,i}}{e_{\text{оп}}} + 1 \right) \right] + \\ + \frac{e^{\text{н}}_{\text{вх}\,i}}{I_{1\,\text{max}} R_{i\text{н}}} |\delta e_{\text{оп}}| \tag{6.26}$$

From expression (6.26) it follows that relative error in current of diode element increases with increase of $e_{\text{вх max}}, e^{\text{н}}_{\text{вх}\,i}$, the steepness of current characteristic $\Delta Y_i - \frac{1}{R_i}$ of the considered diode element and change of resistance of the diode Δr_i relative to average calculating value.

Results of determination of error for other diode elements, received, by the above-stated method, are given in Table IV.

Comparison of these results allows us to make the following conclusions:

1. The influence of internal resistance of diodes and inaccuracy of selection of resistances for diode elements of limiter type, assembled by diagram II, is significantly less than for diode elements with potentially grounded diodes. With increase of $e^{\text{н}}_{\text{вх}\,i}$ other conditions equal, error from r_i decreases, whole for diode elements with potentially grounded diodes it increases. Influence of inaccuracy of setting of $e_{\text{оп}}$ shows only during the turned off state of a diode element of limiter type. In connection with this it is expedient during construction of circuits for $e_{\text{вх}} > e_{\text{оп}}$ to use diode elements of limiter type, assembled by diagram II.

Table IV

Type of diode element	Expression for current at integrating point	State of diode element	Expression for maximum relative error of current at integrating point	Note
1. Diode element with potentially grounded diode	$I_{1l} = \dfrac{e_{ex} - \dfrac{R_l}{r_l} e_{on}}{R_l}$	On	$\delta I_{1l} = \dfrac{e_{ex.\,max}}{I_{1\,max} R_{l\,n}} \lvert \delta R_l \rvert + \dfrac{e^{n}_{ex\,l}}{I_{1\,max} R_{l\,n}} \lvert \delta r_l \rvert + \dfrac{e^{n}_{ex\,l}}{I_{1\,max} R_{l\,n}} \lvert \delta e_{on} \rvert$	Influence of resistance of diodes considered in δR_l and δr_l
	$I_{1l} = 0$	Off		
2. Diode element of limiter type by diagram I operating on switching off	$I_{1l} = \dfrac{e_{on} \dfrac{r_s}{r_l} + e_{ex}}{r_s + \dfrac{r_s}{r_l} R_l + R_l}$	On	$\delta I_{1l} = \dfrac{e_{ex.\,max}}{I_{1\,max} R_{l\,n}} \lvert \delta R_l \rvert + \dfrac{\dfrac{e_{on\,n}}{e^{n}_{ex\,l}} \left[e_{ex.\,max} - e^{n}_{ex\,l} \right] \left(\dfrac{e_{on\,n}}{e^{n}_{ex\,l}} - 1 \right)}{I_{1\,max} r^{2}_{l\,n}} r_s$	For diode element of limiter type, working on switching on by diagram I (when $e_{..} < e_{..}$) expressions for current and for error will be the same
	$I_{1l} = \dfrac{e_{on}}{r_l + R_l}$	Off	$\delta I_{1l} = \dfrac{e_{on\,n} R_{l\,n}}{(r_{l\,n} + R_{l\,n})^2 I_{1\,max}} \lvert \delta R_l \rvert + \dfrac{e_{on\,n} r_{l\,n}}{(r_{l\,n} + R_{l\,n})^2 I_{1\,max}} \lvert \delta r_l \rvert + \dfrac{e_{on\,n}}{(r_{l\,n} + R_{l\,n}) I_{1\,max}} \lvert \delta e_{on} \rvert$	
3. Diode element of limiter type by diagram II operating on switching off	$I_{1l} = \dfrac{e_{ex}}{R_{1l} + R_{2l}}$	On	$\delta I_{1l} = \dfrac{e_{ex} (R_{1l})_n}{I_{1\,max} (R_{1l} + R_{2l})^2_n} \lvert \delta R_{1l} \rvert + \dfrac{e_{ex} (R_{2l})_n}{I_{1\,max} (R_{1l} + R_{2l})^2_n} \lvert \delta R_{2l} \rvert$	For diode element of limiter type, working on switching on by diagram II, expressions for current and for error will be the same.
	$I_{1l} = \dfrac{e_{on} + \dfrac{r_s}{R_{1l}} e_{ex}}{r_s + r_s \dfrac{R_{2l}}{R_{1l}} + R_{2l}}$	Off	$\delta I_{1l} = \dfrac{(e_{on})_n}{I_{1\,max} R_{2l}} \lvert \delta e_{on} \rvert + \dfrac{(e_{on})_n}{I_{1\,max} (R_{2l})_n} \lvert \delta R_{2l} \rvert + \dfrac{\lvert e_{ex.\,max} - e^{n}_{ex\,l} \rvert r_s}{\left(\dfrac{e^{n}_{ex\,l}}{(e_{on})_n} - 1 \right) I_{1\,max} (R_{2l})^2_n}$	

2. Diode elements of limiter type, working by diagram I, do not give essential advantages with respect to lowering of error when compared with diode elements with potentially grounded diodes.

3. For all circuits without exception error increases with growth of the given steepness of the current characteristic of the diode element.

Since total current of the diode circuit, send to the integrating point, is equal to the sum of currents of separate diode elements, then total error of the diode circuit can be calculated for the worst case as the sum of errors, introduced in the current by every diode element. Thus, for example, for diode circuit (Fig. 85a), consisting of diode elements with potentially grounded diodes, total relative error will be maximum, when all diode elements are switched on:

$$\delta I_1 = \sum_{1}^{7} \frac{e_{ex\ max}}{I_{1\ max} R_{in}} |\delta R_i| + \sum_{1}^{7} \frac{e^n_{ex\ i}}{I_{1\ max} R_{in}} |\delta_{ri}| + \sum_{1} \frac{e^n_{ex\ i}}{I_{1\ max} R_{in}} |\delta e_{in}|$$

When in the circuit there are used diode elements of limiter type, then it is necessary to consider error, introduced by diode elements, in both on and off state. It is necessary to seek to build a circuit of nonlinear converters, so that the number of simultaneously "on" diode elements is minimum.

Total error of a functional device can be calculated by expression (6.21) and values of δI_1 and δI_2 found as shown.

3. Methods of Construction of Circuits of Diode Functional Generators

As was shown above (see Chapter V, Section 3, page 156), parallel connection of various diode elements does not ensure possibility of reproduction of a sufficiently broad class of functions. For the purpose of expansion of possibilities of diode circuits it is necessary to apply artificial methods, for example shift of current characteristics by connection at the integrating point of an additional constant current component, summation of current characteristics of separate diode circuits and so forth. However often, for reproduction of steep functions one must have current characteristics with great steepness and increase the number of simultaneously working diode elements which unfavorably affects accuracy of work of the considered devices.

We will estimate the limit of the given steepness of current characteristics. Taking as base quantities $i_{1\,\max} = i_{2\,\max} = 1$ ма, $e_{\text{вх}\,\max} = e_{\text{вых}\,\max}$ = 100 v and limiting limits of resistances in circuits of input and feedback to quantities 10 kilohm < R < 5 megohm, we receive

$$0,02 < S < 10.$$

When in input or feedback circuits there is connected a constant resistance, the steepness accordingly will be

$$S_{I_1} = 1, \quad S_{I_2} = 1.$$

When the diode circuit is connected only at the input, $S_{I_2} = 1$ and

$$S_e = -S_{I_1}.$$

i.e., steepness of current characteristic of diode circuit should be equal to the steepness, given for reproduction of the function.

Since steepness of current characteristic of diode circuit by conditions of physical realization is limited as indicated above, then the class of functions, reproduced by such functional device, will be limited by these limits of change of steepness.

During connection of the diode circuit only in the feedback circuit

$$S_{I_2} = \frac{1}{S_e}, \quad S_{I_1} = -1.$$

and the class of reproduced functions will be limited by conditions

$$S_{I_2} > 0 \quad \text{и} \quad 0.02 < S_{I_2} < 10.$$

Therefore, such functions, as, for example, $e_{\text{вых}} = \frac{10}{e_{\text{вх}}}$. which within limits of change 0.1 v $< e_{\text{вх}} <$ 100 v give change of steepness $-1000 < S_e < -0.001$, cannot be in general, reproduced with such a method of connection of diode circuits.

It is possible to offer a number of methods of synthesis of diode elements, allowing us to reproduce a given nonlinear dependence with steepness, varying in a wide range. These methods allow us to make diode converters in such a manner that steepness of current characteritics of separate diode circuits does not go beyond permissible limits and that it is provided by the least number of simultaneously switched-on diode elements. They are based on the earlier derived dependence (6.18), which can be written in the form

$$S_e = -\frac{S_{I_1}}{S_{I_2}}. \qquad (6.27)$$

Quantity S_e and the characrer of its change in functions of the input signal usually are given by the initial nonlinear dependence and the problem consists in such distribution of quantity S_e between S_{I_1} and S_{I_2}, so that each of them does not exceed permissible limits. During simultaneous connection of diode circuits in input and feedbacks steepness of the given function S_e, as expression (6.27) shows, can be realized by S_{I_1} and S_{I_2}, where each of them will change in a narrower range.

We will show in the example of reproduction of function $e_{вых} = \frac{10}{e_{вх}}$ the method of determination of current characteristics for circuits of nonlinear converters, utilized simultaneously at the input and in feedback circuits.

Characteristic of steepness of the function for the considered case will be

$$S_e = -\frac{10}{e_{вх}^2} = -\frac{S_{I_1}}{S_{I_2}}.$$

For the purpose of decreasing steepness S_{I_1} and S_{I_2} as compared with S_e we will decompose expression S_e into factors. Such decomposition can be very diverse. Rejecting decompositions leading to increase of the degree of $e_{вх}$ we receive a number of possible variants:

$$\frac{10}{e_{вх}^2} = \frac{1}{e_{вх}}\,\frac{10}{e_{вх}}, \quad \frac{10}{e_{вх}^2} = \frac{1}{e_{вх}\sqrt{e_{вх}}}\,\frac{10}{\sqrt{e_{вх}}}, \quad \frac{10}{e_{вх}^2} = \frac{1}{e_{вх}\sqrt[3]{e_{вх}}}\,\frac{10}{\sqrt[3]{e_{вх}^2}}.$$

Hence expressions for steepness of current characteristics accordingly will be:

$$1.\ S_{I_1} = \frac{1}{e_{вх}}, \qquad S_{I_2} = \frac{1}{\frac{10}{e_{вх}}} = \frac{1}{e_{вых}}.$$

$$2.\ S_{I_1} = \frac{1}{e_{вх}\sqrt{e_{вх}}}, \qquad S_{I_2} = \frac{1}{\frac{10}{\sqrt{e_{вх}}}} = \frac{1}{\sqrt{10 e_{вых}}}.$$

$$3.\ S_{I_1} = \frac{1}{e_{вх}\sqrt[3]{e_{вх}}}, \qquad S_{I_2} = \frac{\sqrt[3]{e_{вх}^2}}{10} = \frac{1}{\sqrt[3]{10 e_{вых}^2}}.$$

Current characteristics can be found by integration of expression S_{I_1} and S_{I_2}. Indeed, when $I_{1\,max} = I_{2\,max} = 1$ ma and $e_{вх.\,макс} = e_{вых.\,макс.} = 100$ v:

$$I_1 = 0.01 \int_{e_{вх.\,н}}^{e_{вх}} S_{I_1}\, de_{вх}\ [ма]. \tag{6.28}$$

$$I_2 = 0.01 \int_{e_{вых.\,н}}^{e_{вых}} S_{I_2}\, de_{вых}\ [ма]. \tag{6.29}$$

Here $e_{вх.\,н}$ and $e_{вых.\,н}$ are initial values of voltages of considered sections of current characteristics.

Values of S_{I_1}, S_{I_2}, I_1 and I_2 and limits of change of S_{I_1} and S_{I_2} with change of $e_{вх}$ and $e_{вых}$ in the interval from 0.1 to 100 v are brought in Table V.

Comparison of results, brought in the table, shows that in all three cases limits of change of steepness of S_{I_1} and S_{I_2} are significantly narrower than limits of change of steepness S_e of the initial nonlinear characteristic. Such an approach actually leads to functional generation of initial characteristics, indicating at the same time a regular way of finding the form of the generation.

First variant of generation of current characteristics one should recognize as the most rational, since it allows us to use identical circuits of nonlinear converters at the input and in feedback circuits.

On the basis of expression (6.27) there can be offered another method of reproduction of a given function with a wide range of change of steepness, with which the given steepness of the current characteristics of the diode circuits does not exceed one. Essence of the method consists of the fact that the given function

Table V

	Expressions for steepness		Limits of change of steepness with change of $e_{вх}$ and $e_{вых}$ from 0.1 to 100 v		Expressions for current characteristics	
	S_{I_1}	S_{I_2}	S_{I_1}	S_{I_2}	I_1 (ма)	I_2 (ма)
1	$\frac{1}{e_{вх}}$	$\frac{1}{e_{вых}}$	from 10 to 0,01	from 10 to 0,01	$I_1 = 0{,}01\,[\ln e_{вх} - \ln e_{вх\cdot н}]$	$I_2 = 0{,}01\,[\ln e_{вых} - \ln e_{вых\cdot н}]$
2	$\frac{1}{e_{вх}\sqrt{e_{вх}}}$	$\frac{1}{\sqrt{10 e_{вых}}}$	from $10\sqrt{10}$ to 0,001	from 1 to $\frac{1}{10\sqrt{10}}$	$I_1 = -0{,}02\left[\frac{1}{\sqrt{e_{вх}}} - \frac{1}{\sqrt{0{,}1}}\right]$	$I_2 = \frac{0{,}02}{\sqrt{10}}\,[\sqrt{e_{вых}} - \sqrt{0{,}1}]$
3	$\frac{1}{e_{вх}\sqrt[3]{e_{вх}}}$	$\frac{1}{\sqrt[3]{10 e^2_{вых}}}$	from $10\sqrt{10}$ to $\frac{1}{100\sqrt{100}}$	from $\sqrt[3]{10}$ to $\frac{1}{10\sqrt[3]{100}}$	$I_1 = -0{,}03\left[\frac{1}{\sqrt[3]{e_{вх}}} - \frac{1}{\sqrt[3]{0{,}1}}\right]$	$I_2 = \frac{0{,}03}{\sqrt[3]{10}}\,[\sqrt[3]{e_{вых}} - \sqrt[3]{0{,}1}]$

by steepness is broken down into two sections; in general on one the steepness will change from 0 to 1 and on the second from 1 to

Reproduction of the first section of the curve one should place in the diode circuit, connected at the input, the second section, on the diode circuit, coupled in the feedback circuit. If one were now to make these circuits work in that sequence, with which change of steepness of the current characteristic at the input occurs with constant steepness of the feedback circuit (equal to 1) and, vice versa, change of steepness of the current characteristic, coupled in the feedback circuit, is carried out with steepness of the input current characteristic constant and equal to 1, then this task will be carried out.

As an illustration let us consider reproduction of function $e_{\text{вых}} = 10^{-4} e_{\text{вх}}^3$. Steepness of the function will be

$$S_e = \frac{de_{\text{вых}}}{de_{\text{вх}}} = 3 \cdot 10^{-4} e_{\text{вх}}^2 = \frac{S_{I_2}}{S_{I_1}}.$$

S_e reaches one when $e_{\text{вх}} = \sqrt{\frac{10^4}{3}} = 57.8$ v.

Consequently, up to $e_{\text{вх}} = 57.8$ v the given curve should be reproduced by diode converters reformers with current characteristics

$$I_1 = -\nu_1 \cdot 10^{-4} e_{\text{вх}}^3 \qquad (6.30)$$

and

$$I_2 = \nu_2 e_{\text{вых}}, \qquad (6.31)$$

wherein $\nu_1 = \nu_2$

Quantity $\nu_2 = \nu_1$ we determine from the condition that on the considered interval of change of argument $S_{I_2} = 100 \cdot \frac{dI_2}{de_{\text{вых}}} = 1$. Since $\frac{dI_2}{de_{\text{вых}}} = \nu_2$, then $\nu_2 = \nu_1 = 0.01$. On the boundary of the first section $I_{1\,\text{гран}} = -0.192$ ма, and $I_{2\,\text{гран}} = 0.192$ ма. On the remaining interval of change of argument, i.e., from $e_{\text{вх}} = 57.8$ v to 100 v, $S_{I_1} = -1$, and $S_{I_2} = \frac{1}{S_e}$.

Current characteristics on this interval of change of the argument can be found by integration of corresponding characteristics of change of steepness.

Indeed, $S_{I_1} = 100 \quad \frac{dI_1}{de_{\text{вх}}} = -1$, whence

$$I_1 = -[I_{1\,\text{гран}} + 0{,}01\,(e_{\text{вх}} - e_{\text{вх. гран}})]. \tag{6.32}$$

Since

$$S_{I_2} = \frac{1}{S_e} = \frac{1}{3\cdot 10^{-4} e_{\text{вх}}^2} = \frac{1}{0{,}3\sqrt[3]{0{,}1 e_{\text{вых}}^2}},$$

then

$$I_2 = I_{2\,\text{гран}} + \sqrt[3]{0{,}01 e_{\text{вых}}} - \frac{e_{\text{вх. гран}}}{100}. \tag{6.33}$$

The graph of current characteristics received thus is shown in Fig. 89. Knowledge of the law of change of current characteristics and their steepness allows one most rationally to select types of diode elements. For a circuit, connected at the input, it is rational to use diode elements with potentially grounded diodes, working on switching on. After $e_{\text{вх}} = e_{\text{вх. гран}}$ all diode elements of the input circuit should be switched on and should provide the given steepness of the input current characteristic, equal to 1. In feedback circuits it is rational to use diode elements of limiter type, working on switching off by diagram II. When $e_{\text{вх}} < e_{\text{вх. гран}}$ all diode elements are connected in the feedback circuit and provide a steepness $S_{I_2} = 1$. By measure of increase of $e_{\text{вых}}$ diode elements are consecutively turned off.

A complete fundamental circuit of the considered nonlinear generator is shown in Fig. 90. In it is foreseen the possibility of work with two signs of input and output signals.

The number of diode elements and their parameters are determined on the basis of piecewise-linear approximation.

In certain cases lowering of the steepness of current characteristics of diode generators, coupled only to input, can be carried out also by preliminary shift of the curve of differential conductance a constant quantity and connection to the input signal of the equivalent constant compensating conductance.

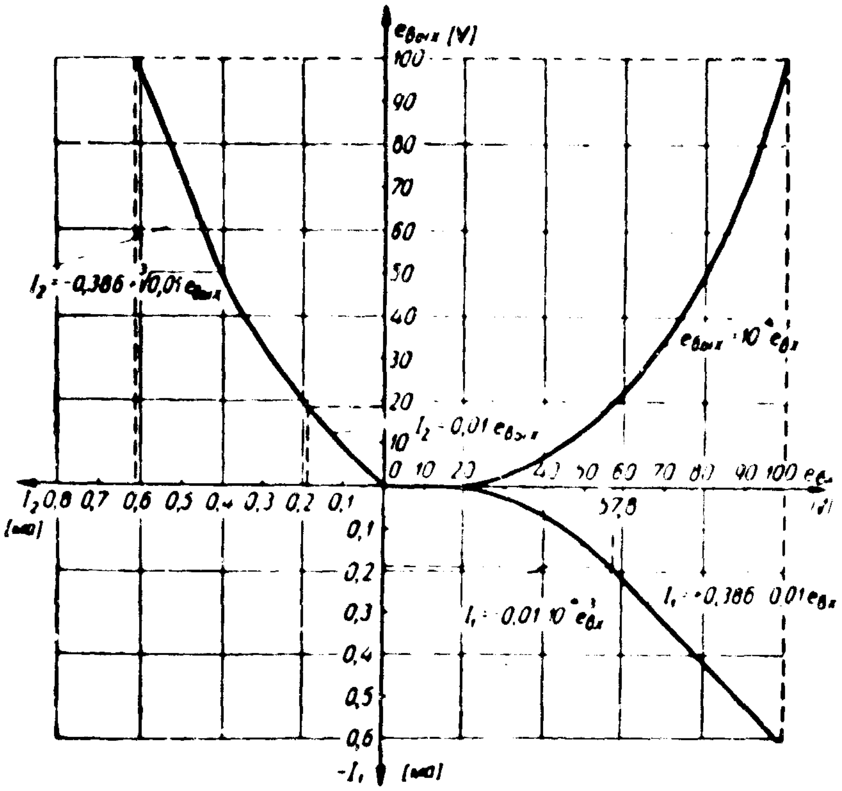

Fig. 89. Current characteristics of the nonlinear generator of $e_{вых} = 10^{-4}\cdot e_{вх}^3$

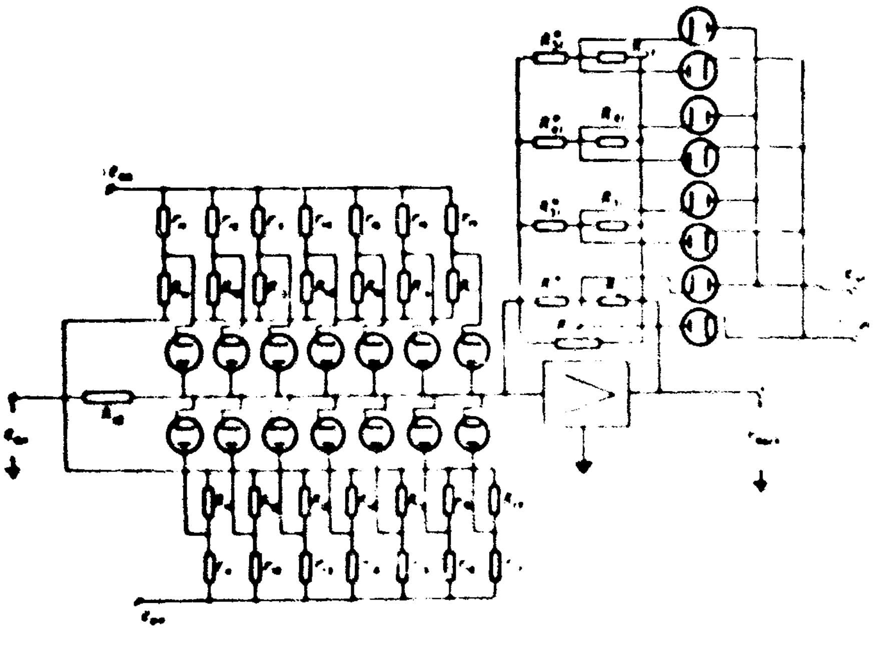

Fig. 90. Fundamental circuit of nonlinear generator of $e_{вых} = 10^{-4}\cdot e_{вх}^3$.

Let us imagine steepness S_{I_1} as consisting of sum

$$S_{I_1} = S_{I_{1н}} + S_{I_{1в}}. \qquad (6.34)$$

Then

$$S_{I_u} = S_{I_1} - S_{I_2}.$$

Thus, if $S_e S_{I_2}$ does not change sign on the whole interval of change of the argument, then steepness $|S_{I_u}| < |S_e S_{I_1}|$ Such a shift of the characteristic of differential steepness of a constant quantity is equivalent to turn of the current characteristic a certain angle. It is obvious that the greatest lowering of steepness when $S_{I_2} = 1$ constitutes $\frac{S_e}{2}$. Therefore, by such method one can obtain current characteristics with steepness, not exceeding one, only for a quadratic function.

Considered methods of lowering steepness of current characteristics remain in force also during reproduction of nonmontonic functions. Of specific is reproduction of a trigonometric function of the form

$$e_{вых} = 100 \sin \frac{\pi}{100} e_{вх}.$$

Characteristic for steepness of this function will have the form

$$S_e = \pi \cos \frac{\pi}{100} e_{вх}. \tag{6.35}$$

Steepness will be positive in the interval $0 < e_{вх} < 50$ v and negative when 50 v $< e_{вх} < 100$ v.

According to (6.27) we obtain

$$\pi \cos \frac{\pi}{100} e_{вх} = -\frac{S_{I_1}}{S_{I_2}}. \tag{6.36}$$

As expression (6.36) shows, reproduction of this function can be carried out both by connecting a nonlinear converter to input, and also in the feedback circuit.

In the first case

$$\left.\begin{aligned} S_{I_1} &= -\pi \cos \frac{\pi}{100} e_{вх}, \\ S_{I_2} &= 1. \end{aligned}\right\} \tag{6.37}$$

Here the maximum given steepness of the diode circuit at input will constitute $S_{I_1 \max} = \pi$.

In the second case characteristic for steepness S_{I_2}

$$S_{I_2} = \frac{-S_{I_1}}{\pi \cos \frac{\pi}{100} e_{вх}} = \frac{-S_{I_1}}{\pi \sqrt{1 - \left(\frac{e_{вых}}{100}\right)^2}}$$

is decomposed depending upon the value of the voltage of the argument on the three sections (offered by A. A. Maslov).

when

$$0 < e_{вх} < 47.1\ \text{v} \qquad S_{I_1} = -0.8.$$

$$0 < e_{вых} < 99.6\ \text{v} \qquad S_{I_2} = \frac{0.8}{\pi \sqrt{1 - \left(\frac{e_{вых}}{100}\right)^2}}.$$

$$47.1 < e_{вх} < 52.9\ \text{v} \qquad S_{I_1} = 0.$$

$$99.6 < e_{вых} < 100\ \text{v} \qquad S_{I_2} = S_{I_2\,max} = \frac{0.8}{\pi \sqrt{1 - \left(\frac{99.6}{100}\right)^2}} \approx 3.$$

$$52.9 < e_{вх} < 100\ \text{v} \qquad S_{I_1} = 0.8.$$

$$0 < e_{вых} < 99.6\ \text{v} \qquad S_{I_2} = \frac{0.8}{\pi \sqrt{1 - \left(\frac{e_{вых}}{100}\right)^2}}.$$

Introduction of average interval is caused by the fact that steepness of initial nonlinear characteristic when $e_{вх} = 50$ v turns into zero. It is expedient that S_e turns into zero not due to S_{I_2} seeking infinity, but due to passage to $S_{I_1} = 0$.

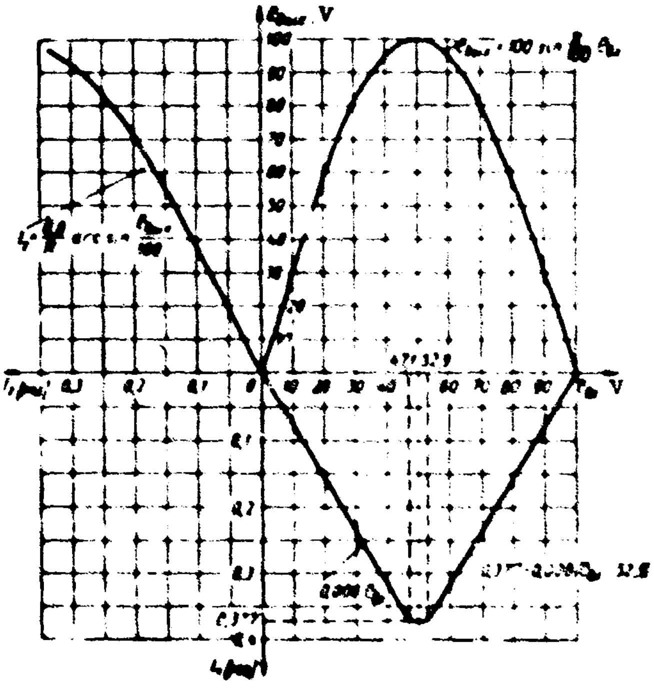

Fig. 91. Graph of current characteristics of functional generator for reproduction of function $e_{вых} = 100 \sin \frac{\pi}{100} e_{вх}$.

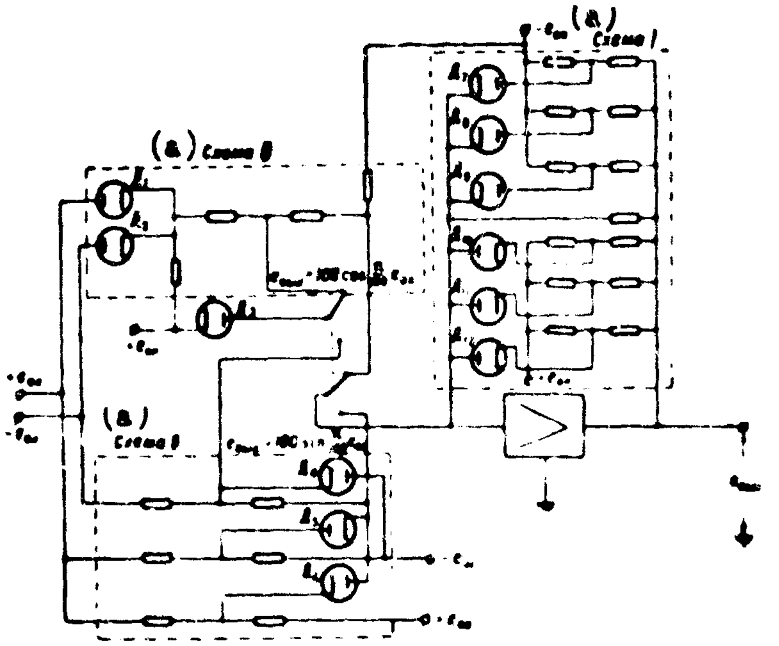
(a) Схема I
(a) Схема II
(a) Схема II
$e_{вых_1} = 100\cos\frac{\pi}{100}e_{вх}$
$e_{вых_2} = 100\sin\frac{\pi}{100}e_{вх}$
$+e_{вх}$
$-e_{вх}$
$e_{вых}$

Fig. 92. Fundamental circuit of double functional generator for reproduction of functions $e_{вых} = 100\sin\frac{\pi}{100}e_{вх}$ and $e_{вых} = 100\cos\frac{\pi}{100}e_{вх}$.
KEY: (a) Circuit.

Current characteristics, found by integration of above mentioned expressions for S_{I_1} and S_{I_2}, will be:

$$I_2 = \frac{0.8}{\pi}\arcsin\frac{e_{вых}}{100}; \tag{6.38}$$

$$I_1 = \begin{cases} -0{,}008e_{вх} & \text{when } 0 < e_{вх} < 47.1, \\ -0{,}377e_{вх} & \text{when } 47.1 < e_{вх} < 52.9, \\ -0{,}377 + 0{,}008(e_{вх} - 52.9) & \text{when } 52.9 < e_{вх} < 100 \end{cases} \tag{6.39}$$

The graph of these characteristics is shown in Fig. 91.

To obtain function $e_{вых} = 100\cos\frac{\pi}{100}e_{вх}$ it is possible to use the same current characteristic I_2. Current characteristic of input circuit should be to the left 50 v. The fundamental circuit fo functional converter, intended for reproduction of these functions, is presented in Fig. 92 (V. A. Trapeznikov, B. Ya. Kogan, V. V. Gurov, A. A. Maslov [1]). Here circuit I (built on diode elements with potentially grounded diodes, working on switching on) reproduces current characteristic I_2, and circuits II and III reproduce current characteristics I_1, correspondingly for reproduction of function

$$e_{out} = 100 \sin \frac{\pi}{100} e_{in},$$

$$e_{out} = 100 \cos \frac{\pi}{100} e_{in}.$$

In circuits I and II are applied double diode elements, where in circuit II for construction of double diode elements are used diodes which commutate sign. Diode is common for circuits II and III.

4. Diode Functional Generators of Two or More Arguments

The necessity of reproduction of functions of two or more arguments is usually met during solution of various ballistic problems, investigations of systems of control of flying objects, for which coefficients of moments, lift and lateral aerodynamic forces are functions simultaneously of angular displacements and Mach numbers, in complicated interconnected systems of automatic control of industrial processes, in systems of automatic adjustment and control, designed for maintaining an optimum regime.

Functions of two variables can be directly reproduced by three-dimensional cams (N. Ye. Kobrinskiy [1]) or their electric analogs (H. F. Meissinger [1]). However, application of these devices usually requires realization of mechanical displacements which causes sharp reduction of the passband. Their manufacture is extraordinarily labor-consuming and is connected with great expense.

Functions of three and more variables generally cannot be reproduced in such a manner. Therefore, we usually resort even in the case of a function of two variables to various methods of approximation with the help of combinations of functions of only one variable.

For functions, given in table form, it is possible to use decomposition (E. W. Pike and T. R. Silverberg [1]) of the form

$$f(x_i, y_j) = g(x_i) + h(y_j) + g'(x_i)h'(y_j) + g''(x_i)h''(y_j) + \ldots \qquad (6.40)$$

Here it is assumed that the mean value of function $f(x_i, y_j)$ equals zero. If the mean value o. the function does not equal zero, then it can always be brought to such a form, subtracting from every tabular value of the function its mea.. value $F_0 = \frac{1}{mn}\sum_{i}^{m}\sum_{j}^{n} F(x_i, y_j)$ of arguments x_i, y_j it is assumed that they are connected only by operations of addition and subtraction. With these assumptions series (6.40) turns out to be convergent.

The desired functions of the expansion are determined from the condition that the sum of squares of errors of approximation is minimum. When approximation is conducted only by the sum of functions

$$f(x, y_j) = g(x_i) + h(y_j).$$

solution of this variational problem leads to the following expressions for $g(x_i)$ and $h(y_j)$:

$$\left.\begin{aligned} g(x_i) &= \frac{1}{n}\sum_{j}^{n} f(x_i, y_j). \\ h(y_j) &= \frac{1}{m}\sum_{i}^{m} f(x_i, y_j). \end{aligned}\right\} \qquad (6.41)$$

where n is the number of values of y in the table, m is the number of values of x in the table.

In case of use of more complete decomposition

$$f(x_i, y_j) = g(x_i) + h(y_j) + g'(x_i)\,h'(y_j) + \ldots \qquad (6.42)$$

values of functions $g'(x_i)$ and $h'(y_j)$ are determined by solution by successive approximations of the system of equations

$$g'(x_i) = \frac{\sum_{j}^{n} R(x_i, y_j)\,h'(y_j)}{\sum_{j}^{n} [h'(y_j)]^2}, \quad h'(y_j) = \frac{\sum_{i}^{m} R(x_i, y_j)\,g'(x_i)}{\sum_{i}^{m} [g'(x_i)]^2}. \qquad (6.43)$$

where

$$R(x_i, y_j) = f(x_i, y_j) - [g(x_i) + h(y_j)].$$

System of equations (6.43) is a result of minimizing the sum of the squares of errors during approximation for $R(x_i, y_j)$ with the help of

$$g'(x_i)\,h'(y_j).$$

Designating error of this approximation by

$$R'(x_i, y_j) = R(x_i, y_j) - g'(x_i)\,h'(y_j),$$

it is possible to approximate it in turn by product $g''(x_i)\ h''(y_j)$, and find function $g''(x_i)$ and $h''(y_i)$ by minimization of the sum of the squares of resulting errors.

Appraisal of acsuracy of approximation can be made by the value of the mean quadratic deflection, which is determined as the square root of the ratio of dispersion of the last remainder to dispersion of the original function

$$\sigma = \sqrt{\frac{\sum_i \sum_j [R(x_i, y_j)]^2}{\sum_i \sum_j f^2(x_i, y_j)}}. \qquad (6.44)$$

Thus, for the given $f(x_i, y_j)$ we find functions $g(x_i)$, $h(y_j)$, $g'(x_i)$, $h'(y_j)$, etc., for fixed values of x_i, y_j. During reproduction of the found functions by diode converters it is natural to use linear interpolation between tabular values of the found functions.

This method is useful for representation of functions of many variables, which here are reduced to functions of one and two variables (see for more detail E. W. Pike and T. R. Silverberg [1]).

Functional diagram of a functional generator for a function of two variables, using the decomposition (6.40), is shown in Fig. 93. As follows from the figure, with such approximation there are required six functional generators, two factor devices and one adder.

To a somewhat different composition of the required equipment leads consideration of given function $f(x, y)$ by sections for given fixed values of y_i. In every section here we get a function of one variable $f_j(x, y_j)$.

Transition from $f_j(x, y_j)$ to the curve of the following section can be carried out by application of one or another intepolating function. Usually here there is used linear interpolation. An example of such electromechanical device, made with functional potentiometers, is shown in Fig. 94.

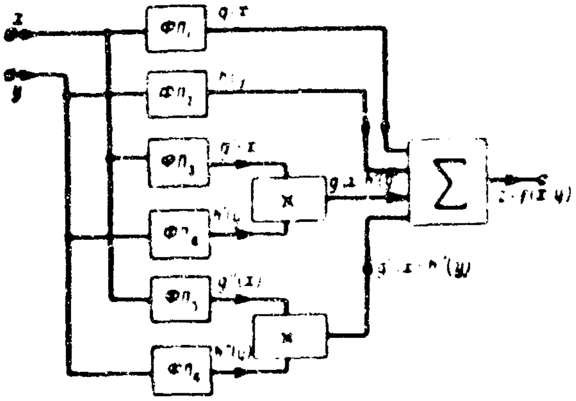

Fig. 93. Functional diagram of functional device for reproduction of functions of two variables.

During application of diode generators for reproduction of each function $f_j(x, y_j)$ it is necessary to resort to their piecewise-linear approximation. Here transition from one function to another is best carried out by "triangular functions."*

Skeleton diagram of such a converter is shown in Fig. 95.

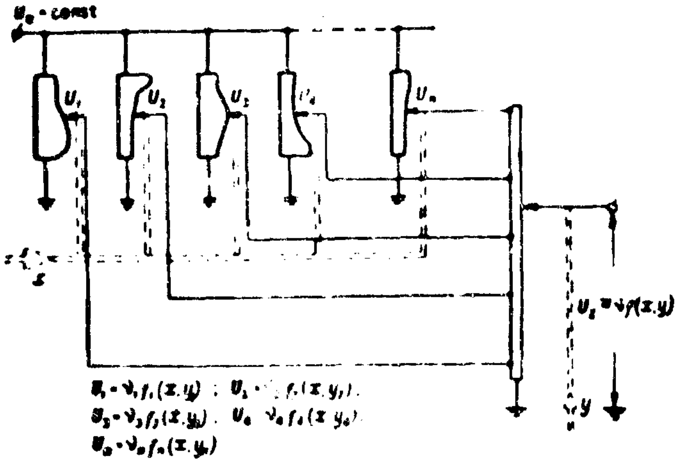
U0 = const
U1
U2
U3
U4
Un
x
Uz = νf(x,y)
y
U1 = ν1f1(x,y1) ; U2 = ν2f2(x,y2)
U3 = ν3f3(x,y3) ; U4 = ν4f4(x,y4)
Un = νnfn(x,yn)

Fig. 94. Skeleton diagram of electromechanical device with functional potentiometers for reproduction of a function of two variables.

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*A. A. Fel'daum and A. I. Manukhin, Electronic apparatus for obtaining functions of two variables, Auth. cert. No. 100891, priority from October 16, 1951.

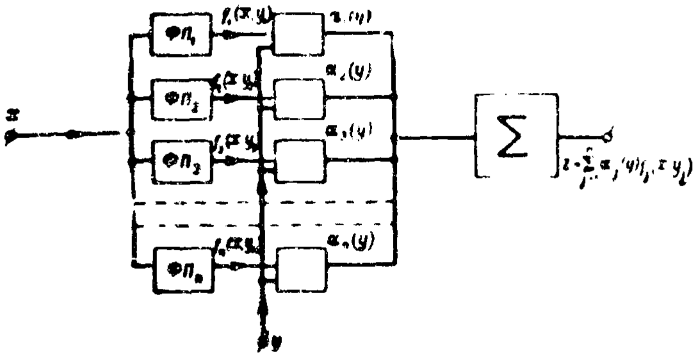

Fig. 95. Skeleton diagram of functional generator for reproduction of functions of two varibles.

Variable x moves to the input of diode functional generators designed for reproduction of functions $f_j(x, y_j)$. Voltage, representing in the device the value of these functions, is passed accordingly through dividers $\alpha_j(y)$, so that at the output of each divider we obtain the product of $\alpha_j(y) f_j(x, y_j)$, where $\alpha_j \leq 1$.

These products then are summed so that the output quantity will be equal to

$$z = \sum_{1}^{n} \alpha_j(y) f_j(x, y_j). \tag{6.45}$$

Functions $\alpha_j(y)$ have triangular form. Their form is shown in Fig. 96. When $y = y_1$ $\alpha_1(y) = 1$, and $\alpha_2(y) = 0$, therefore, $z = f_1(x, y_1)$; when $y = y_2$ $\alpha_1(y) = 0$, and $\alpha_2(y) = 1$

$$z = f_2(x, y_2).$$

In the interval between y_1 and y_2

$$z = \alpha_1(y) f_1(x, y_1) + \alpha_2(y) f_2(x, y_2).$$

If one were to fix a certain value $x = x_1$, then during change of y inside the considered interval, z will change linearly between values $z_1 = f_1(x_1, y_1)$ and $z_2 = f_2(x_1, y_2)$.

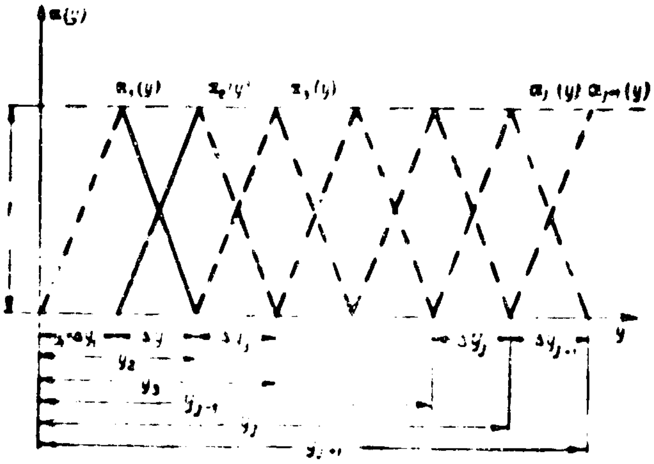

Fig. 96. Graph of function $a_j(y)$.

Thus, application of "triangular" functions also leads to linear interpolation. Here as can be noted from comparison of Figs. 95 and 96, in reproduction of given function simultaneously participate only two adjacent functional generators of $f_j(x, y_j)$ which significantly reduces total error as compared with the above-considered method of representation of a function of two variables, with which there must simultaneously work six functional generators and two factor devices. The fundamental circuit of such a functional generator, developed by A. A. Fel'dbaum and A. I. Manukhin [1], is shown in Fig. 93. Here multiplication of $f_j(x, y_j)$ and $a_j(y)$ is carried out by a device based on the time-pulse principle. Quantity y will be converted in the relative duration of pulses, controlled by connection of outputs of separate functional generators $\Phi\Pi_1 \ldots \Phi\Pi_n$ *) by diodes $Д_1, Д_2, \ldots, Д_n$ to the integrating point of output operational amplifier 1. Control occurs in such a way that with increase of y within limits adjacent sections Δy_j and Δy_{j+1} the relative duration of switching on of $\Phi\Pi_j$ at first grows from zero to one, and then toward the end of section Δy_{j+1} again falls to zero. Here the mean value of voltage at the output of integrating amplifier 1 from this functional converter will change according to the law of the triangle. Control of diodes $Д_1, \ldots, Д_n$

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*Functional generators $\Phi\Pi_1 \ldots \Phi\Pi_n$ consist of separate diode elements (see page 142), whose currents are summed by amplifier 1.

is produced in the circuit of Fig. 97 by triggers T_1, $T_2, \ldots, T_n$. Voltage, representing variable y, is fed to auxiliary amplifier 2 through resistance R_{02}. Resistances of feedback for this amplifier are resistances R_1, $R_2, \ldots, R_n$, coupled to plates of triggers, having potentials U_1', $U_2', \ldots, U_n'$. Output voltage U_y of amplifier 2 through voltage dividers R_{T11} -- R_{T21}, R_{T12} -- $R_{T22}, \ldots, R_{T1n}$ -- R_{T2n} moves to grids of the triggers. Here also move reference voltages (U_{01}, $U_{02}, \ldots, U_{0n}$) from divisor ДН and voltage U_n of sawtooth form with frequency of the order of 1000 cps.

In the absence of an input signal the reference voltage displaces voltage U_n so much that not one trigger works. In this position on some output of triggers there will be large positive potentials U_1, $U_2, \ldots, U_n$, and on others negative potentials U'_1, $U'_2, \ldots, U'_n$. Resistances R_1, R_2, R'_3 and R'_n of the circuit of auxiliary commutating diodes $Д_1$ are selected with such magnitude that here both diodes are locked and at points M_1, $M_2, \ldots, M_n$ of the circuit there is fed a positive potential. As a result all diodes $Д_1, Д_2, \ldots, Д_n$ turn out to be locked, and on the output of the integrating amplifier voltage is zero. With increase of input voltage resultant voltage reaches the threshold of operation of the trigger. This occurs at the end of the period of change of voltage U_n first on the grid of trigger T_1, whereupon the trigger is switched on for very short interval of time τ (Fig. 98). With switching on of the trigger potential U_1 becomes negative and both diodes $Д_1$ are unlocked, fixing here, the potential of the ground at point M_1. Diode $Д_1$ is unlocked and voltage, representing function $f_1(x, y_1)$, passes to amplifier 1. With durther increase of y the time of the switched-on state of trigger T_1 increases and when $y = y_1$ it equals period T of the change of sawtooth voltage U_n. Further increase of the input signal leads to operation of trigger T_2 at first for a very short interval of time. With switching on of the second trigger diodes $Д_1$ are locked and diode $Д_1$ is locked. Thus, with increase of input signal above y_1, relative duration of switched-on state of diode $Д_1$ starts to decrease. The remaining parallel circuits work similarly.

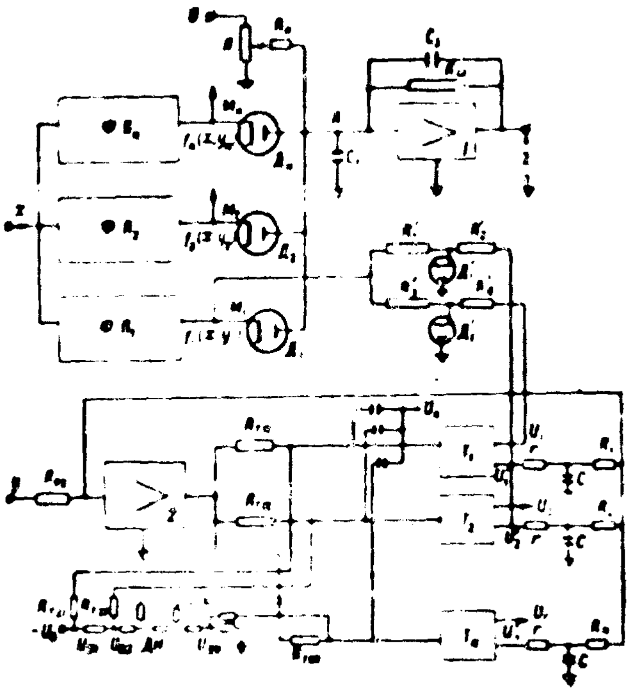

Fig. 97. Fundamental circuit of functional generator.

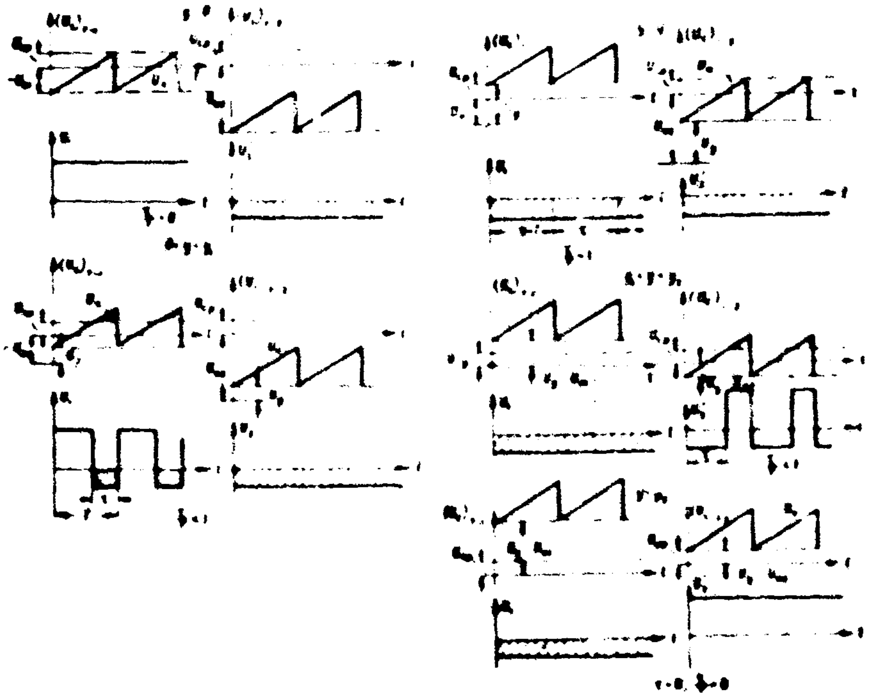

Fig. 98. Explanation of work of circuit of conversion of output signal into relative pulse length.

With the help of feedback through resistances R_1, R_2,..., R_n we can establish proportionality between input signal and variable mean value of voltage at output of triggers, and consequently, the relative duration of switching on independently of variation of parameters of the circuit and linearity of change of voltage U_n. Capacitors C_1 and C_2 are connected to decrease the variable component in output voltage.

A deficiency of the considered circuit, besides large volume of required equipment, is also the comparatively small passband, which is limited, as for most systems based on the time-pulse principle, to a frequency of 10 — 15 cps.

Of great interest in this connection is study of the possibility of use of the previously considered diode generators of one variable for reproduction also of functions of two variables.

If in these generators we make the reference voltage variable also, then there can be obtained a family of nonlinear dependences with parameter $e_{оп}$. If we use a functional generator with potentially grounded diodes, then during linear change of reference voltage in function $e_{вх2}$ ($e_{вх2}$ represents the second variable) there results a family of equidistant nonlinear curves. This follows the fact that for diode elements with potentially grounded diodes change of reference voltage does not lead to change of steepness of the current characteristic, but only to its parallel displacement. If one were to change $e_{оп}$ according to a certain nonlinear dependence $e_{оп} = K(e_{вх2})$, then it is possible to reproduce a more complicated function of two variables. In Fig. 99 is brought the family of curves and circuit of a diode functional generator for their reproduction.*

As it was shown in the work of Meissinger, in general during reproduction of an arbitrary function of two variables it is necessary to change reference voltage of every diode element by its own law depending upon $e_{вх2}$. Therefore, for example, with 10 diode elements there will be required ten auxiliary functional generators.

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*The figure is borrowed from the work of H. F. Meissinger [2]. It is necessary to note that this circuit possesses the deficiency that toward the end of the scale of the argument all diode elements turn out to be switched-on in the circuit.

However thorough analysis of character of flow of functions given for reproduction by separate sections allows us to reduce the number of required functional generators by feeding from one functional generator $g_i(e_{\text{вх}2})$ of those diode elements, which participate in formation of sections of the curve, having identical steepness.

It is most convenient in studying the initial dependences to use the method of isoclines receiving wide use in the qualitative theory of differential equations.

Determination of functions of reference voltage $g_1(e_{\text{вх}2}), \ldots, g_n(e_{\text{вх}2})$, its required for reproduction of a given function of two variables, can be carried out in the following manner: the function given for reproduction $e_{\text{вых}} = f(e_{\text{вх}1}, e_{\text{вх}2})$ will be placed on coordinates $e_{\text{вых}}, e_{\text{вх}1}$ or $e_{\text{вых}}, e_{\text{вх}2}$ for various fixed values of $(e_{\text{вх}2})_j$ in the first case and of $(e_{\text{вх}1})_j$ in the second.

By one of the curves of this family are determined current characteristics of input circuit of the operational amplifier and feedback circuits, which are subjected to piecewise-linear approximation, and then there is determined the number of diode elements, their type and operating regime and switching voltage.

In case of application of diode elements with potentially grounded diodes for the i-th diode element on the basis of (5.19) this equality is correct:

$$e_{\text{вых}\,i} = [e^{н}_{\text{вх}\,i}]_{(e_{\text{вх}2})_j} \frac{r_i}{R_i}. \tag{6.46}$$

where i is the number of the diode element, j — the number of interval of decomposition for variable $e_{\text{вх}2}$

Since from approximation of the given curve we know $e^{н}_{\text{вх}\,i}$ for every fixed value of $(e_{\text{вх}2})_j$, then, consequently, we know the relationship

$$e^{н}_{\text{вх}\,i} = \bar{g}_i[(e_{\text{вх}2})_j]. \tag{6.47}$$

Substituting (6.47) in (6.46), we find

$$e_{\text{вых}\,i} = g_i(e_{\text{вх}2}) = \frac{r_i}{R_i}\bar{g}_i[(e_{\text{вх}2})_j].$$

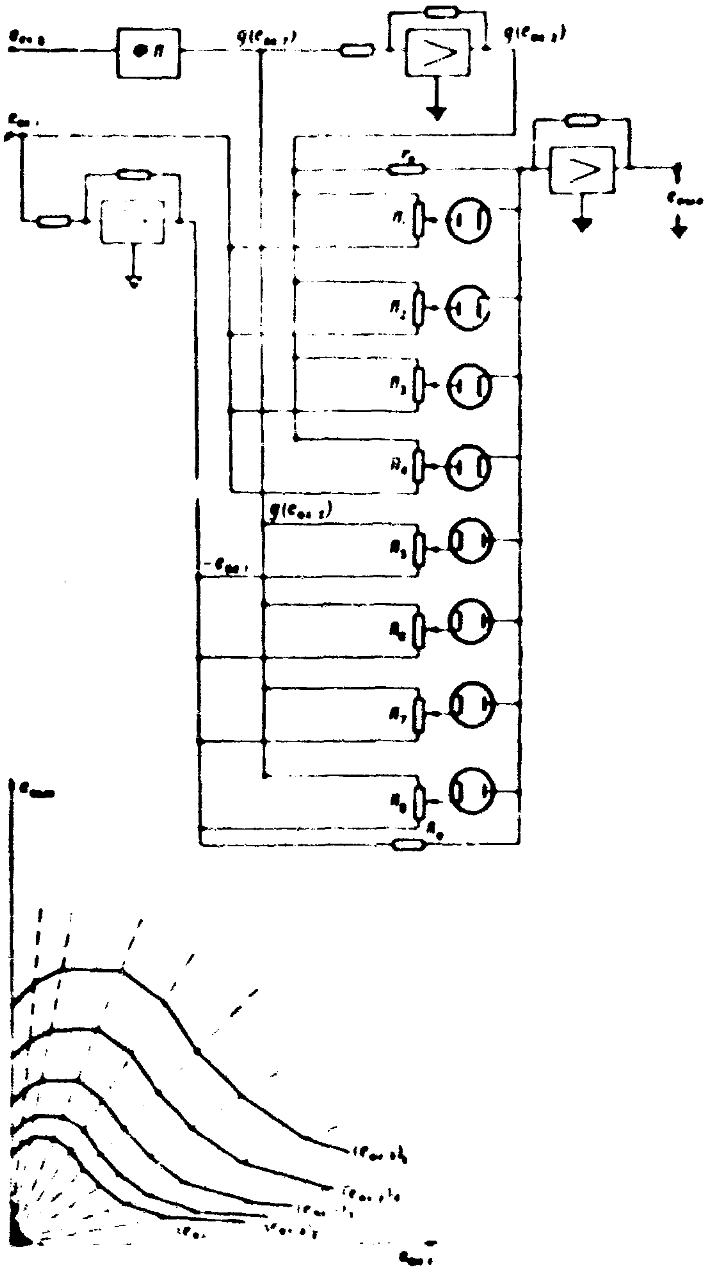

Fig. 99. Fundamental circuit of functional generator of two variables and the family of nonlinear dependences reproduced by them.

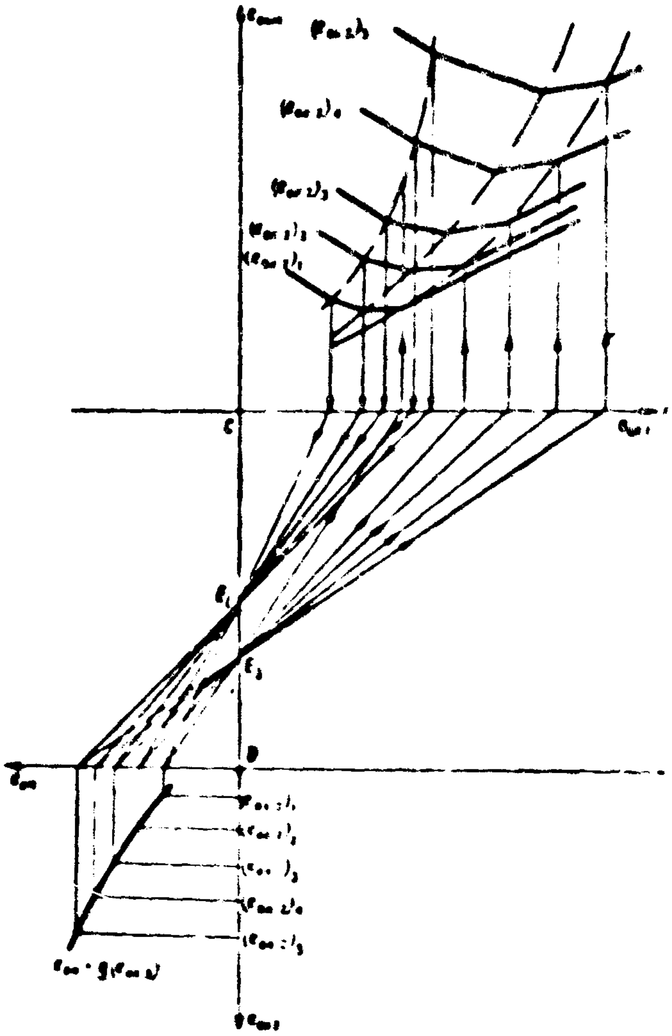

Fig. 100. Graphic method of determination of functions $\epsilon(e_{ex1})$

In Fig. 100 is brought a method of determination of these functions (H. F. Meissinger [2]), based on graphic construction of relationship (6.46).

Distance CD in a certain scale represents a unit of relative resistance. On this straight line we mark point E_1 in such a manner that distance CE_1 is equal to $\frac{R_i}{R_i + r_i}$. Passing straight lines, connecting points E_1 and e''_{ex1} (for one and the same diode element, but for various values of $(e_{ex1})_i$) to intersection with a straight line, parallel to axis e_{ex1} and passing from it a distance CD, we will receive the desired values of e_{o1}. This directly ensues from the similarity of the triangles thus obtained. Making like construction for switching voltages of the second diode element, we find for it the function of the reference voltage.

In many cases with proper selection of $\frac{r_i}{R_i}$ switching voltages of separate diode elements will be so connected among themselves that functions $g_i(e_{x,y})$ will appear to coincide. Executing graphic construction in reverse order, there can found by the given $g(e_{x,y})$ for one diode element the switching voltages for the remaining.

The presented method of reproductions of functions of two variables can be extended also to functions of more variables, if as the function of the reference voltage we use in turn a function of two variables.

Material brought in this paragraph* show that the problem of reproduction of a function of two or more variables presents great difficulties. Presently known solutions lead to significant complications of equipment and lowering of accuracy connected with this, and also require a large volume of preliminary calculation.

For these reasons it is possible to consider that development of functional devices for reproduction of a function of two or more variables still is far from completion.

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*In recent years receiving application are methods of synthesis of functional generators of many variable from diode logical elements, offered independently by T. E. Stern and G. A. Philbrick and further developed by S. A. Ginsburg (see, for example, T. E. Stern Piecewise—linear network analysis and synthesis, proceedings of the symposium on nonlinear circuit analysis, New York—London (1957); S. A. Ginsburg, Logical method of synthesis of functional generators, Trans. of First Congress IFAK, Vol. IV, Publishing House of Academy of Sciences of USSR (1961).

CHAPTER VII

FUNCTIONAL GENERATORS USING CATHODE-RAY TUBES

With necessity of rapid transition from one form of reproduced nonlinear dependence to another, and also in case of reproduction of functions with many extrema and functions, possessing great steepness, as is known from literature,* general-purpose functional generators, based on the use of the cathode-ray tube, can be applied.

Well-known types of such functional generators can be divided into two groups:

1. Devices of closed type with negative feedback, using principle of servo system.
2. Pulse devices of open type.

1. Devices of Closed Type, Based on Principle of Static Servo System.

In Fig. 101 is brought functional diagram of a functional generator based on a cathode-ray tube, built by principle of static servo system. In front of screen of electron-beam tube 1 with electrostatic control is placed opaque plate (mask), whose profile is executed in accordance with the nonlinear dependence given for reproduction. A certain distance from the mask is placed photo multiplier 2. Electronic beam of tube is deflected from axis y to an upper boundary determined by the horizontal side of the inscribed square, by voltage U_0.

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*D. Mynall [1], D. M. Mackay [1], D. E. Sunstein[1], G. D. McCann, C. H. Wilts, B. N. Locanthi [1], H. W. Shults [1], E. J. Hancock [1].

During switching on of the photomultiplier the luminous flux, formed by the luminescent point on the screen, will cause appearance of photocurrent. Voltage drop from photocurrent in output resistor of photomultiplier after amplification by electronic amplifier 3 is fed to vertical plates Y-Y of the tube with such polarity that beam begins to lower toward the mask. After arriving at the boundaries of the mask further lowering of the beam causes decrease of the area of the luminescent point on screen, and together with this decrease of voltage U_y , fed on the vertical plates. This decrease will occur until the difference between the initial value of voltage U_0 and voltage drop from the photocurrent with the given

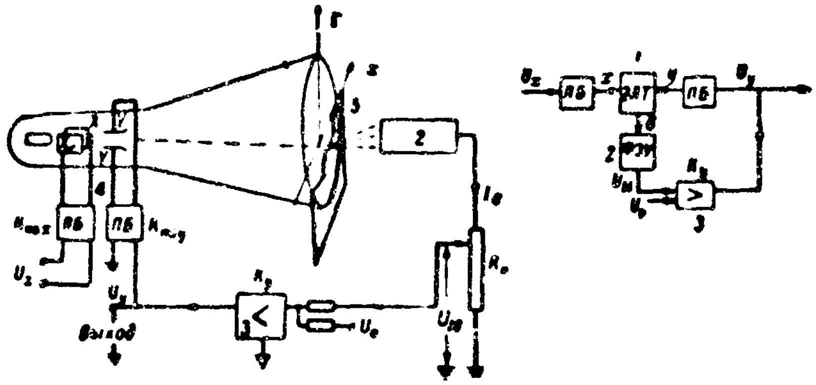

Fig. 101. Principal functional diagram of cathode ray functional generator with photomultiplier, 1--electron-beam tube, 2--photomultiplier, 3--amplifier, 4--transition units, 5--mask.

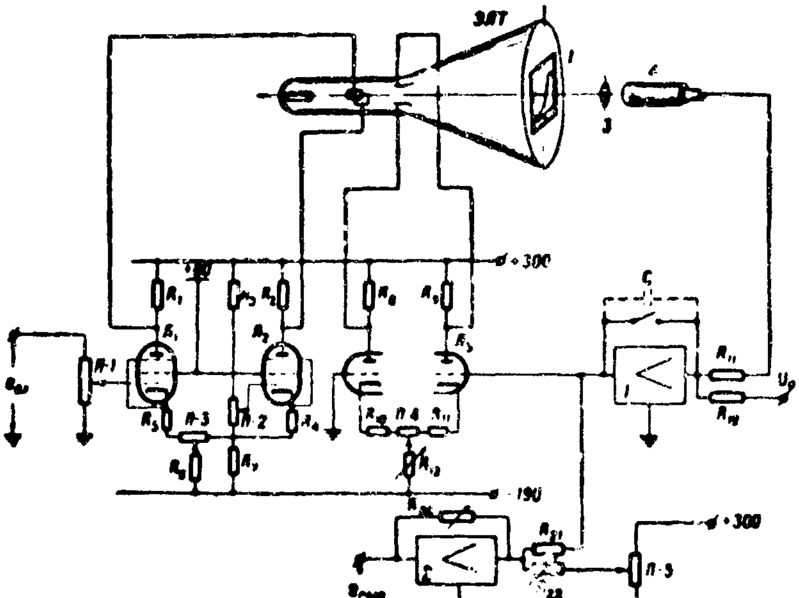

Fig. 102. Fundamental circuit of functional generator, 1--mask; 2--photomultiplier, 3--optics.

amplification factor of amplifier provides on the vertical plates the voltage, necessary for setting the beam on the edge of the mask. Obviously, this voltage will also depict the value of the function for the considered value of the argument. If one were to start to change voltage U_x on horizontal plates X—X, then accordingly the beam will be transferred along the edge of the mask, and voltage on plates Y—Y will change by the given nonlinear curve. Fundamental circuit of such functional generator as made by Institute of Automation and Telemechanics of Academy of Sciences of USSR is presented in Fig. 102. Here the transition unit controlling horizontal deflection of the beam is made from two tubes Л_1 and Л_2, and the transition unit block controlling vertical deflection is made from double triode Л_3. Amplifier 1 serves as the amplifier of main channel, and amplifier 2--for setting the scale of the function. The scale of the argument can be set by potentiometer П-1. Potentiometer П-5 serves to compensate the constant component, appearing during shift of the curve given for reproduction with respect to axes of the tube for the purpose of maximum use of the area of the screen.

2. Error of the Functional Generator.

We will derive the basic relationship for estimating error of the given circuit. For this purpose it is convenient to present the functional diagram of Fig. 101 in the form of a block-diagram (Fig. 103a). Here are designated:

ПБГ -- bridging amplifier of horizontal deflection,

$k_{\text{пбг}}$ -- amplification factor of bridging amplifier,

ГП -- system of deflection of beam in horizontal,

$\lambda_{\text{гп}}$ -- sensitivity of system of beam deflection in the horizontal,

Mask -- nonlinear dependence, given by profile of mask,

ВП -- system of beam deflection in vertical,

$\lambda_{\text{вп}}$ -- sensitivity of system of beam deflection in vertical,

$U_{\text{вп}}$. $U_{\text{гп}}$ -- voltages on horizontal and vertical plates, ПБВ -- bridging amplifier of vertical deflection,

$\Psi(\lambda)$ -- characteristic of screen,

$\varepsilon\,(y_{\text{зад}}\ \ y)$ -- error in setting of beam in vertical,

S -- area of undarkened part of fluorescent spot,

F -- luminous flux,

E -- transmission factor of luminophor,

T_1 -- time constant of luminophor,

$I_\text{ф}$ -- current of photomultiplier,

$I_\text{т}$ -- dark current,

$\sigma(I)$ -- integral sensitivity of photomultiplier,

$R_\text{н}$ -- load resistor of photomultiplier,

U_1 -- voltage on load resistor,

U_0 -- voltage of initial displacement of beam,

e_0 -- voltage of zero drift of amplifier, brought to the input,

α -- angle of inclination of profile of mask with horizontal.

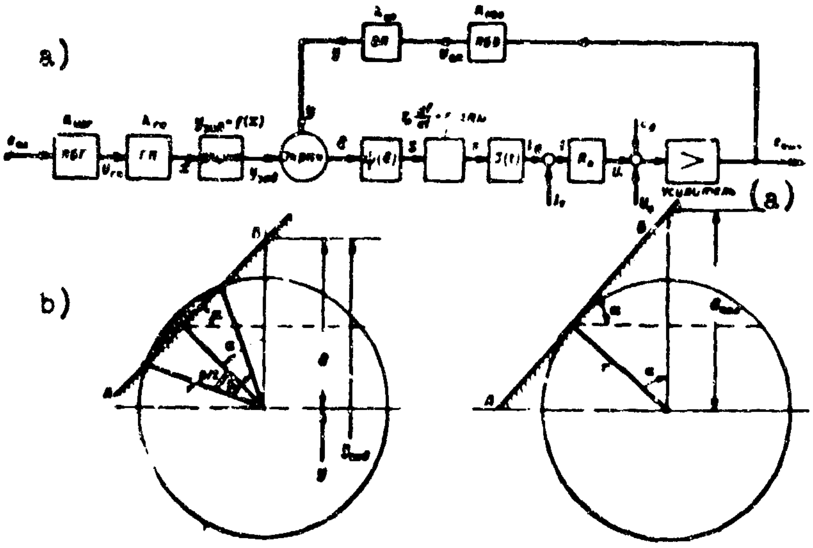

Fig. 103. Block-diagram for derivation of fundamental equation of functional generator.
KEY: (a) Amplifier.

Equations of separate units of block-diagram, Fig. 103, will be:

1. Equation of input circuit

$$\left.\begin{aligned} x &= \lambda_{\text{тп}} k_{\text{ибг}} e_{\text{вх}}, \\ y_{\text{зад}} &= f(x). \end{aligned}\right\}. \qquad (7.1)$$

2. Equations of main channel of amplification of error in turn breaks up into:

a) Equation, determining dependence of unshaded part of spot on error $\delta = y_{\text{зад}} - y$

$$S = \Psi(\delta)\delta.$$

Form of function $\Psi(\delta)$ depends on given idealization of the shape of the spot and character of curvature of mask in the region of the spot. With a well focused spot it is possible with accuracy sufficient for practice to consider that the spot has the shape of a circle, and the mask in the region of the spot can be replaced by a sloping straight line AB (Fig. 103b).

With these assumptions

$$S = r^2\left[\arccos\left(\frac{\delta}{\delta_{\max}}\right) - \frac{\delta}{\delta_{\max}}\sqrt{1 - \left(\frac{\delta}{\delta_{\max}}\right)^2}\right].$$

Assuming movements of the light spot about the mask to be small, after expansion of quantity arccos ($\frac{\delta}{\delta_{\max}}$) in a series and disregarding all terms $\frac{\delta}{\delta_{\max}}$ of a degree higher than the first, we will receive*

$$S = \frac{\pi r^2}{2}\left[1 - \frac{4}{\pi}\frac{\delta}{\delta_{\max}}\right].$$

b) Equation of luminous flux

$$T_a \frac{dF}{dt} + F = E(t) \cdot S.$$

c) Equation of photomultiplier

$$I = I_\text{т} + I_\text{ф} = \sigma(t)F + I_\text{т}.$$
$$U_1 = IR_\text{н} = R_\text{н}\sigma(t)F + I_\text{т}R_\text{н}.$$

d) Equation of amplifier

$$e_{\text{вых}} = K_\text{у}(-U_0 + U_1 \pm e_\partial).$$

Solving the given equations for $e_{\text{вых}}$, we receive finally equations of the main channel of amplification in the form

$$\left.\begin{aligned} e_{\text{вых}} &= K_\text{у}[R_\text{н}\sigma(t)F + I_\text{т}R_\text{н} - U_0 \pm e_\partial]. \\ T_a\frac{dF}{dt} + F &= E(t)\frac{\pi r^2}{2}\left(1 - \frac{4}{\pi}\cdot\frac{y - y_{\text{зад}}}{\delta_{\max}}\right). \end{aligned}\right\} \qquad (7.2)$$

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*Derivation of this dependence belongs to A. D. Talantsev.

3. Equation of feedback circuit

$$y = \lambda_{\text{вп}} \cdot k_{\text{пбв}} \cdot e_{\text{вых}}. \qquad (7.3)$$

Considering that $\lambda_{\text{max}} = \frac{r}{\cos\alpha}$, we find from equations (7.1), (7.2) and (7.3) when $T_1 \to 0$

$$e_{\text{вых}} = \frac{f(x)}{\lambda_{\text{вп}} k_{\text{пбв}}} \cdot \frac{1}{\frac{1}{\cos\alpha\, K_1 R_{\text{н}} \sigma(t) E(t)\, 2r \lambda_{\text{вп}} k_{\text{пбв}}} + 1} + K_y \frac{\left[U_0 - R_{\text{н}} \sigma(t) E(t) \frac{\pi r^2}{2}\right] + I_{\text{т}} R_{\text{н}} \pm e_{\text{д}}}{1 + K_1 R_{\text{н}} \sigma(t) E(t)\, 2r \lambda_{\text{вп}} \cdot k_{\text{пбв}} \cdot \cos\alpha}. \qquad (7.4)$$

In a steady-state regime in ideal conditions, when voltage $U_0 = R_{\text{н}} \sigma(t) E \frac{\pi r^2}{2}$, $I_{\text{т}} = 0$, $e_{\text{д}} = 0$, with very large K_y and $f(x) = f(x_{\text{н}})$, from expression (7.4) we obtain

$$e_{\text{вых.н}} = \frac{f(x_{\text{н}})}{\lambda_{\text{вп}} \cdot k_{\text{пбв}}}. \qquad (7.5)$$

It follows from this that error of work of such functional generator will be caused by inaccuracy of making the mask $\Delta f = f_{\text{мз}}(x_{\text{н}}) - f(x_{\text{н}})$, the finite value of the amplification factor of the main channel, time constant of luminophor, presence of dark current of photomultiplier, inaccuracy of setting of required quantity U_0, by drift of the amplifier and instability of sensitivity of the deflecting system.

For the purpose of decreasing error one should take special measures for accurate adjustment and manufacture of the mask. As amplifier of main channel it is desirable to use amplifier with automatic stabilization of zero level.

With very large amplification factor and ideal manufacture of mask the error, as expression (7.4) shows, will have the form

$$\Delta e_{\text{вых}} = e_{\text{вых}} - e_{\text{вых.н}} = \frac{\left(U_0 - R_{\text{н}} \sigma(t) E(t) \frac{\pi r^2}{2}\right) + I_{\text{т}} R_{\text{н}} \pm e_{\text{д}}}{R_{\text{н}} \sigma(t) E(t)\, 2r \lambda_{\text{вп}}\, k_{\text{пбв}} \cdot \cos\alpha}. \qquad (7.6)$$

It is obvious that this error will be less, the greater the magnitude of the denominator. This indicates a way of selecting parameters of photomultiplier, properties of mask and transmission factors of transition units.

Analysis of work of such functional generator shows that changes of dark current of photomultiplier and brightness of fluorescent point of screen in time in essence determined the natural zero drift of functional generator, and inasmuch they act almost directly on the amplifier input, increase of the amplification factor of the latter in the presence of negative feedback does not lead to decrease of indicated drift.

In connection with this there were attempts (by C. N. Pederson, A. A. Gerlach, R. E. Zenner [1]) to surmount these difficulties by increasing dimensions of screen of the tube, transition to another type of screen and application of several photomultipliers. In Fig. 104 is brought a skeleton diagram of such a functional generator. In this generator the luminescent spot, appearing on the screen of the tube, is projected with the help of optics on black sheet of special paper, on which by white paint there will be depicted the graph of the function given for reproduction . Into the circuit of vertical beam deflection there is fed biased voltage, which is sufficient to move the beam to the upper edge of the tube. Light, emitted by the luminescent spot of the screen, passing through the white line on the black screen, is reflected from it and hits the group of photomultipliers. Here at the output of the amplifier there appears voltage of such sign, with which the beam seeks to drop down. As a result the beam will be

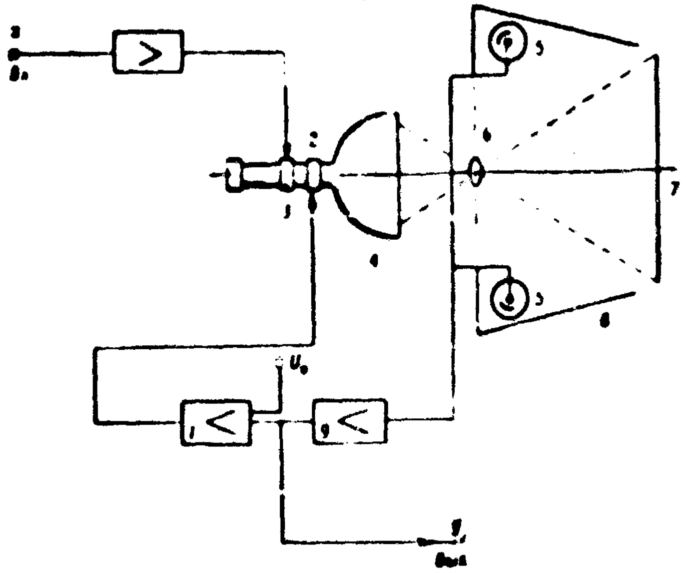

Fig. 104. Fundamental circuit of functional generator with photomultipliers and cathode-ray tube with magnetic control. 1--amplifiers of deflecting systems, 2--coil of vertical deflection, 3--coil of horizontal deflection, 4--cathode-ray tube; 5--photomultipliers, 6--optical system, 7--graph of given function (white on black background), 8--light absorber, 9--amplifier of signal.

held on boundary of the white line and with change of voltage in the system of horizontal deflection will track the boundary of the white line. Thickness of the white line, by which the graph of the function given for reproduction is depicted, should increase with increase of steepness of this function. With an angle of inclination, approaching 90°, for retention of the beam on the white line its thickness should be in the order of 3 mm. By application of four photomultipliers fluctuations of total dark current are sharply reduced.

According to the data of C. N. Pederson, A. A. Gerlach, R. E. Zenner [1] the described device ensures accuracy of 0.5%, a passband of 100 c, ease and speed of set up.

Of great interest also are functional generators with application of cathode-ray tubes, which do not require conversion of the beam current into luminous flux. From literature we know the principle of construction of such a generator (A. C. Munster [1]). In a cathode-ray tube, equipped with the usual gun with electrostatic deflection of beam, there is placed a plate-target, made from material, possessing a significant coefficient of secondary emission (for example, aluminum oxide).

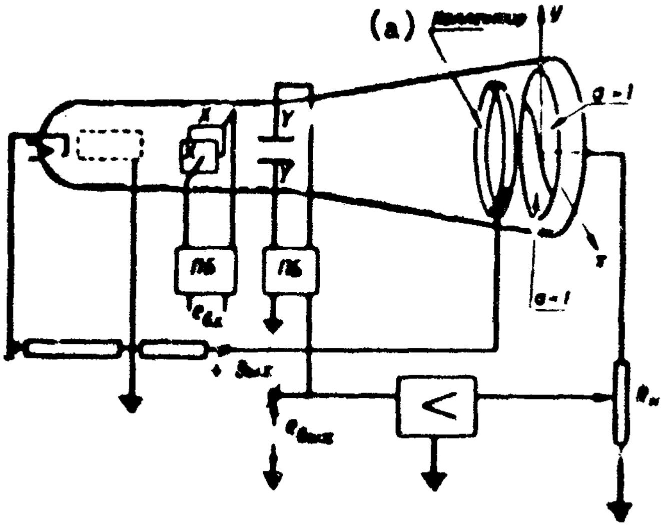

Fig. 105. Fundamental circuit of functional generator with internal receiver of beam.
KEY: (a) Collector.

Part of the target is covered with carbon ink in such a way that the curve, bounding the carbon dye, depicts the function given for reproduction. In the tube is placed an additional electrode-collector (Fig. 105), gathering secondary electrons, flying from the target under the influence of electron bombardment of the beam. Aluminum and carbon have at selected accelerating voltages different coefficients of secondary emission (for aluminum oxide $\sigma = 2$, and for carbon dye $\sigma < 1$). Therefore the direction of current in resistor R_H will vary depending upon what part of the target the electron beam strikes. If after preliminary amplification we use the voltage drop at R_H to control deflection of the beam by plates Y—Y, then it is possible with feeding of the input signal to plates X—Y to force the electron beam to follow the boundary between the carbon dye and aluminum oxide. Here voltage after the amplifier will represent the desired value of the function.

Such a functional generator, as follows from this description, does not require conversion of current of the beam into a luminous flux. However there is required accurate adjustment of the target inside the tube, which presents known difficulties, and there also appears an additional source of error due to instability of coefficients of secondary emission along the boundary of the division.

By principle of action the considered functional generator cannot be general-purpose. Replacement of functions signifies replacement of the electron-beam tube.

For construction of a functional generator based on a cathode-ray tube, which would not require conversion of beam current into a luminous flux, and then into photomultiplier current, it is possible to use the phenomena occurring in the cathode-ray tube with accumulation of charges (B. Chance, V. Huges [1]). In these tubes the screen should possess a coefficient of secondary emission $\sigma > 1$ in range of operating accelerating voltages and very high resistance. These requirements are met, for example, by Prex glass, from which envelopes of electron-beam tubes are usually prepared. For use of this phenomenon on the outer surface of the tube (Fig. 106) is put a signal electrode, and inside is placed an additional electrode (collector), under a positive potential, somewhat exceeding the potential of the second plate. As collector there can serve, for example, an Aquadage electrode, utilized for post deflection acceleration.

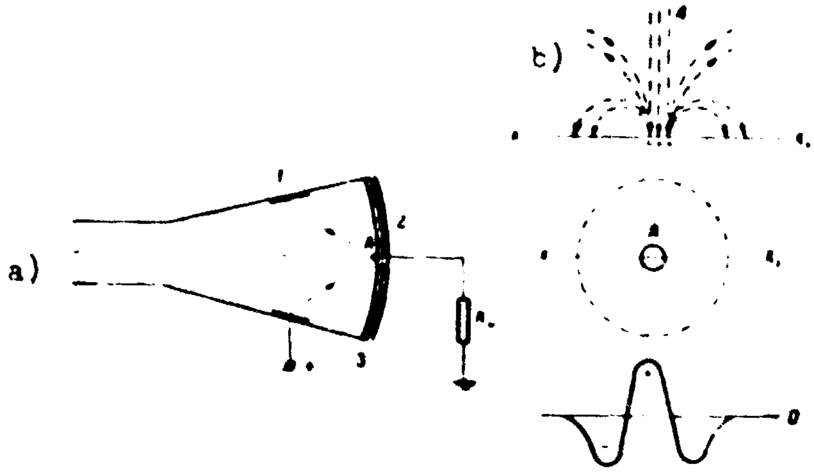

Fig. 106. Principle of action of a cathode-ray tube with accumulation of charges. 1--capacitor; 2--signal electrode; 3--glass; 4--electron beam.

The surface of the screen, under the influence of the electron beam, will be charged positively until its potential comes near the potential of the collector. Certain secondary electrons, attracted by the field of the charged positive spot, will settle near it and will create a negatively charged region (Fig. 106b). Changing periodically the intensity (brightness) of the beam by the influence on potential of a modulating electrode, it is possible to carry out periodic change of total charge at a given place of the screen which causes appearance of capacitive currents through the signal electrode. During use of the described phenomenon for construction of a functional generator the signal electrode is made of two parts, divided by a slot, made in the form of the function given for reproduction (B. Ya. Kogan [2]). Alternating current, removed from each part of the signal electrode, is amplified by high-frequency amplifier and is rectified. The rectified voltage is added in anti-phase to the input of the operational d-c amplifier, covered by negative feedback through the electron-beam tube (Fig. 107). When the beam is in the middle of the slot, current of each half of the conducting layer is identical and voltage at the output is equal to zero. With displacement of beam in the

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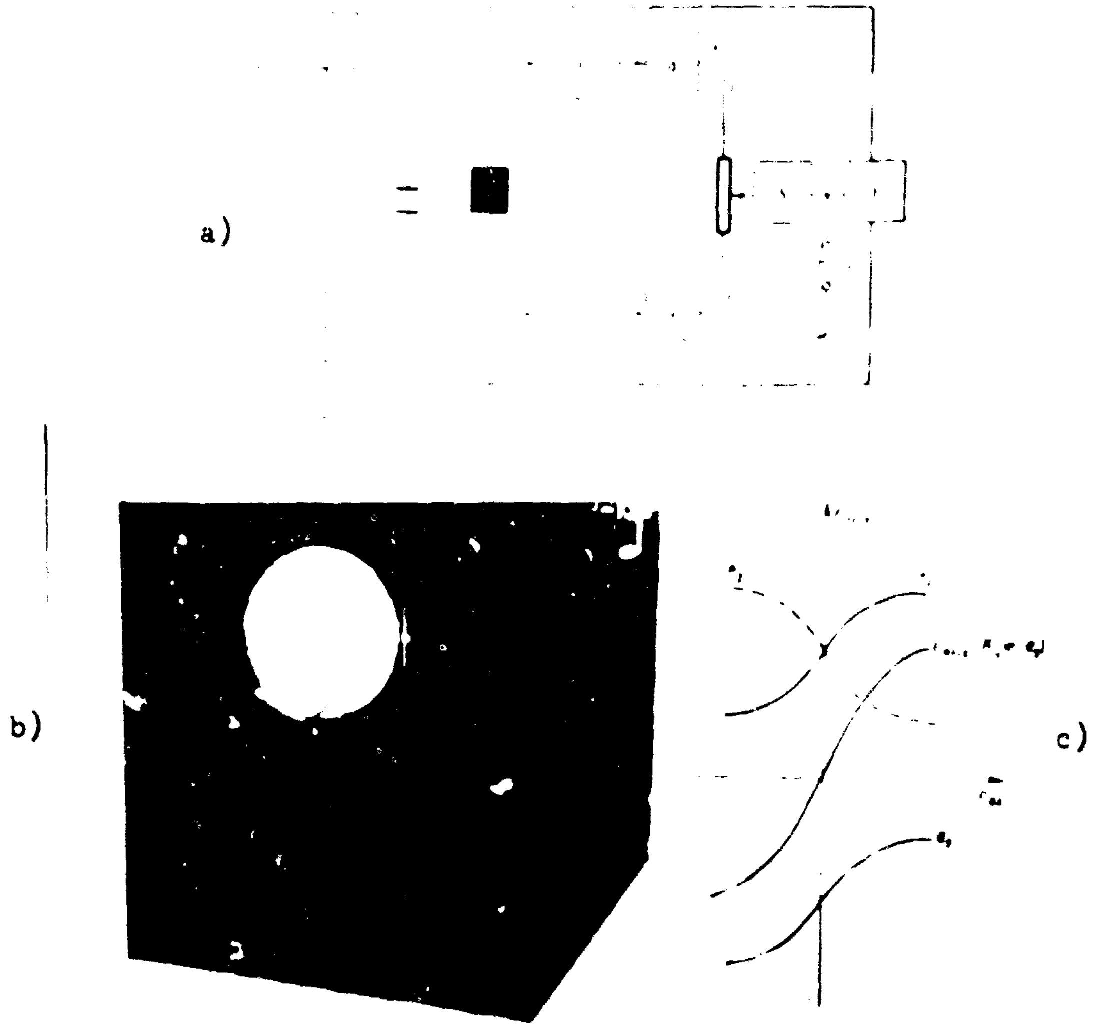

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Fig. 107. Functional generator based on cathode-ray tube without photomultiplier. a) fundamental circuit; 1--transition units, 2--high-frequency oscillator; 3--high-frequency amplifiers; 4--detectors; 5--d-c amplifier; b) general view: c) characteristic.

direction of one or the other plate at output there appears voltage of one or the other sign, and of the magnitude, necessary for holding the beam on the slot level. Obviously, this voltage will also represent the value of the desired function. The diagram and general view of a functional generator, built on such a principle by the Institute of Automation and Telemechanics of Academy of Sciences of USSR, is shown in Fig. 108, a and b) and its characteristic in Fig. 108c.*

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*Functional generator developed by colleague of Institute, V. V. Gurov.

Basic data of this functional generator are such: input and output voltage ±100 v, output current--of the order of 10 ma, input impedance--greater than 200 kilohm, accuracy--of the order of 1.5-2% of total scale, passband--greater than 100 c.

3. Functional Generators with Controlled Beam Scanning

Principle of action of such functional generator is based on use of dynamic compensation (F. E. Temnikov [1]). It can be comprehended by the functional diagram in Fig. 108.*

Functional generator consists of cathode-ray tube 1, opague mask 9, iterating the curve given for reproduction, photomultiplier 2, located a certain distance from screen of the tube, pulse shaping unit 3, generator 4 for scanning of the beam of cathode-ray tubs along y-y axis, peak detector 5 and integrating amplifier 6. To these elements are added (Fig. 108b) a source of constant compensating voltage 7 (ЗН) and key 8, protecting break-down of the beam of tube with sharp changes of voltage of the agrument.

Before beginning operation of functional generator output clamps of generator of scanning are closed and when $e_{\text{вх}}$ = 0 the electronic beam of tube is at point O.

With starting of functional generator output voltage of generator of scanning deflects the beam to one of the edges of the mask. With departure of the beam beyond the edge of the mask, thanks to illumination of the photomultiplier there occurs a sharp change of voltage on its load resistor due to change of current of photomultiplier. This change of voltage is shaped by unit БФМ into a standard pulse, which starts the scanning generator in the opposite direction, by force of which the beam will move to the opposite edge. As a result there will be established natural oscillation of the beam between edges of the mask. Amplitude of these oscillations will be directly proportional, and frequency inversely proportional to the distance between edges, when the beam moves with constant speed along the y—y axis.

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*Offered and developed in Academy of Sciences of USSR by A. D. Talantsev [3], [4]; see also R. Haviland [1].

Usually in such functional generators we use the dependence of amplitude of natural oscillations on distance between edges, since here rigid requirements are not put on linearity of change of voltage of generator of scanning. During change of $e_{\text{вх}}$ amplitude of natural oscillations will change by the same law, by which the distance between edges changes. When both edges of mask are symmetric with respect to axis x, amplitude of oscillations will reflect the ordinate of the function given for reproduction with accuracy up to a constant component. Amplitude of natural oscillations will be converted into constant voltage by a demodulator. As demodulator there is used a special circuit of a peak detector, ensuring linearity of rectifying in the operating range of frequencies with accuracy ±0.1%. As generator of scanning we apply symmetric trigger, whose plates are loaded by integrating networks.

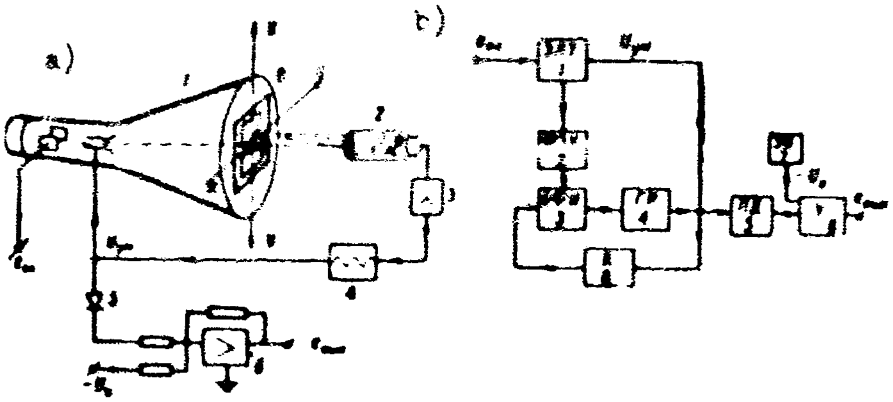

Fig. 108. Functional diagram of functional generator, based on principle of dynamic compensation. 1--cathode-ray tube; 2--photomultiplier; 3--pulse forming unit; 4--generator of scanning, 5--peak detector; 6--integrating amplifier; 7--source of stabilized voltage for compensation of constant component; 8--key; 9--opaque mask.

Presence of initial distance b_0 between edges of mask (Fig. 109) is caused by necessity of obtaining negative values of output voltage, and also the difficulty of obtaining small amplitudes of natural oscillations with very great frequency due to inertness of system and nonlinearity of operation of peak detector at small signal amplitudes. Constant component caused by this initial amplitude of natural oscillations, is compensated at output by voltage U_0.

During operation of the generator there is possible random nonoperation of the scanning generator or break-down of natural oscillations owing to sharp change of input voltage e_{ex} . Therefore in the generator is provided special protection from break-down of natural oscillations. In case of nonoperation of scanning generator after appearance of beam on edge of mask output voltage of scanning generator will increase, until key K opens which will cause change of voltage at the input of BFI and in turn will cause appearance of impulse at output of BFI and operation of scanning generator. The next operation will occur when the beam reaches the opposite edge of mask, since with setting of beam beyond screen at output of photomultiplier appears a voltage jump, opposite in sign to that, which takes place with emergence of beam from behind the mask. Pulse shaping unit does not react to voltage jumps of this polarity. The very same occurs during nonoperation of ГР at the other edge of mask. In absence of mask screen there take place natural oscillations, amplitude of which is greater than amplitude, corresponding to maximum width of mask, which is attained by corresponding adjustment of the key.

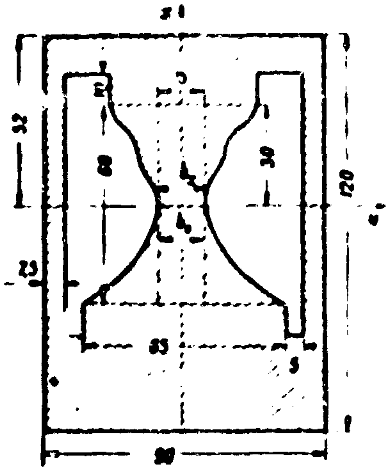

Fig. 109. Example of mask for functional generator, built on principle of dynamic compensation.

Frequency of natural oscillations for obtaining the widest possible passband for the device should be as high as possible. With a fixed maximum distance between edges of mask the lower limit of frequency of natural oscillations is determined basically by properties of the luminophor of the tube (time lag of glow and time of afterglow). In connection with this it is expedient in such generators to use cathode-ray tube with screen made of calcium tungstate, possessing comparatively small afterglow and giving a blue glow, for which sensitivity of photomultiplier turns out to be the greatest.

With the taken dimensions of mask (Fig. 109) minimum frequency of natural oscillations is approximately 1000 c which provides a passband of about 100 c.

Main sources of error of functional generator are: inconstancy of moment of starting and switching off of scanning due to instability of brightness of the spot, nonlinearity and instability of peak detector, instability of power supplys (especially for compensation of constant component) and inaccuracy of manufacture of screen. Error due to inconstancy of moment of starting and switching off of generator of scanning significantly decreases with application of optics, projecting on photocathode the image of luminescent spot and screen. General view of such functional generator as made by Academy of Sciences of USSR is shown in Fig. 110. In generator is provided possibility of rapid transition from circuit with controlled scanning beam to circuit, operating on static principle.

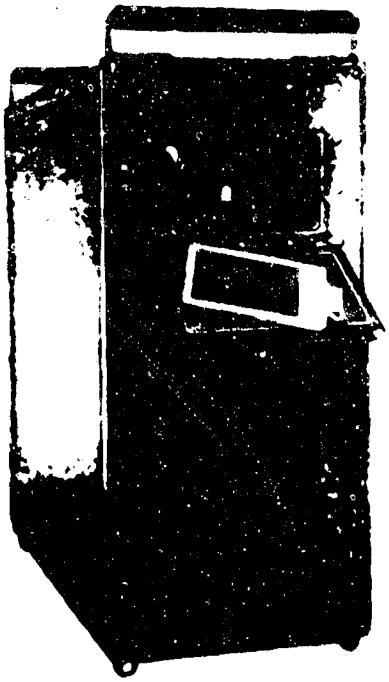

Fig. 110. General view of cathode-ray functional generator with controlled scanning of beam.

Results of experimental checking of operation of functional generator with controlled unfolding of beam showed that

1) error of reproduction of functions with hand manufacture of mask lies within 1.5%;

2) drift of output voltage of order 0.4 v in 15 min;

3) passband -- 30 c;

4) limits of change of output voltage ±120 v with 10 kilohm load.

Comparison of functional generators, made of cathode-ray tubes on the static principle and those with controlled scanning of beam shows that the latter, with other technical indices equal, require for their realisation more electronic equipment and give the worst use of the useful area of the mask. However along with functional generation these devices allow us to carry out also frequency modulation and several other cofunctions. Continuous movement of the beam on the screen in these devices gives a certain increase of period of service of cathode-ray tube, ensuring more uniform wear of screen.

By cathode-ray functional generators it is possible to reproduce also functions of two variables (G. Korn and T. Korn [1]). With this aim between the screen of cathode-ray tube and photomultiplier is placed a mask, whose transparency changes from point to point. The quantity of luminous flux, and consequently, the output current of photomultiplier will be proportional to transparency at a given point of the mask. Making the mask in such a manner that its transparency changes with respect to the given function, from the coordinates of the beam (or voltages on deflecting plates). One can obtain a functional generator of two independent variables

$$I_{\text{фэу}} = f(e_{\text{вх}\,1}, e_{\text{вх}\,2}).$$

Application of such functional generator simplifies obtaining of complicated functions of the form

$$e_{\text{вых}} = \nu\sqrt{e^2_{\text{вх}\,1} + e^2_{\text{вх}\,2}}.$$

However accuracy of their work is low and constitutes 2-10% and the device itself is complicated, since it requires very stable operation of the screen of the tube and photomultiplier. Furthermore, technology of obtaining of semitransparent masks with given distribution of transparency is sufficiently labor-consuming.

During construction of specialised functional generators of two variables semitransparent mask can be located inside the electron-beam tube. Here the need of a photomultiplier, naturally, disappears.

4. Pulse Functional Generators of Open Type.

In recent years there have been offered a number of circuits (L. G. Polimerov [1], E. E. Newholt [1]) of pulse functional generators of open type using cathode-ray tubes. In these functional generators, just as in those described above (page 239), a photomultiplier serves only as an indicator of appearance of beam in slits of mask.

Functional diagram of a device is shown in Fig. 111. Here is depicted also a cathode-ray tube with mask, provided with two slits. Slit I corresponds to the function given for reproduction, and slit II serves to fix the axis of the argument. The beam with the help of scanning generator periodically deviates from the horizontal. At the moment of appearance of beam in slit II or slit I at the output of photomultiplier 2 appear pulses which change the position of trigger 6. As a result at the output of the trigger there can be singled out square pulses, duration of which will be proportional to the distance between edges of slits. So that measurement of this distance is made only during forward movement of beam (from slit II to slit I) slits are made of different width. This establishes a definite sequence of pulses during forward movement and other sequence during reverse. In the pulse selection unit 5 there is passed only the sequence of forward movement.

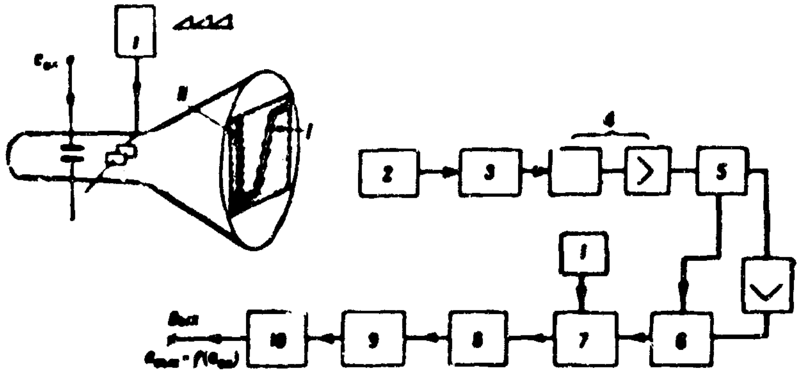

Fig. 111. Functional diagram of pulse functional generator of open type. 1--generator of scanning; 2--photomultiplier; 3--limiter; 4--pulse shaper; 5--pulse selection unit; 6--trigger; 7--key; 8--RC circuit; 9--peak detector; 10--cathode follower.

Conversion of duration of square pulses into voltage is carried out by integrating network 8, and voltage on capacitor is fixed by peak detector 9. With frequency of scanning 500 c operating frequency of input signals does not exceed 5 c.

Such functional generator is yielding to generators of closed type with controlled scanning of beam, since, other conditions equal, its accuracy depends on linearity of generator of scanning, and the passband turns out to be considerably narrower.

Essential improvement of performance of the considered functional generations was attained in the work of A. I. Petrenko [1], who offered a method of combining of pulse and continuous circuits.

CHAPTER VIII

MULTIPLIERS AND DIVIDERS

1. Classification and Short Survey of Principles of Construction of Multipliers.

Various constructions of multipliers and dividers are conveniently classified on the diagram, brought in Fig. 112. Here all devices are broken down into two groups: devices of direct and indirect action.

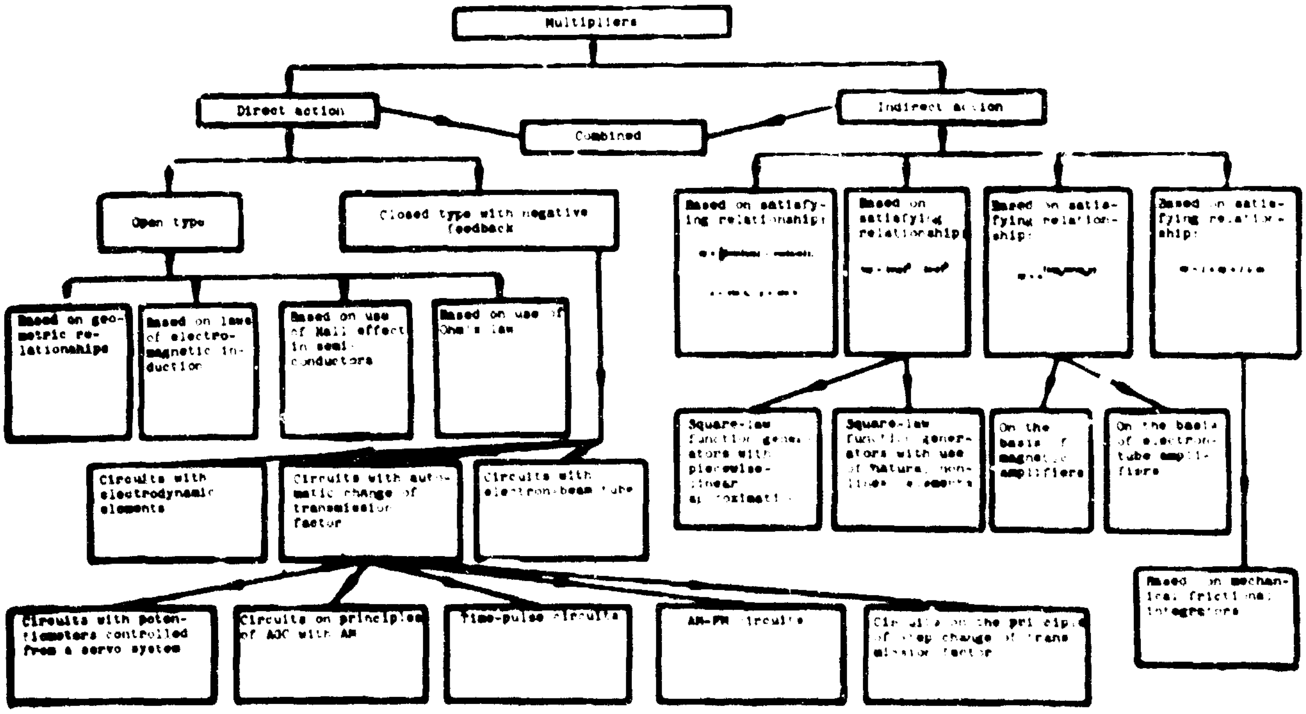
Multipliers
Direct action
Combined
Indirect action
Open type
Closed type with negative feedback
Based on geometric relationships
Based on laws of electromagnetic induction
Based on use of Hall effect in semiconductors
Based on use of Ohm's law
Circuits with electrodynamic elements
Circuits with automatic change of transmission factor
Circuits with electron-beam tube
Circuits with potentiometers controlled from a servo system
Circuits on principles of AOC with AM
Time-pulse circuits
AM-FM circuits
Circuits on the principle of step change of transmission factor
Based on satisfying relationship:
Based on satisfying relationship:
Based on satisfying relationship:
Based on satisfying relationship:
Square-law function generators with piecewise-linear approximation
Square-law function generators with use of natural non-linear elements
On the basis of magnetic amplifiers
On the basis of electron-tube amplifiers
Based on mechanical frictional integrators

Fig. 112. Classification of multipliers.

In direct action devices the operation of multiplication (division) of two independent variables is carried out directly by use of various physical laws.

In indirect action devices the operation of multiplication is carried out by a number of other mathematical operations. Here usually the transition to other mathematical operations is carried out on the basis of known relationships of analysis or algebra and requires, as a rule, realization of functional generation. Thus, for example, using relationship

$$4xy = (x + y)^2 - (x - y)^2, \qquad (8.1)$$

it is possible to replace operation of multiplication by operations of algebraic addition and squaring.

Functional diagram of device, reproducing equation (8.1) is shown in Fig. 113.

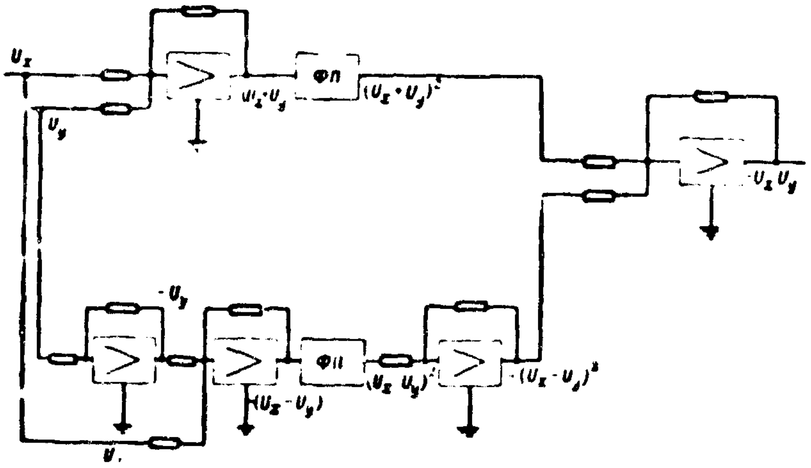

Fig. 113. Functional diagram of multiplier with square-law function generators. ФП -- square-law function generators.

As follows from the figure, for realization of operation of multiplication there are required linear computing elements for summation and change of sign and two functional generators, carrying out squaring.

For construction of such functional generator, there can be used, for example: quadratic dependence of plate current on grid voltage of three-electrode electron tube in regions of small plate currents, nonlinear characteristics of pentodes (J. M. Dukes [1]), and cathode-ray or diode functional generators.

As another example let us consider obtaining of a product on the basis of relationship

$$xy = a^{(\lg_a x + \lg_a y)}. \qquad (8.2)$$

Here it is already required to carry out other functional generation-finding a logarithm of an independent variable. The antilogarithm here is often obtained by coupling a logarithmic generator in the feedback circuit of the operational amplifier.

Since logarithmic function is not definite for negative values of argument, then multipliers of this type are useful directly only if the co-factors do not change sign. Otherwise it is necessary to add constant positive quantities to co-factors x and y, and then by subtraction of corresponding quantities single out the desired product.

Thus, for example,

$$(x + a)(y + b) = \mathrm{anti}\,[\lg(x + a) + \lg(y + b)].$$

whence

$$xy = -ab - ay - bx + \mathrm{anti}\,[\lg(x + a) + \lg(y + b)]. \qquad (8.3)$$

Logarithmic multipliers allow us to execute not only multiplication of several co-factors (two or more) and also involution and extraction of root.

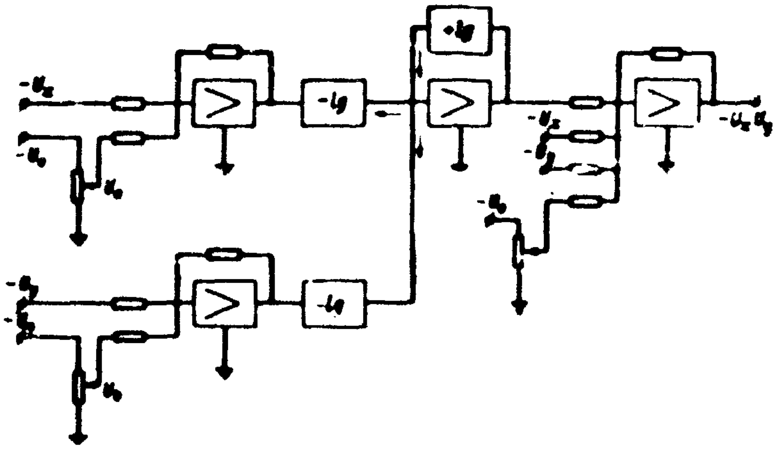

Fig. 114. Functional diagram of multiplier with logarithmic generators.

In Fig. 114 there is presented the functional diagram of such a multiplier. Logarithmic functional generators can be based on the diverse principles. In certain cases there are used natural nonlinear characteristics of electron tubes.

In Fig. 115 is brought circuit (see B. Chance, V. Huges [1]) for obtaining product on the basis of expression (8.2) with use of known property of grid characteristic of triode:

$$U_g = k \log I_g \tag{8.4}$$

which is correct with low grid currents, i.e., with large values of resistors, connected in grid circuit. For integrating amplifier it is possible to write

$$U_1 = U_z = -K_y[(U_{a_0})_1 - \mu U_{g_1} + (U_{a_0})_2 - \mu U_{g_2} - (U_{a_0})_3 - \mu U_{g_3} - U_0]$$

or

$$(U_{a_0})_1 - \mu U_{g_1} + (U_{a_0})_2 - \mu U_{g_2} - (U_{a_0})_3 + \mu U_{g_3} - U_0 + \frac{U_1}{K_y} = 0.$$

where subscripts 1, 2, 3 signify the number of the tube in the circuit.

Selecting U_0 in such a manner that $(U_{a_0})_1 + (U_{a_0})_2 - (U_{a_0})_3 = U_0$, and assuming $K_y \to \infty$, we receive

$$-U_{g_1} - U_{g_2} + U_{g_3} = 0,$$

whence it follows

$$\log I_{g_1} + \log I_{g_2} = \log I_{g_3} \quad \text{or} \quad U_1 = U_z = \beta U_x U_y. \tag{8.5}$$

Consequently, voltage U_z is the measure of product xy.

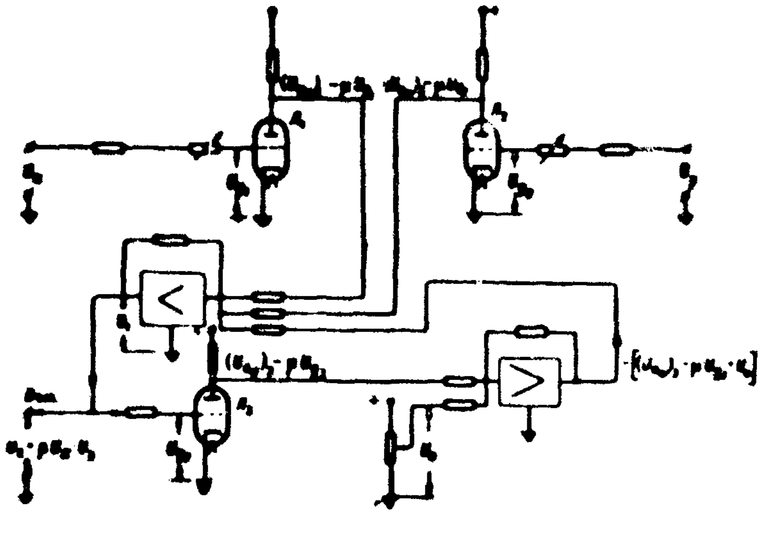

Fig. 115. Diagram of multiplier based on logarithmic dependence of plate current on grid voltage of triode.

Instability of characteristics of electron tubes leads to increase of error of such devices. In connection with this lately widely used are multipliers (B. Davis, I. Swift [1]) in which reproduction of logarithmic function is carried out on the basis of piecewise-linear approximation.

If one were to use the expression for total differential of product of functions

$$d(xy) = x\,dy + y\,dx, \tag{8.6}$$

after integration of both parts of equation we receive

$$xy = \int x\,dy + \int y\,dx. \tag{8.7}$$

In this case operation of multiplication is reduced to integration and summation. Since integration here should be produced for every independent variable, in the general case not being time, this method of obtaining a product cannot be realized on electronic integrators, carrying out, as we know, integration only with respect to time. This method, however, is successfully used in solving problems on mechanical integrators (I. S. Bruk [1], V. Bush [1]).

Comparative complexity of devices of indirect action and dependence of their error on accuracy of fulfillment of separate elements caused development of devices which execute the operation of multiplication (division) directly.

Direct action devices by principle of construction can be divided into two groups: compensational devices, either closed (with negative feedback), or parametric--open.

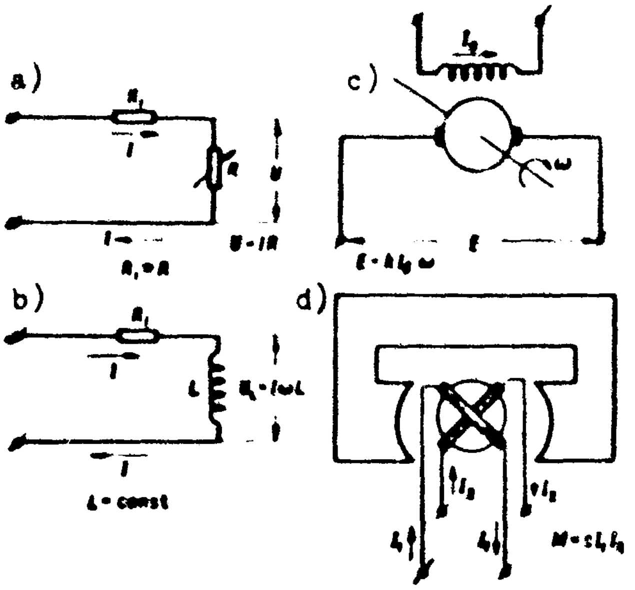

Fig. 116. Various principles of construction of multipliers.

In parametric devices are used elements, whose physical properties ensure fulfillment of multiplication. Examples are linear d-c and a-c circuits (Fig. 116, a and b), for which the voltage drop on a circuit element is the product of two independent variables--current and resistance, or a wattmeter element (Fig. 116d), for which the moment developed by the mobile system is the measure of the product of two independent variables -- two currents or current and voltage. Likewise, the electromotive force is developed by a generator on idle running (Fig. 116c), is proportional to the product of two independent variables: speed of rotation of armature and quantity of magnetic flux.

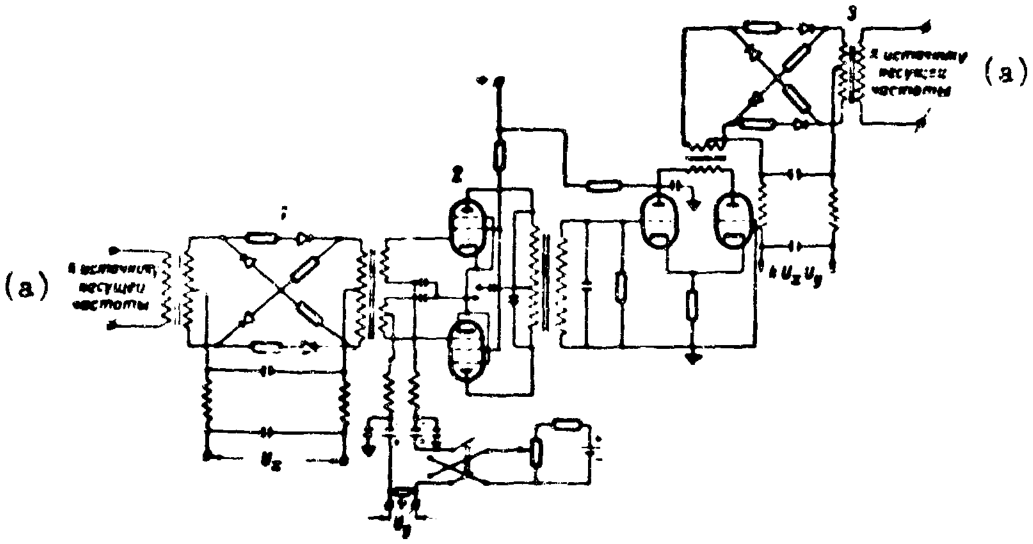

Fig. 117. Multiplier based on amplitude modulation of signals. 1--first balanced modulator; 2--second balanced modulator; 3--demodulator.
KEY: (a) To carrier frequency source.

For obtaining a product there may also be used the phenomenon of amplitude modulation. In Fig. 117 is depicted the fundamental circuit of one such device (G. D. McCann, C H. Wilts, and B. N. Locanthi [1]), made on the basis of two balanced modulators: one, assembled in a annular circuit of dry-disc rectifiers, and the other--tube, using property of variable-mv tubes to change steepness of grid characteristic (in definite limits) directly proportionally to voltage applied to grid.

Modulated in the first modulator the input signal will form as it were the voltage of the carrier frequency for the second modulator, where this carrier frequency a second time is modulated by a second input voltage. Double-modulated voltage of carrier frequency after amplification is rectified by phased rectifier (demodulator). Magnitude of rectified voltage turns out to be proportional to product of U_x and U_y.

In compensational devices the operation of multiplication is executed with coverage of the device or its main elements by negative feedback. Here it turns out that the result of operation of these devices under certain conditions does not depend on change of characteristics of elements, covered by feedback. Compensational multipliers are constructed on diverse principles. They can be divided into three main groups:

1) devices, based on automatic change of transmission factor of a certain network;

2) devices with electrodynamic elements;

3) devices, based on application of electron-beam tube with transverse electric and magnetic fields.

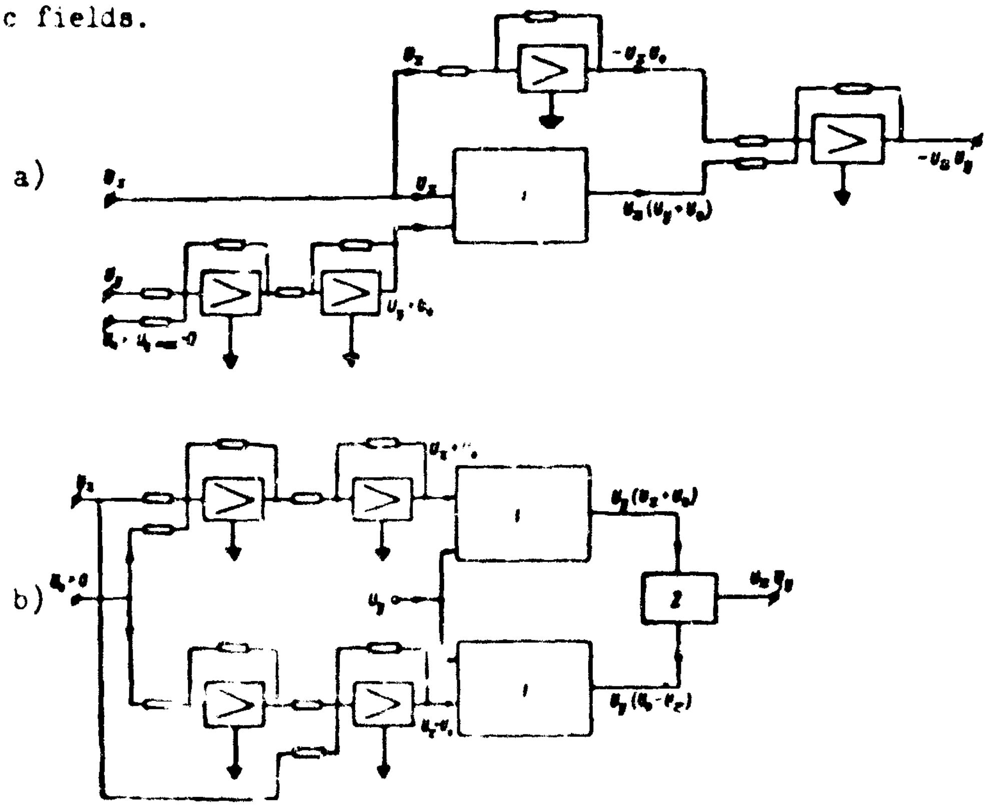

Fig. 118. Methods of realization of multiplication of sign-alternating signals. 1--multipliers, working with one sign-alternating co-factor; 2--differential amplifier.

Multipliers, made in the form of purely electronic systems, provide a sufficiently wide passband of signals and error within 0.1 - 2%. Electromechanical device although they ensure in principle large accuracy, have a narrow passband (up to 1 c).

Usually to multipliers is presented the requirement to carry out the operation of multiplication of two co-factors, each of which can take both positive and negative values. Above it was shown how, with the help of a multiplier, working in principle only with positive values of co-factors (multiplier with logarithmic generator), it is possible to carry out multiplication of sign-alternating co-factors. In Fig. 118a and 118b are brought methods (G. Korn and T. Korn [1]) of realization of multiplication of two sign-alternating co-factors by multipliers, in principle allowing change of sign for only one co-factor.

In recent years most widely applied are multipliers based on the time-pulse principle, and multipliers made from square-law functional generators. The first ensure comparatively high accuracy 0.1 to 0.2% in a relatively narrow passband, the latter differ by a wide passband and accuracy within 0.5 to 1%.

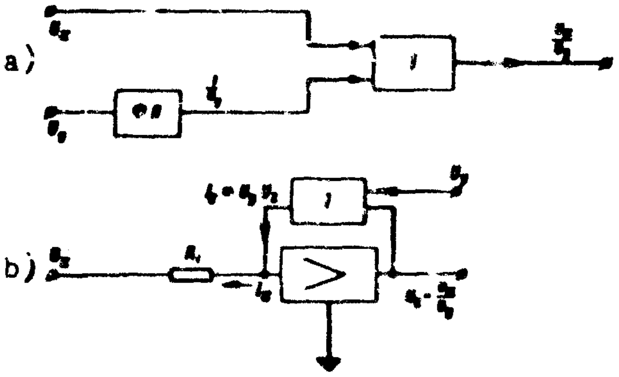

Fig. 119. Methods of construction of dividers. ФП --functional generator, 1--multiplier.

Operation of finding a quotient usually is executed either by a multiplier in combination with a functional generator, giving the reciprocal (Fig. 119a), or by coupling a multiplier in the feedback circuit of an amplifier with large amplification factor (Fig. 119b).

In the last case these equalities are correct

$$I_1 = kU_zU_y, \qquad I_2 = k_1U_x$$

and since $I_1 = I_2$ then

$$U_z = \frac{k_1}{k}\frac{U_x}{U_y}. \tag{8.8}$$

The latter method of obtaining a divider requires less equipment and gives the possibility to obtaining the operation of division in a wider range of change of U_y as compared with application of a functional generator reformer for obtaining of quantities, reciprocal to U_y.

2. Multipliers Based on Principle of Automatically Regulated Transmission Factor.

These devices usually consist of a network, to whose input is fed a voltage, representing one co-factor, and the transmission factor automatically changes lenearly with change of voltage, representing the second co-factor.

Depending upon peculiarity of physical realization of this network we distinguish:

a) devices, made from an operational amplifier with a transmission factor changed by steps;

b) time-pulse devices, or, as they are otherwise called, devices with pulse dividers;

c) devices, based on double amplitude modulation;

d) AM-FM devices.

For increase of accuracy of conversion of one of the co-factors into the transmission factor of the network there is used introduction of negative feedback.

On skeleton diagram, Fig. 120, by Д-1 is designated the divider, to whose input is fed constant voltage U_0, and from the output is removed part αU_0 (where α is the instantaneous value of the divider's transmission factor). This signal is compared by differential amplifier 3 with signal U_x, representing one of the co-factors. In case of mismatch, to the divider is fed a signal which

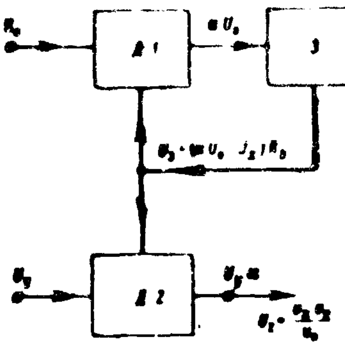

Fig. 120. Method of automatic control of transmission factor by negative feedback. Д-1, Д-2 --dividers; 3--differential amplifiers.

changes its transmission factor in such direction as to eliminate this mismatch. In a steady-state regime with very large K_3 this relationship is correct:

$$aU_0 = U_x,$$

whence

$$a = \frac{1}{U_0} U_x.$$

If the coefficient of division of the second divider Д-2 changes owing to the same mismatch signal, then during feeding its input with voltage of second co-factor U_y we receive

$$U_z = aU_y = \frac{U_x U_y}{U_0}. \qquad (8.9)$$

Let us consider certain practical circuits of the above-mentioned multipliers.

a) Multiplier based on step change of transmission factor. In the multiplier, based on step change of transmission factor of operational amplifier, there is used the basic ratio for an operational amplifier:

$$e_{вых} = -\frac{Y_1}{Y_2} e_{вх\,1}.$$

When Y_2 = const output voltage is proportional to product of conductance Y_1 and $e_{вх\,1}$. If one were to make conductance Y_1 according to law

$$Y_1 = ke_{вх\,2}. \qquad (8.10)$$

then such a device would execute the operation of multiplication.

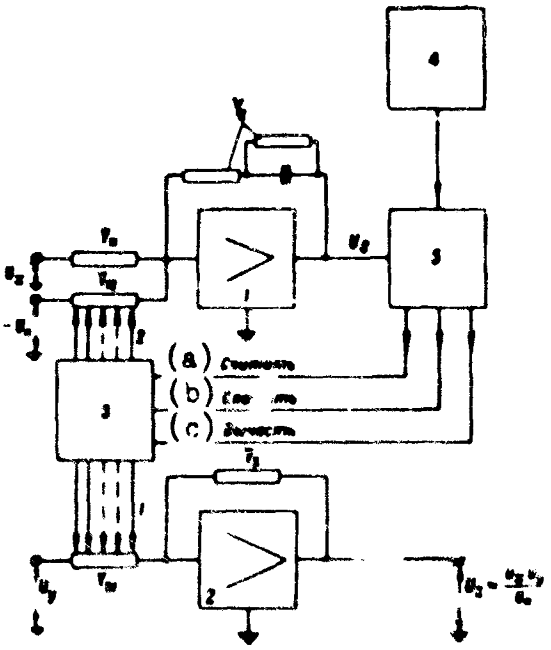

Fig. 121. Skeleton diagram of multipliers with step change of transmission factor. 1, 2--amplifiers; 3--reversible binary counter; 4--pulse generator and shaping cascade; 5--command output, sensitive to polarity and magnitude of mismatch signals.
KEY: (a) Count; (b) Add; (c) Subtract.

However it presents great difficulties to execute conductance, linearly changing under the influence of voltage and not introducing here its own emf to the circuit.

Therefore we usually resort to approximate realization of relationship (8.10), replacing linear dependence by step or changing to linear dependence between mean values.

In Fig. 121 is brought skeleton diagram of one variant of such a multiplier (E. A. Goldberg [1]). Conversion of one of co-factors into a proportional change of conductance is carried out here by a unique system of negative feedback, including a digital device.

For the first operational amplifier this equality is correct:

$$U_4 = -\left[U_x \frac{Y_{11}}{Y_0} - U_3 \frac{Y_{12}}{Y_0}\right]. \qquad (8.11)$$

If this voltage exceeds a specific positive value $U_{4_1} > 0$, the device delivering commands lets pulses pass to the binary counter, which for each pulse takes away one from the number earlier fixed on it. To each digit of the binary counter there corresponds a relay and a conductance (ΔY_{12}) connected by this relay to the input of the operational amplifier. This process will occur until Y_{12} changes so much that with constant U_3 U_4 will be less than U_{4_1}. In this case access of pulses to binary counter will stop. When $U_4 < U_{4_1} < 0$, then device delivering commands again lets pulses pass to the binary counter, but now every pulse already adds one to the number on the counter and thereby switches the relay in such a manner that

$$|U_4| < |U_{4_1}|.$$

When $U_4 \to 0$ in a steady-state regime this relationship is correct:

$$U_x \frac{Y_{11}}{Y_0} - U_3 \frac{Y_{12}}{Y_0} = 0,$$

whence

$$Y_{12} = \frac{Y_{11}}{U_3} U_x \qquad (8.12)$$

If binary counter simultaneously controls several identical systems of relays, and the latter commute conductance to input of other operational amplifiers, then at the output, for example, of operational amplifier 2 we will receive

$$U_z = -U_y \frac{Y_{21}}{Y_0}.$$

If we assume that $Y_{[illegible]} = Y_{[illegible]}$, and $Y_{[illegible]} = Y_2$, then

$$U_1 = \frac{U_x U_y}{U_0} \qquad (8.[illegible])$$

Since input impedance, commuted by the relays, is constituted from passive elements, then for realization of multiplication with sign-alternating co-factor U_x we use addition to the input of first operational amplifer of a positive fixed voltage U_0, in magnitude exceeding the maximum possible value of voltage U_x. Here in the product there is a superfluous component $\frac{U_0 U_y}{U_0}$, which must be deducted by feeding the input of the operational amplifier 2 an additional component after sign-inverter 3 (Fig. 122).

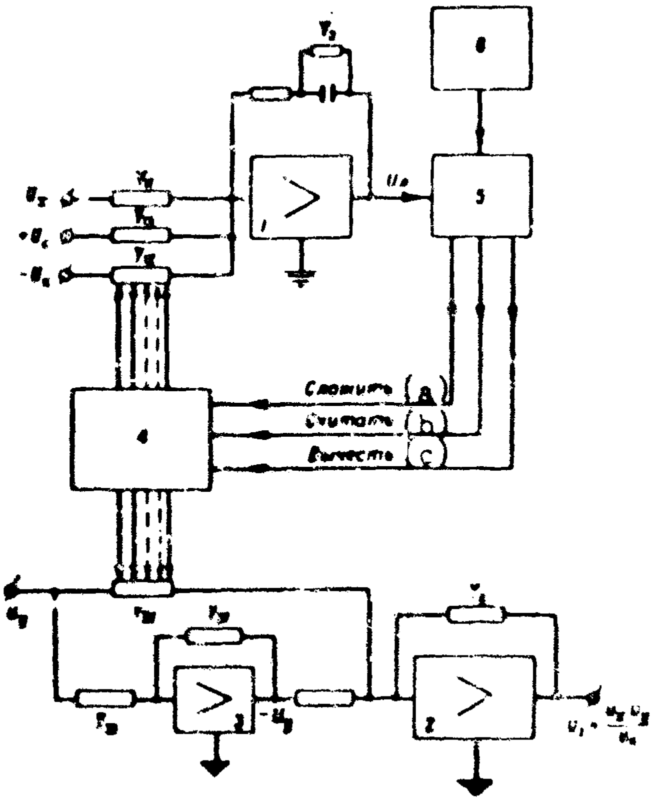

Fig. 122. The same as in Fig. 121 but for multiplication of sign-alternating signals. 1, 2, 3--amplifiers; 4--reversible binary counter; 5--command output; 6--pulse generator and shaping cascade.
KEY: (a) Add; (b) Count; (c) Subtract.

In the described device there were provided 11 binary digits in the binary counter and correspondingly 11 passive circuits, commuted to input of operational amplifier which allowed us to set conductance at the output with accuracy of 0.0025% in reference to maximum value.

For acceleration of operation of device as commuting devices there were used special high speed relays with a response time, not exceeding 100 microseconds. At a frequency of pulse repetition of 1000 c the time, required to change Y_{12} with intermittent change of U_x by a total magnitude, constituted 1 sec.

The considered device allows us to realize simultaneous multiplication of magnitude U_x by several variables U_{y1}, U_{y2}, U_{y3},...Here we need additional conductances, controlled from relays, and corresponding quantity of operation amplifiers. Other equipment will be common for all multiplication circuits.

The device as a whole possesses a low passband and requires a large number of electron tubes.

b)Multipliers, based on application of pulse voltage dividers (time-pulse devices). Multipliers considered in this section are based on change of spacing of periodic sign-alternating rectangular pulses proportional to one of the co-factors and of the amplitude of these pulses proportional to the second.

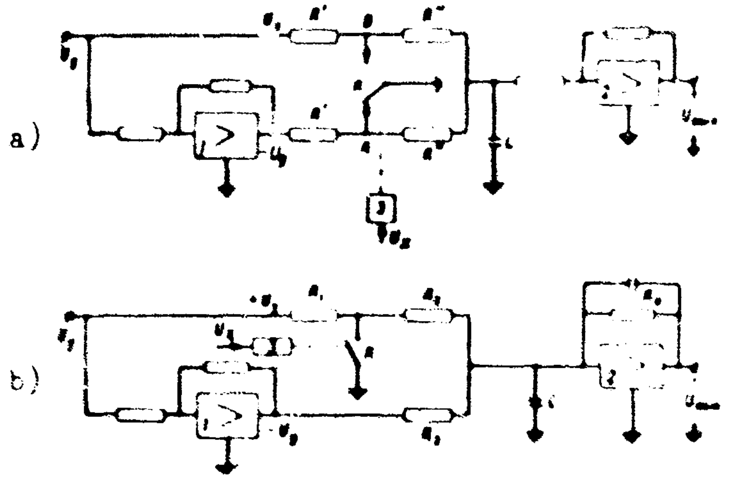

Fig. 123. Principles of construction of time-pulse multipliers. 3--key controlling units.

In Fig. 123a is a skeleton diagram of the device, explaining the principle of its operation. Key K is periodically switched from contact A to contact B, grounding thereby the lower, and then the upper voltage divider. To the upper voltage divider is fed a positive value of first co-factor, and to the lower--a negative, obtained with the help of sign-inverting amplifier 1.

To the output of the device moves voltage [illegible], when key is on contact A and-- $2U_y$ with transition of key to contact B.

Let time of stay of key on contact A be t_1, on contact B, t_2, and total period of operation of key--T.

We will calculate mean value of output voltage of device for period T:

$$U_{\text{вых ср}} = 2U_y \frac{t_1 - t_2}{T}. \tag{8.14}$$

From formula it follows that mean value of output voltage will be proportional to product of U_y by relative duration (spacing) of operation of key:

$$\xi = \frac{t_1 - t_2}{T}. \tag{8.15}$$

If one were to construct circuit of control of key K in such a manner that ξ k_1U_x, then

$$U_{\text{вых ср}} = U_yU_x2k_1. \tag{8.16}$$

So that such a device operates correctly, it is necessary that frequency of switching of key is significantly higher than frequency of change of input signals U_x and U_y, i.e., that for the period of operation of key K these voltages can be considered practically constant. Capacitor C serves to smooth the sign-alternating high-frequency pulses.

For simplification it is possible to execute the circuit of the multiplier with sign-alternating co-factors, using only a one-way key (Fig. 123b). In this case, so that output voltage of amplifier 2 had equal amplitudes during open and closed state of the key, obviously, it is necessary, that

$$R_3 = 2(R_1 + R_2).$$

In recent years there have been offered a number of devices, working on this principle. These devices differ mainly in method of physical realization of key and circuit of obtaining the dependence $\xi = k_1U_x$. As key frequently there are used polarized relays, fed by alternating current (D. Isle [1], G. Korn and T. Korn [2]).

Simultaneously with feeding by alternating current into coil of relay is introduced magnetizing, created by voltage U_x of second co-factor. Thanks to flow of magnetizing current armature of relay lags in one extreme position longer than in the other, and thus there is attained change of spacing of rectangular pulses.

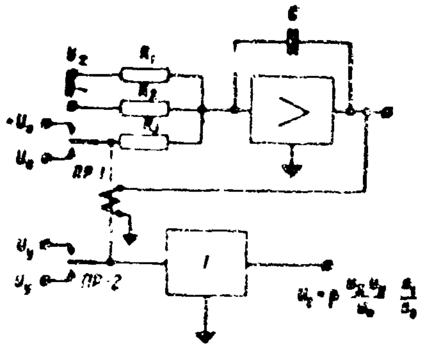

Fig. 124. Electromechanical time-pulse multipliers with negative feedback. ПР -- polarized relays, β --static transmission factor of filter, 1--smoothing filter.

To improve linear dependence of spacing of pulses on voltage U_x there is applied the compensational circuit (G. Korn and T. Korn [2]) shown in Fig. 124.

This circuit can work also in the absence of external alternating voltage due to natural oscillations.

Such multipliers are outstanding in their simplicity and comparatively low cost. Error of their work is 3-3.5%, and passband is not higher than 1 cycle.

Desire to expand passband of such devices and increase accuracy of their operation lead to replacement of relays by the electronic key.

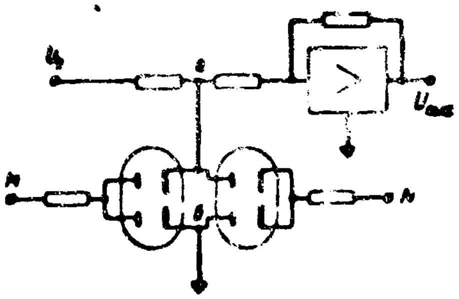

Fig. 125. Circuit of diode key.

The most wide-spread circuit of diode key, applied in these devices, is shown in Fig. 125 for the case of a one-way key. To terminals M—N is fed alternately voltage from control unit by the key first ±150 v and then ∓150 v. When there is fed ±150v, both pairs of diodes open, whereupon the difference of potentials between points a and b becomes equal to zero; the key is closed. In the case of opposite polarity diodes are closed and the key is open; the input signal passes into the operational amplifier.

In Fig. 126a is the functional, and in Fig. 126b the fundamental diagram of a multiplier, developed at the Academy of Sciences of USSR.*

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*Figs. 126 and 127 are borrowed from work of I. S. Bruk and N. N. Lenov [1].

Stable frequency oscillator creates sinusoidal oscillations, which with the help of pulse shaping circuit ФИ will be converted into pulses with frequency 2.5 kc, starting oscillator of saw-tooth oscillations ПГ .

The pulse shaper at the beginning of each operating period of the saw-tooth oscillator gives a pulse to trigger T, bringing it to initial position. ПГ for each cycle of operation gives a voltage linearly variable in time with a small return time (of the order of 0.05 T), symmetric relative to zero (Fig. 126b). Voltage $U_{\text{пг}}$ moves to gain comparator CA, where it is compared with input voltage U_x. At the moment of equality of $U_{\text{пг}}$ and U_x the gain comparator sends a pulse to second input of trigger T and transfers it to another position. Output voltage of the trigger controls the diode key, consisting of four double diodes and two voltage dividers (Fig. 127).

From the diagram of change of voltages at separate points of the circuit, in Fig. 126b, it follows that spacing of rectangular pulses at output will be

$$\xi = \frac{t_1 - t_2}{T} = \frac{U_x}{U_{\text{пг}}}.$$

Therefore according to expression (8.14) mean value of output voltage will constitute

$$U_{\text{вых ср}} = \frac{1}{U_{\text{пг}}} U_x U_y. \qquad (8.17)$$

Stable frequency oscillator, pulse shaper and saw-tooth oscillator are mounted separately and are equipment, common to all multipliers the simulator.

Voltage from ПГ moves to gain comparator through cathode follower, reproducing input signal with error, not exceeding 0.05% (tubes $Л_1$ and $\frac{1}{2}Л_2$ on the diagram of Fig. 127), and eliminating interaction of separate multipliers of the installation. The gain comparator is built by a circuit with positive feedback through transformer ИТ (tube $Л_4$). Trigger consists of double triode 6N8 ($Л_5$). Diode key consists of four 6N8 tubes ($Л_6$ -- $Л_9$), working in diode regime. With the help of toggle switch T it is possible to change the sign of output voltage of the device.

Sign-changing amplifier and output amplifier are standard operational amplifiers of an analog.

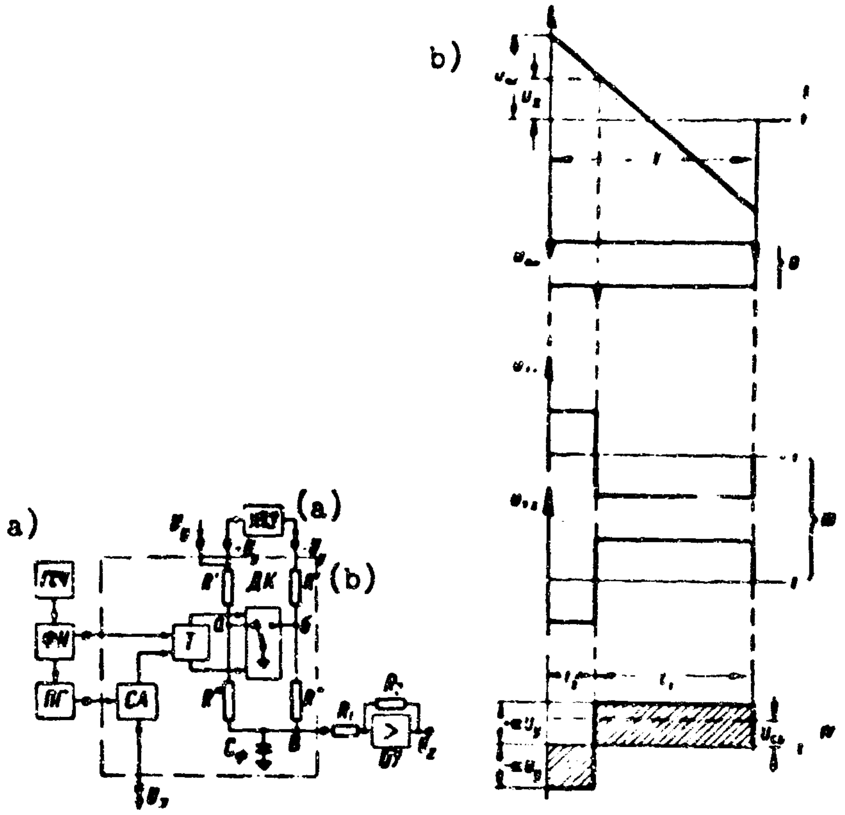

Fig. 126. Multiplier of analog of Academy of Sciences of USSR.
KEY: (a) Sign-changing amplifier; (b) Diode key.

As expression (8.17) and analysis of work of such devices show, main sources of error for them will be: unequalness of steepness of leading and trailing edges of pulses at the trigger output, instability of amplitude, error in linearity of voltage U_{sr} , and also imprecise work of keys with increase of the commutation frequency. According to the data of N. N. Lenov [1], total error of such a device constitutes 1%, and the passband is 19 c.

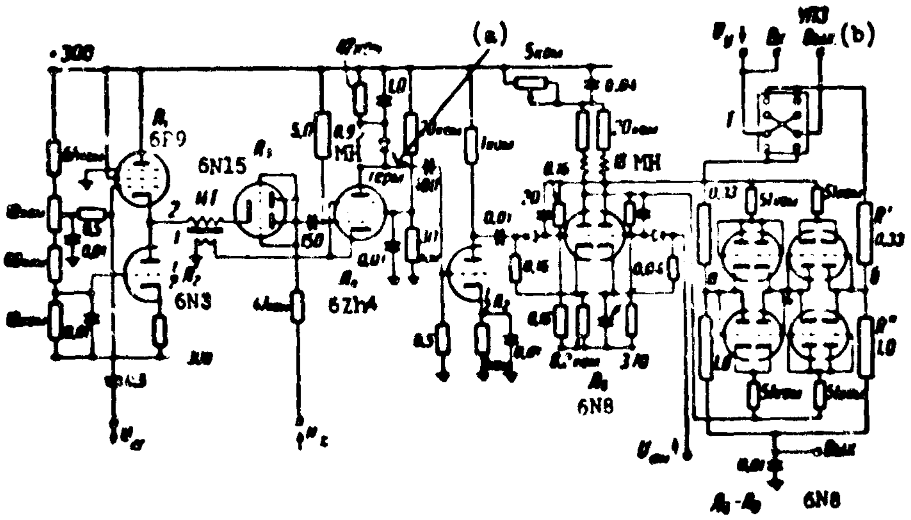

Fig. 127. Operation of multiplier of Fig. 126.
KEY: (a) Germanium; (b) Sign-changing amplifier.

It is possible to significantly decrease influence of above-mentioned factors on the device's error, if the circuit of conversion of co-factor U_x into spacing of rectangular pulses is executed on the principle of negative feedback, as this is offered in Vol. 21 of Trans. Massachusetts Institute of Technology and in further developed by A. A. Fel'dbaum and A. I. Manukhin [1].

In distinction from earlier considered circuit here trigger T simultaneously controls two diode one-way keys (Fig. 128). Key K_1 passes voltage of one of the co-factors U_y to output operational amplifier. To key K_2 is fed a constant 'reference' voltage U_0. Output voltage U_2 of key K_2 is averaged by filter R_ϕ, C_ϕ and is fed into amplifier 1 through resistor R_2 with a sign, opposite the sign of voltage U_x.

Thus, for amplifier 1 there will be formed a negative feedback circuit through trigger T, key K_2, filter and resistor R_2. When $U_x = 0$ there is established voltage U_1 of such magnitude that $\xi = 0$. Voltage at integrating point Σ of amplifier 1 will be

$$U_\Sigma = U_x \frac{R_2}{R_1 + R_2} - \frac{R_1}{R_1 + R_2} U_{cp}. \tag{8.18}$$

With a very large amplification factor of amplifier 1 magnitude U_Σ can be disregarded as compared with components of the right side of equation (8.18), and

$$U_{cp} = U_x \frac{R_2}{R_1}.$$

On the other hand, according to (8.14)

$$U_{cp} = aU_0\xi.$$

whence

$$\xi = \frac{R_2}{R_1} \frac{1}{aU_0} U_x. \tag{8.19}$$

From expression (8.19) it follows that considered circuit with large meaning of amplification factor of amplifier 1 establishes proportionality between input signal and spacing of rectangular pulses at the key output.

Instability of amplitude of voltage of saw-toothed oscillations U_{nr} , amplification factor K: amplifier 1 and value of initial spacing does not affect accuracy of conversion. Equally small deflections in linearity of voltage U_{nr} are immaterial.

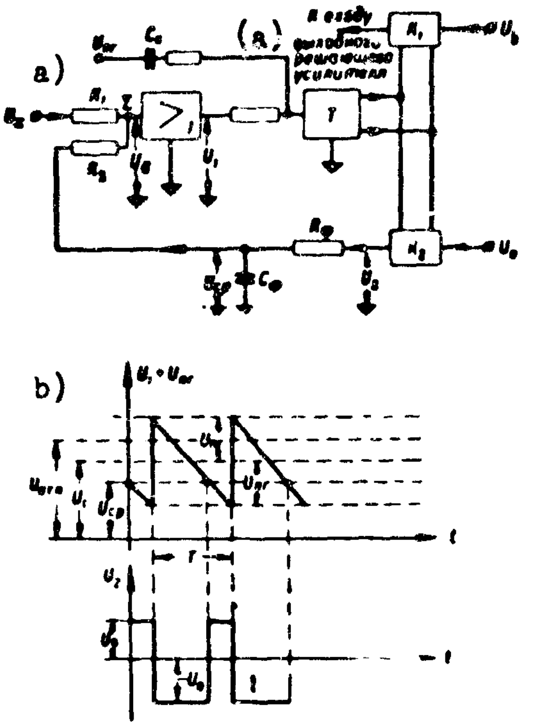

Fig. 128. Skeleton diagram of time-pulse multiplier with negative feedback and diode keys. KEY: (a) To input of output operational amplifier.

Accuracy of result will depend on stability of $U_0 \cdot \frac{R_2}{R_1}$ and τ.

This investigation carries a qualitative character, since by force of the finite value of amplification factor there will be introduced certain error. Furthermore, unequalness of time constant of charge and discharge of capacitor $C_ф$ during operation of the key will also affect the departure from linear dependence (8.19).

Fundamental circuit of a multiplier is shown in Fig. 129. Six envelopes in the circuit are distributed as follows: $Л_1$ and $Л_2$ constitute d-c amplifier 1, $Л_3$ will form an asymmetric trigger with cathode coupling, $Л_4$, $Л_5$ and $Л_6$ --diode keys K_1 and K_2. Tube $Л_5$ is common to diode keys K_1 and K_2. Capacitor C_2 serves to prevent of generation of the d-c amplifier, but capacitor C_1 will form integrating feedback, improving filtration of voltage in the circuit. Setting of trigger in intial position is carried out by a voltage pulse, appearing at the moment of a jump of saw-toothed voltage. Output voltage of considered multiplier is determined on the basis of (8.14) and (8.19) by expression

$$U_{вых. ср} = U_{ф ср} = \frac{R_2}{R_1} \frac{U_x U_y}{U_0}.$$

If reference voltage U_0 and voltage U_y change places, such a time-pulse device will execute the operation of division.

Main parameters of the device (according to the data of A. A. Fel'dbaum and A. I. Manukhin [1] with $U_{пг}$ frequency equal to 1000 c are such:

1) time of initiating the operating mode 12 min;

2) maximum drift of output voltage in 100 sec 80-100 mv, in 10 min—140 mv;

3) maximum error of product ±0.4v;

4) maximum background at output 1 mv;

5) gain-frequency response has a slump of less than 5% for channel U_x up to frequency 12 c, for channel U_y — up to frequency 16 c.

Further improvement time-pulse multipliers has been in the direction of increase of static accuracy and expansion of passband.*

In the circuit (C. D. Morrill and R. V. Baum [1]) shown in Fig. 130 this is attained by transition to stabilized electronic keys (K_1 and K_2) and introduction into the self-excited regime a circuit of conversion of U_x into spacing of pulses. Here there drops the need for a source of saw-toothed voltage.

Stabilized electronic key is an operational amplifier with two feedback circuits parallel to the amplifier. Each feedback circuit is commutated by a double triode, controlled through two-cycle amplifying cascade by rectangular pulses of trigger circuit.

Depending upon polarity of rectangular pulses, arriving at grid of tube Л_3 tube Л_1 or Л_2 opens. Output voltage is removed from points A and A' of keys K_1 and K_2 accordingly. When the upper feedback circuit (R_{01}) is closed, the lower is open, and voltage U_A is equal to voltage at integrating point of amplifier 1. In practice one can consider it equal to zero. Upon closing the lower feedback circuit the upper one opens and output voltage obtains the value

$$U_A = -\frac{R_{02}}{R_{11}} U_x. \tag{8.20}$$

With such a principle of construction of a key there is removed drift and influence of natural parameters of commutating tubes on accuracy of work of system, and we can also increase frequency of commutation of voltage U_x and thereby expand the passband.

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*See, for example, E. A. Goldberg [3], E. Flater and K. Fräntz [1].

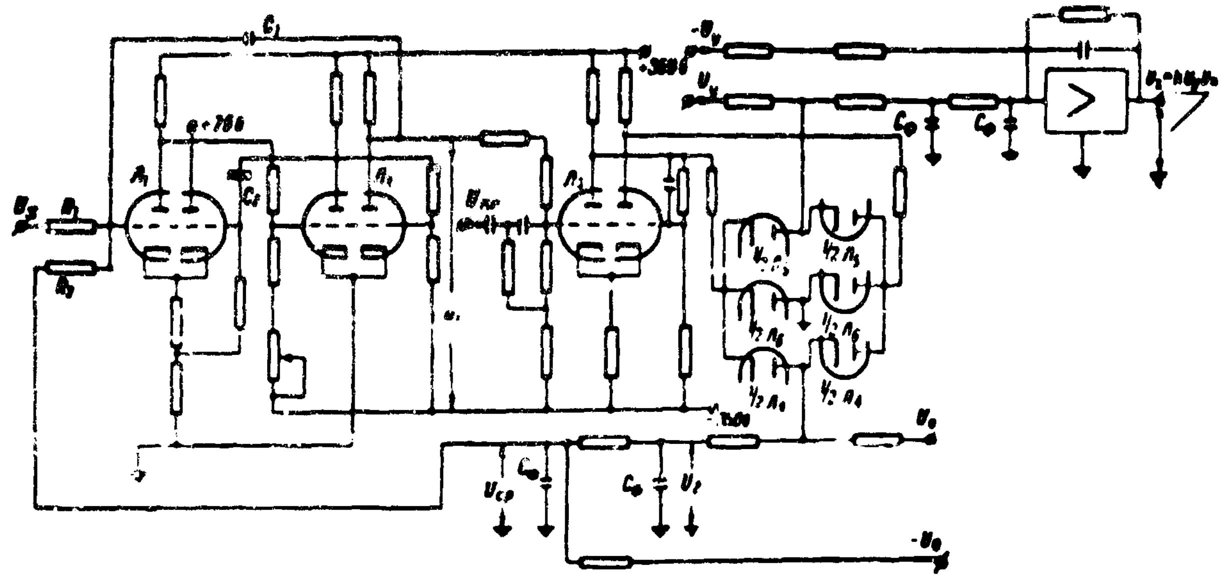

Fig. 129. Fundamental circuit of multiplier.

We will derive basic relationships for the considered circuit (Fig. 130).

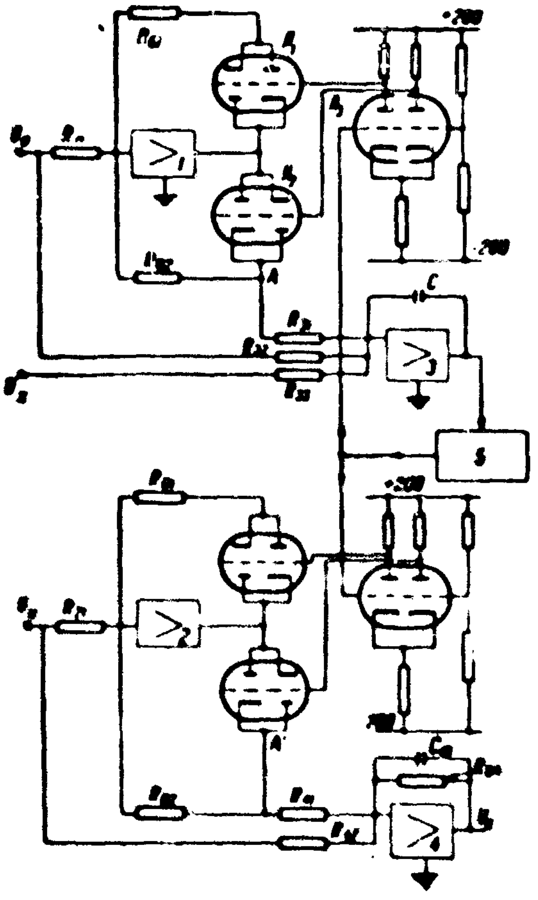

Fig. 130. Time-pulse multiplier with stabilized key. 1, 2, 3, 4--amplifiers; 5--trigger circuit with two stable equilibrium positions.

Let voltages of operation of trigger circuit be e_1 and e_2 ($e_1 > e_2$). Let us assume also that operation of trigger circuit after reaching e_2 corresponds to closing of key (Л_2 is open, and Л_1 is shut), and operation upon reaching e_1 corresponds to opening.

Change of voltage at output of integrator 3 for both cases when $\frac{R_{02}}{R_{11}} = 2$, $R_{31} = R_{32}$ and $U_0 < 0$ will be:

$$\left.\begin{aligned} e_1 - e_2 &= \frac{1}{C}\int_0^{T_1}\left(\frac{U_0}{R_{32}} - \frac{U_x}{R_{31}}\right)dt, \\ e_1 - e_2 &= \frac{1}{C}\int_0^{T_2}\left(\frac{U_0}{R_{31}} + \frac{U_x}{R_{32}}\right)dt \end{aligned}\right\} \quad (8.21)$$

If U_x changes little during the time T_1 and T_2, then duration of the section of build-up of voltage at output of integrator will be

$$T_1 = \frac{(e_1 - e_2)C}{\frac{U_0}{R_{32}} - \frac{U_x}{R_{33}}}. \qquad (8.22)$$

For section of voltage drop we obtain analogously

$$T_2 = \frac{(e_1 - e_2)C}{\frac{U_0}{R_{31}} + \frac{U_x}{R_{33}}}. \qquad (8.23)$$

Spacing of pulses, created by keys K_1 and K_2 when R_{31} R_{32}, is determined by expression

$$\xi = \frac{T_1 - T_2}{T_1 + T_2} = \frac{U_x}{U_0} \cdot \frac{R_{31}}{R_{33}}. \qquad (8.24)$$

Mean value of output voltage of operational amplifier 4 when $\frac{R_{03}}{R_{31}}$ = 2 and R_{41} R_{42} will be

$$U_{x\,cp} = U_y \frac{R_{04}}{R_{03}} \xi = \frac{R_{31}}{R_{33}} \frac{R_{04}}{R_{03}} \frac{U_x U_y}{U_0}. \qquad (8.25)$$

Frequency of natural oscillations of circuit of conversion of voltage U_x can be found when $R_{32} = R_{31}$ from expression

$$f = \frac{1}{T_1 + T_2} = \frac{1}{(e_1 - e_2)CR_{31}} \frac{U_0^2 - \left(\frac{R_{31}}{R_{33}} U_x\right)^2}{2U_0}. \qquad (8.26)$$

which shows that frequency of natural oscillations depends on magnitude of voltage U_0, U_x, time constant CR_{31} and voltages of operation of trigger circuit. When U_0 = 100 v this frequency of natural oscillations according to the data of C. D. Morrill and R. V. Baum [1] changes from f = 15 kilocycle when U_x = 0 to f = 10 kilocycle when U_x = 100 v. Error of product of such a multiplier does not exceed 0.1% and passband lies from 0 to 200 cycles. Total of required envelopes is 18.

Described devices, although possessing comparatively good technical characteristics, still are excessively complicated, unreliable and have exceptionally high cost. In recent years in connection with development of technology of semiconductor instruments and magnetic amplifiers there were repeated efforts to realize the time-pulse principle on tubeless elements (L. A. Finzi and R. A. Mathias [1], L. J. Craig [1], P. L. Van-Allen [1]).

As example let us consider the multiplier offered by R. L. Van-Allen. In it are used pulse dividers, controlled by keys, built on semiconductor triodes (R. L. Bright [1]). Relative duration of switched on state of key changes proportional to voltage, representing one of the co-factors, and amplitude of the voltage, commutated by the key, represents the second co-factor. Mean value of voltage, taken from load resistor here will be proportional to the product of the mentioned voltages of separate co-factors.

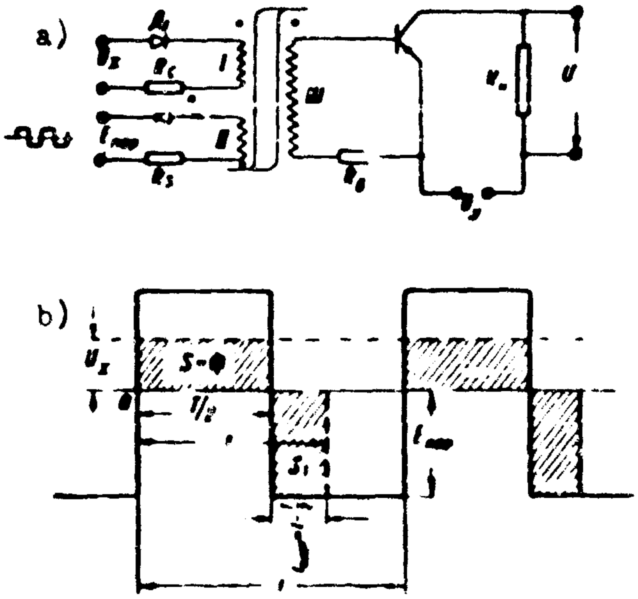

Fig. 131. Principle of action of time-pulse multiplier on semiconductor triodes and transformer with core, possessing a rectangular hysteresis loop.

Conversion of voltage in relative duration of work of semiconductor key is carried out by special circuit, the basis of which is a transformer with core, made from material with a rectangular hysteresis loop. Change of flow in such a core under the influence of applied voltage U_x one can determine from expression

$$\Phi = k\int_0^T U_x\,dt\,[\text{v x sec}]. \qquad (8.27)$$

if one were to ignore voltage drop in ohmic resistance of winding circuit I (Fig. 131a).

If at moment of application of voltage U_x the core already was saturated under the influence, for example, of voltage of rectangular form $F_{\text{пер}}$ and polarity U_x such that the core is here magnetically reversed, then change of flow for a fixed interval of time will be proportional to the mean val · of voltage U_x for the same interval of time. With application of constant voltage of magnetization $E_{\text{пер}}$ and a disconnected U_x the recovery time of saturated state will be proportional the accumulated change of flow, and consequently, to the mean value of voltage U_x.

Thanks to inclusion of diodes $Д_1$ and $Д_2$ in the circuit of windings I and II there is achieved the required sequence of application of voltages U_x and $E_{\text{пер}}$. Indeed, in the half-period, when $E_{\text{пер}}$ is disconnected from winding II, under action of voltage U_x in the core is stored a chan· of flow; in the other half-period after opening diode $Д_2$ there is a return to the former saturated state. Here for the duration of time $\left(t - \frac{T}{2}\right)$ diode $Д_1$ closed under the influence of the voltage induced in the winding. Since S - S_1 (Fig. 131b), then

$$t_{\text{имп}} = U_x \frac{T}{2E_{\text{пер}}}. \qquad (8.28)$$

In the half-period of connection of voltage U_x in winding III there is induced a voltage of such polarity, that the semiconductor triode is closed (potential of base is higher than potential of the emitter) and resistance of the emitter--collector section sharply increases. This state corresponds to opening of the key.

In interval $\left(t - \frac{T}{2}\right)$ polarity of induced voltage changes, the triode opens and to load resistor in practice there is fed total voltage.

Thus, the mean value of voltage on load resistor for period T will be proportional to the desired product

$$e_{\text{вых ср}} = \frac{U_y t_{\text{имп}}}{T} = \frac{U_y U_x}{2E_{\text{пер}}}. \qquad (8.29)$$

In Fig. 132 is brought a full fundamental circuit, designed for multiplication of sign-alternating co-factors. In this circuit for improvement of commutation of circuits of voltage U_x and $E_{\text{пер}}$ there are introduced two semiconductor keys Tp-5 and Tp-6 and an auxiliary transformer with windings W_1, W_2, W_3 and W_4, fed by voltage $E_{\text{пер}}$. Besides, for limitation of current in the circuit $E_{\text{пер}}$ there are introduced diodes $Д_4$ and $Д_3$ with reference-voltage sources E.

To guarantee accurate work there is required high degree of stabilization of voltages E and $E_{пер}$. It is necessary also that sources of U_x and U_y possess constant internal resistance, not exceeding 1000 ohms.

The passband is determined by the frequency of $E_{пер}$, and usually its upper boundary is at least an order lower than the frequency of $E_{пер}$. Static accuracy according to the data of R. L. Van-Allen [1] constitutes ±1% at constant temperature. During change of temperature from 0 to 60° accuracy falls to ±4% (during change of input voltages in an interval from 0 to 16 v).

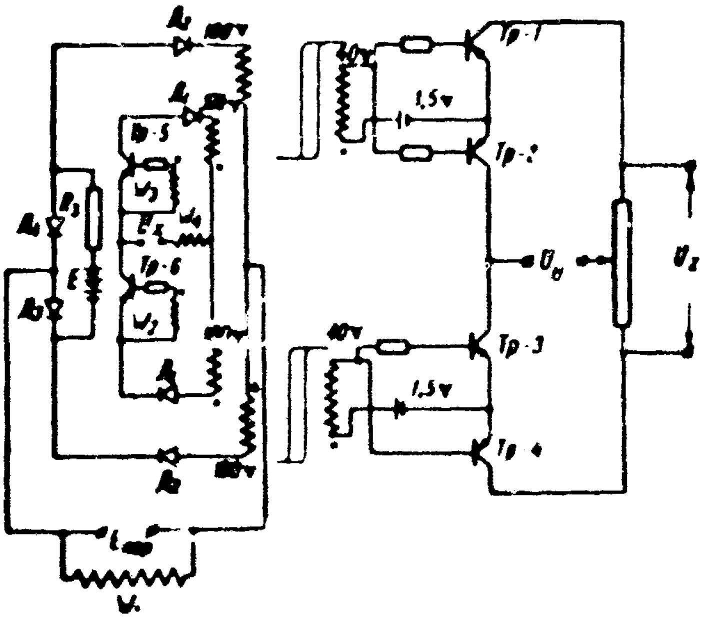

Fig. 132. Fundamental circuit of time-pulse multiplier made of magnetic elements with a rectangular hysteresis loop and semiconductor triode keys.

A dividing time-pulse device, made out of these elements, is based on peculiarities of cores with rectangular hysteresis loops, consisting of the fact that the time of reverse magnetization of core from one state of saturation to the other is reciprocal to the amplitude of the magnetism-reversing voltage (D. Schaefer [1]).

Analysis of work of described devices shows that they still cannot completely replace tube devices. Their further development and improvement are in direct dependence on improvement of technical indices of semiconductor triodes and cores with a rectangular hysteresis loop.

Multipliers based on modulations of input voltages. Application of various methods of modulation of signals allows us practically with that same static accuracy to construct a multiplier with a significantly wider passband as compared with the above considered time-pulse systems.

We distinguish devices, based on amplitude (M. Mehron and W. Otto [1]) and on a combined amplitude and frequency system of modulation of signals.

An example of a device of the first type is the multiplier of the Massachusetts Institute of Technology (see B. Chance, V. Huges [1]), a skeleton diagram of which is shown in Fig. 133.

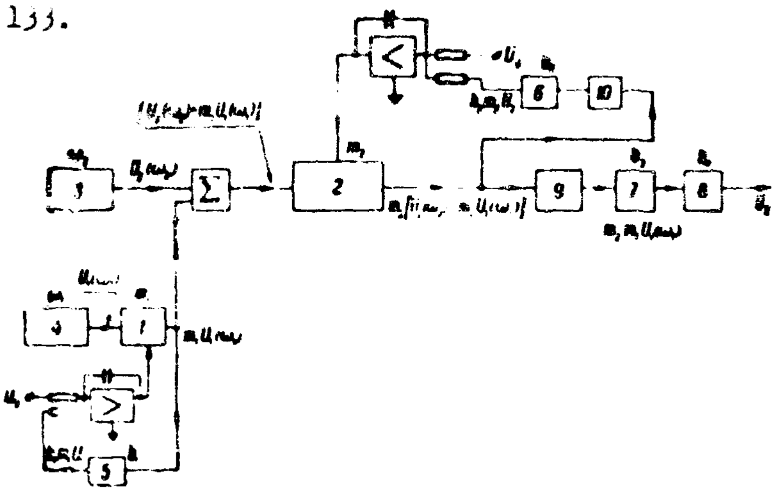

Fig. 133. Multiplier, based on amplitude modulation of signal. 1--controlled amplifier with amplification factor m_1; 2--controlled amplifier with amplification factor m_2, 3, 4--generators with frequency w_2, w_1; 5, 6, 7--rectifiers, 8--cathode follower, 9, 10--filters, tuned to frequencies w_1, w_2.

Basis of this device is two modulators 1 and 2, made in the form of amplifiers with amplification factor, changing proportional to one of the applied voltages. As such amplifiers one can use, for example, cascades of vacuum-tube amplifiers, constructed from "varimu" tubes or multigrid tubes.

To amplifier 1 is fed signal U_1 constant in amplitude and of frequency ω_1. This signal after amplification is rectified and is compared with input signal U_x. If both signals are unequal, then there appears a differential signal at the input of integrator, and to amplifier 1 is fed tension until its amplification factor changes so that

$$U_x = b_1 m_1 U_1(\omega_1). \qquad (8.30)$$

Thus, by the considered negative feedback circuit there is established linear dependence of the amplification factor of the amplifier 1 on voltage U_x of the first co-factor independently of change (within certain limits) of parameters and operating conditions of amplifier 1.

Output signal of amplifier 1 here is a signal with frequency ω_1 and an amplitude, variable proportional to input signal U_x. This signal is summed with a signal of frequency ω_2 and constant amplitude U_2. This sum moves into amplifier 2 with changed amplification factor. Linear dependence between amplification factor of amplifier 2 and voltage U_y, representing the second co-factor, is attained also by application of a negative feedback circuit. Here output voltage of amplifier 2 after separating voltage of frequency ω_2 and rectification is compared with U_y, and the difference after intergration changes the amplification factor of amplifier 2 until

$$U_y = b_2 m_2 U_2(\omega_2). \qquad (8.31)$$

Output voltage of amplifier 2 is simultaneously passed through filter 9, separating signals with frequency ω_1, and then after rectifier 7 moves to the output through cathode follower 8.

Output voltage of cathode follower here will be

$$U_z = b_3 b_4 m_2 m_1 U_1(\omega_1).$$

Substituting in this expression values of m_1 and m_2 from (8.30) and (8.31), we will receive finally

$$U_z = \frac{b_3 b_4}{b_2 b_1} \frac{U_x U_y}{U_2}. \qquad (8.32)$$

Such a multiplier can be made with accuracy up to ±0.1% and a passband with one constant co-factor up to 1000 cycles.

The main deficiency is comparative equipment complexity (there are required more than 12 envelopes), and impossibility of direct fulfillment of operation of multiplication with sign-alternating co-factors, since during conversion of input signals into voltage of carrier frequencies polarity is lost.

The latter deficiency can be eliminated, if we use a special modulator circuit (L. A. Lukashevich [1]), which permits us to obtain also negative values of the modulation factor, i.e., change the phase of the voltage of the carrier frequency depending upon the sign of modulating voltage and replace in the circuit of Fig. 133 ordinary rectifiers with phase-sensitive ones.

Fundamental circuit of such modulator is presented in Fig. 134.

Voltage of carrier freuuency $U(\omega)$ is fed in anti-phase by secondary windings of transformer T to grids of a triode and heptode, which work on the total plate load. Voltage U_0 controls steepness of characteristic of heptode.

If steepness of heptode and triode is identical for a certain value $U_0 = \overline{U}_0$, then the variable output component will equal zero and, consequently, $m = 0$. With increase of voltage U_0 steepness of heptode increases, and output voltage receives phase, determined by halfwinding 1 of the transformer. Here $m > 0$. When $U_0 < \overline{U}_0$ the phase of output voltage changes 180° and the modulation factor becomes $m < 0$.

A multiplier made with such modulators by the general diagram of Fig. 133, ensures accuracy of 0.3% with a band of frequencies up to 10 kilocycles for both co-factors. Frequency of carriers in the circuit of L. A. Lukashevich was selected $f_1 = 1200$ kc and $f_2 = 500$ kilocycles.

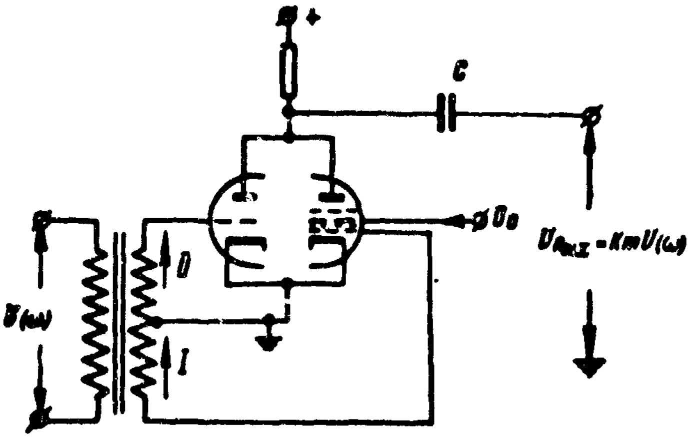

Fig. 134. Fundamental circuit of phase-sensitive modulator.

As compared with time-pulse circuits here there is attained a significantly wider passband, although there is lost simplicity of realization of several dividers with identically changing transmission factor.

Multipliers with combined amplitude and frequency modulation of signals do not have advantages as compared with devices with double amplitude modulation and therefore are not considered here.

Description and detailed analysis of these devices can be found in works of K. E. Erglis and W. A. McCool [1].

3. Multipliers Made from Quadratic Functional Generators.

Methods of constructing multipliers from quadratic functional generators. Structure of multipler, reproducing expression (8.1), in many respects depends on two factors: the method of obtaining sums $(U_x + U_y)$ and $(U_x - U_y)$ and peculiarities of functional generators, utilized for obtaining quadratic dependence.

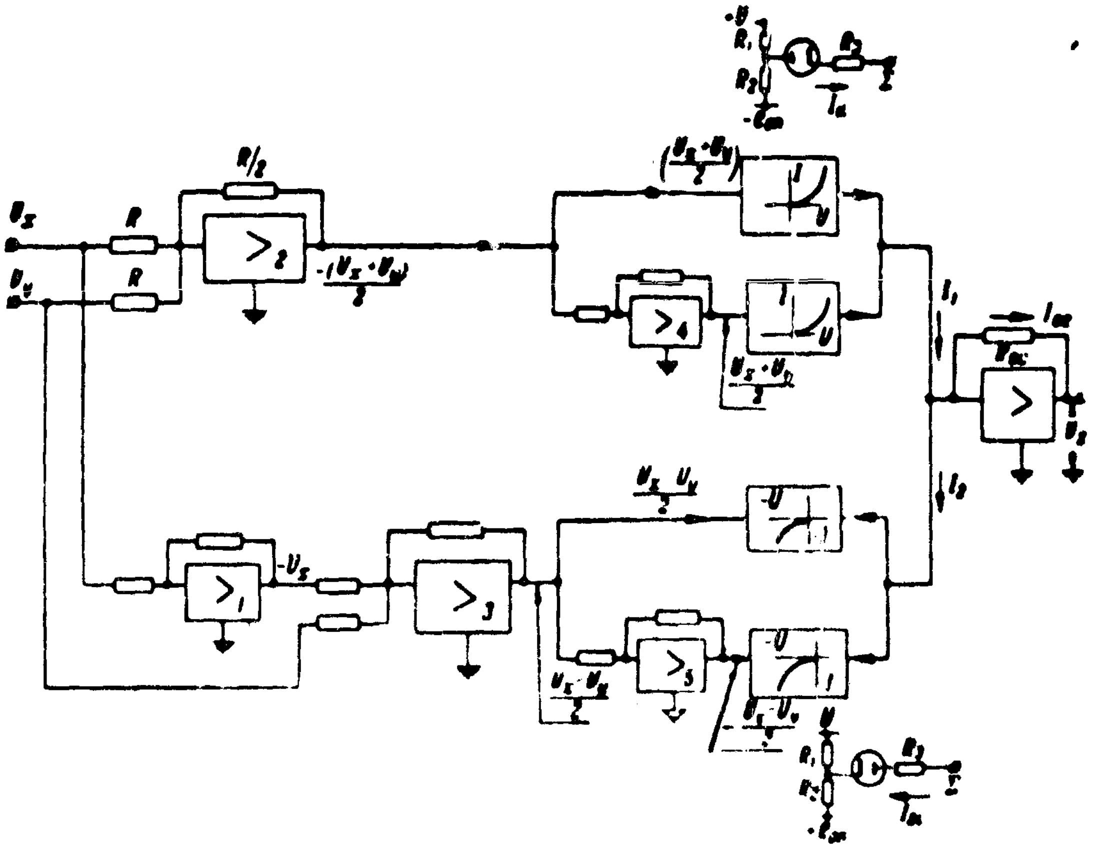

Fig. 135. Multiplier with quadratic functional generators.

In functional generators, made on the basis of piecewise-linear approximation by diode elements (see Ch. V and VI), and also with artificial deformation of nonlinear characteristics of certain semiconductors (thyrite, germanium), quadratic dependence of current of nonlinear element on input voltage is reproduced only with one sign of the input signal. In connection with this in such multipliers appears the problem singling out the modulo of sums ($U_x + U_y$) and ($U_x - U_y$).

The necessity of singling out the modulo of these sums completely drops with use of a quadratic functional generator, in principle allowing work with both signs of input signal. Among such functional generators in first place are devices based on use of electron-beam tubes with an inner or outer screen (see Ch. VII). Application of these devices brings the necessity of inverting of output voltage of one of the quadratic functional generators.

In Fig. 135 is brought a circuit of a multiplier based on diode square-law generators, developed in industry (see I. M. Vittenberg [2], and also L. N. Fitsner [1]). In this circuit for formation of the required sums there are used three operational amplifiers 1, 2, and 3. Amplifier 1 works as a sign inverter. For formation of modulo there is applied an ordinary circuit (see, for example, Fig. 85, page 172), in which as gates are used circuits of quadratic functional generators, working only in one quadrant. Amplifiers 4 and 5 serve to change polarity of input signal. By force of such specifics of use of quadratic functional generators in the circuit there are introduced four generators.

The upper two quadratic generators work only with positive input voltage, and the lower--only with negative. Their output currents have opposite directions. This is attained by various methods of coupling diode elements (Fig. 135).

Multipliers of electronic analogs must give the possibility of establishing the required (positive or negative) sign of output voltage independently of the sign of input voltages. In the considered circuit this is possible to execute by a switch by change of the place of connecting of the upper and lower pair of square-law generators. Such multipliers equipped the electronic analog of type

Simplification of the circuit of Fig. 135 can be conducted by change of the methods of summation to input voltages and methods of singling out the modulo of sums. In the functional diagram (M. A. Shnaydman [2]) in Fig. 136, summation of input signals is carried out by two operational amplifiers, where the halfdifference of U_x and U_y is obtained here by addition to one of co-factors (in the considered circuit to U_y) the halfsum of U_x and U_y with reverse sign, obtained from output of operational amplifier 1. Singling out of modulo is carried out by coupling in each channel of square-law generators with various signs. Depending upon polarity of the signal in the channel one or another quadratic functional generator works. So that output current of upper square-law generators always has a positive direction, and the lower--negative, there is connected an additional sign-inverting amplifier 3. Switch ПП serves to change polarity of output voltage. Further simplification of circuit (Fig. 137a) can be is attained by transition to summation of input signals by resistances and singling out of the modulo of these sums by diode gate circuits (I. M. Vittenberg [2], L. N. Fitsner [1]). Such multipliers are applied in the latest types of nonlinear models (type MN-7).

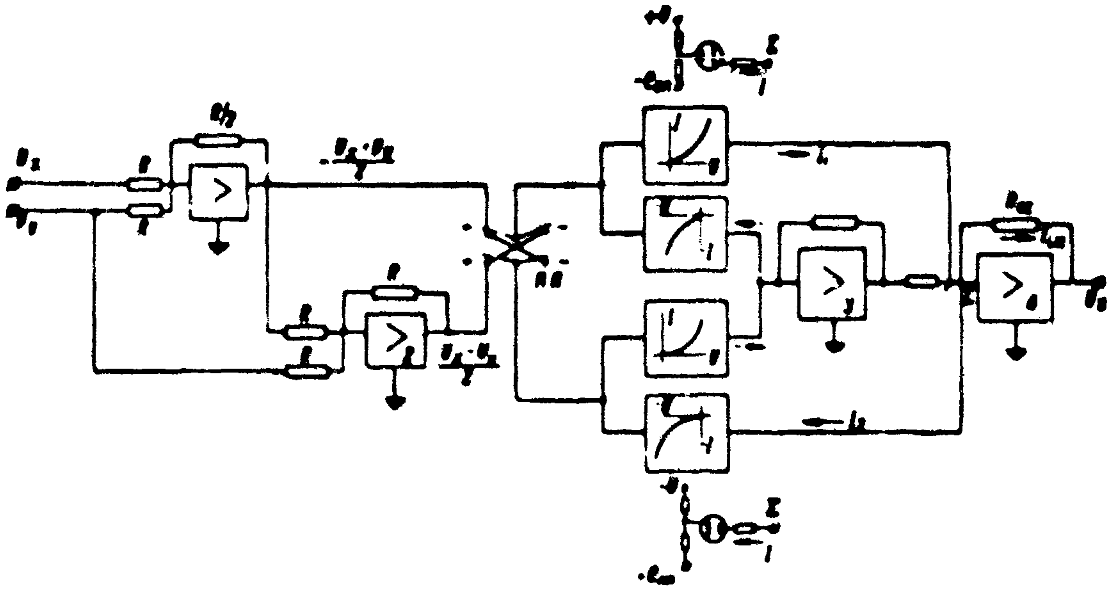

Fig. 136. The same as Fig. 135, but with another method of formation of the sum and singling out of modulo.

Analysis of these variants of functional circuits shows that in the last circuit of Fig. 137a the number of operational amplifiers can be reduced to two by another method of summation of input voltages of Fig. 137b. This variant requires for its realization a minimum of elements and, apparently, represents the limit of simplification of the circuit, which can be achieved.

All circuits in Fig. 137 are built on quadratic functional generators with diode elements, which differ from these applied in diode universal generators only by the fact that in them the output magnitude is current, and steepness of their characteristic is established not by a voltage divider, but by selection of the required value R_3 for given R_1 and R_2.

For the purpose of conserving on resistors for assignment of reference voltage there is used a series, and not a parallel divider. Circuits of square-law generators with positive and negative output currents are shown in Fig. 138.

Error of these multipliers is 0.6-0.8%. Passband is limited to the passband of the operational amplifiers.

Main deficiency of these circuits (especially the two latter ones) consists in difficulties of calculation and adjustment of separate square-law generators. This is caused by the fact that source of signal $\frac{U_x + U_y}{2}$ and $\frac{U_x - U_y}{2}$ has significant internal resistance and is loaded with variable resistance of diode circuits, depending on magnitude of voltages applied to them. Furthermore, to guarantee correct summation it is necessary to introduce secondary voltage for compensation of influence of reference voltage on integrating circuits. To decrease influences of resistance of diodes of the gating circuit in conducting direction it is necessary to increase resistance of series divider which in turn requires transition to higher ratings of resistances in the remaining circuit.

Application for construction of square-law generators for multipliers of diode elements with potentially grounded diodes, possessing integrating properties, allows us to eliminate above-indicated deficiencies. In Fig. 139 is brought functional diagram of multiplier with potentially grounded diodes of the analog

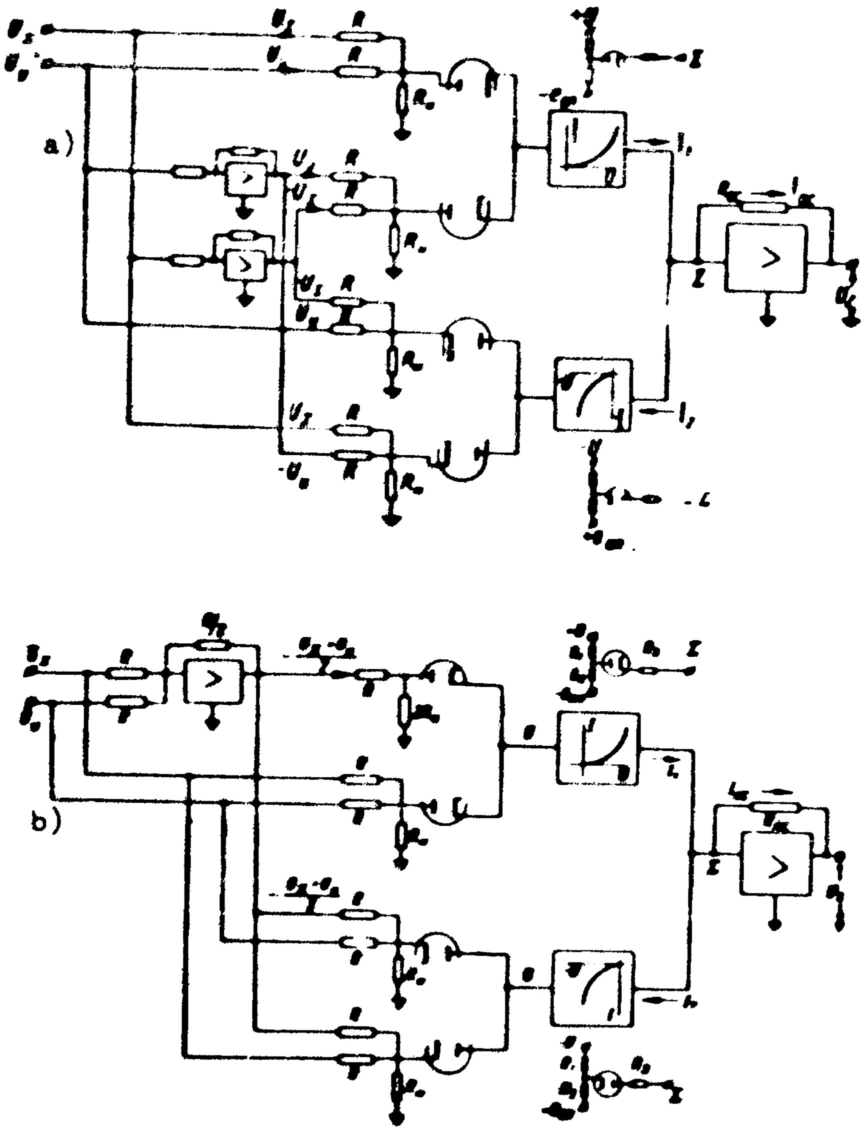

Fig. 137. Modification of circuits of Figs. 136 and 135.

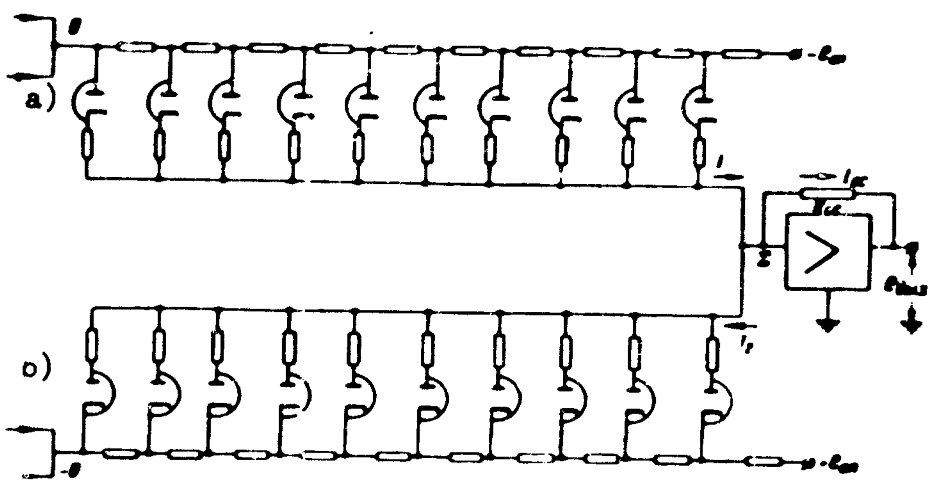

Fig. 138. Circuits of diode square-law generators.

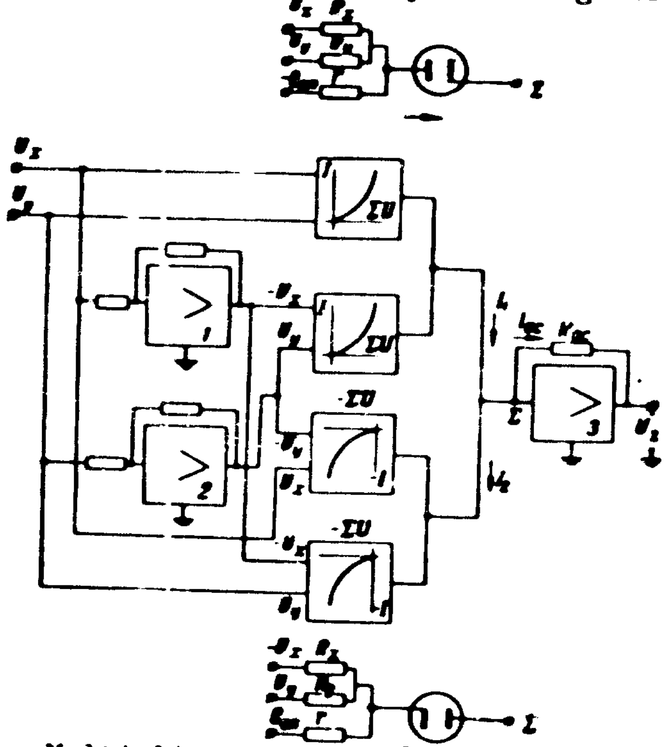

Fig. 139. Multiplier, made from square-law generators with potentially grounded diodes.

EMU-5 (V. V. Gurov, B. Ya. Kogan, A. D. Talantsev, V. A. Trapeznikov [1]). Summation of voltages of separate co-factors is transferred directly to diode elements of square-law generators.

Functional diagram, shown in Fig. 140, indicate a way of further simplification of multipliers constructed from square-law generators, possessing integrating properties.

Of principal interest is the method of singling out the modulo of the sum and difference of input magnitudes, offered in the work of A. A. Maslov [1] and depicted in Fig. 140b.

On diode elements of the upper square-law generator here are summarized the following quantities:

$$\Sigma U = (U_x + U_y)\frac{1}{2} + \text{when} + \left\{\begin{matrix} 0 & & U_x + U_y \geq 0 \\ -2\left[\dfrac{U_x + U_y}{2}\right] & \text{when} & U_x + U_y < 0 \end{matrix}\right\} = \frac{|U_x + U_y|}{2}. \qquad (8.33)$$

On diodes of the lower square-law generator

$$\Sigma U = -\frac{U_x + U_y}{2} + \min(U_x, U_y) = -\frac{|U_x - U_y|}{2}. \qquad (8.34)$$

Coefficient $\frac{1}{2}$ in the first component of formula (8.33) is obtained by summation on the diode element of voltages U_x and U_y with weight $\frac{1}{2}$; second component, included in braces, is obtained by switching on diode Д_1 , wherein voltage after diode Д_1 is summed on resistors of the main diode element with weight 2. Second component in expression (8.34) is obtained by application of special circuit of singling out the least of the two voltages U_x and U_y.

As is known from theory of rectifying circuits, during connection of two rectifiers by like-sign electrodes and feeding of remaining electrodes from two independent sources that rectifier will pass the current, on whose plate the potential is higher and on the cathode, lower. Therefore during supplying cathodes

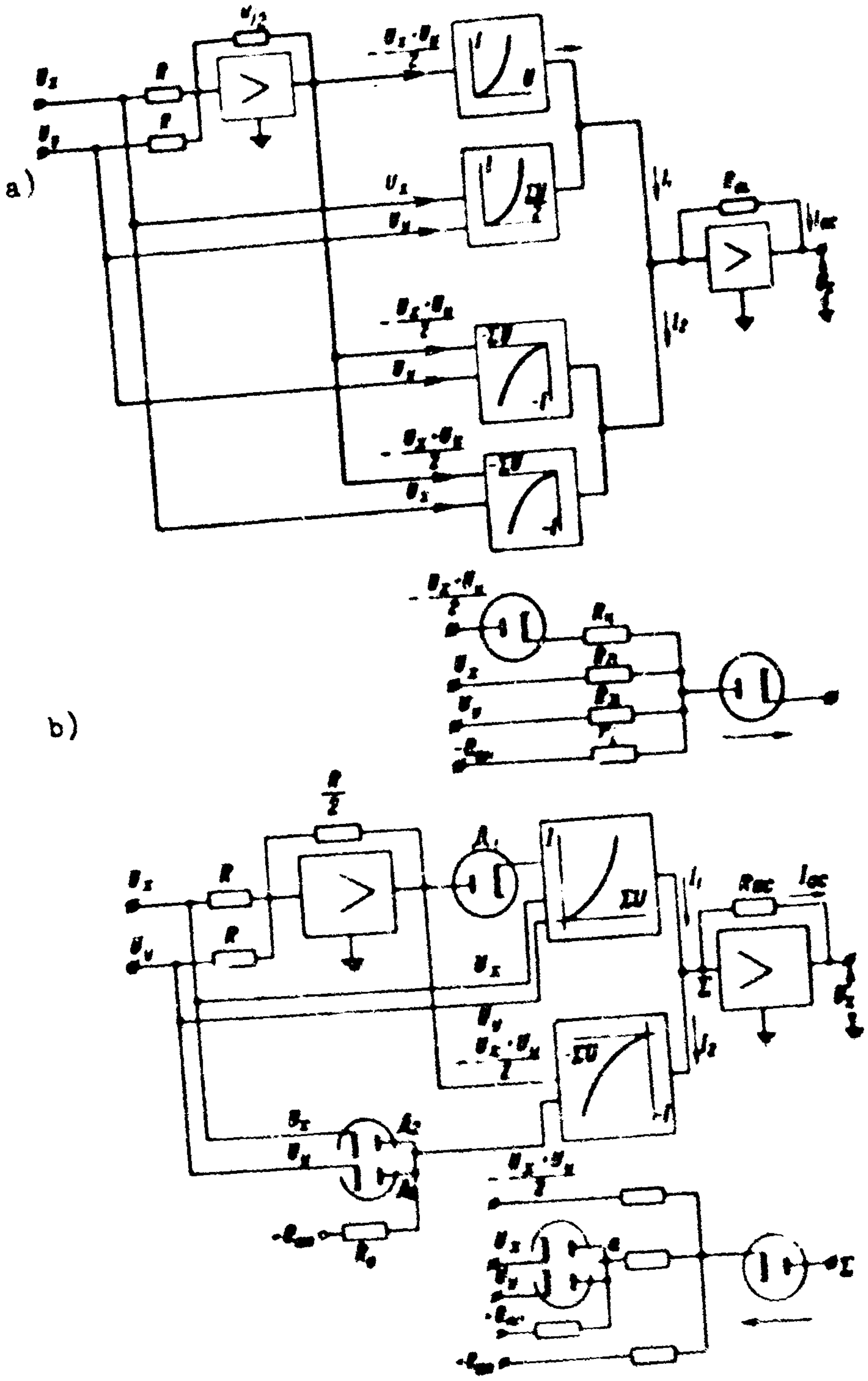

Fig. 140. Modification of a multiplier according to Fig. 139.

of diodes $Д_2$ and $Д_3$ voltages U_x and U_y there will be transmitted to the circuit that voltage, which at the given moment has the least value. So that the device worked also during positive voltages U_x and U_y, the potential of point a is raised by voltage [illegible].

When the formation of modulo fo a difference with a plus sign is indispensable expression (8.34) is replaced by

$$\sum U = -\frac{U_x + U_y}{2} + \max(U_x, U_y) = \frac{|U_x - U_y|}{2}. \qquad (8.35)$$

Here it is necessary to single out the greater of the two voltages U_x and U_y. Obviously, for that it is sufficient to change the circuit diagram of diodes, coupling diodes by cathodes, and change the sign of the reference voltage.

We will calculate the value of the potential at point a when $U_x > U_y > 0$ in the circuit of separating min (U_x, U_y). Considering equivalent the resistance of the diode circuit of the square-law generator R_H and resistance of diode $Д_3$ in conducting direction $R_Д$, we will receive

$$e_a = \frac{\frac{R_Д}{R_0} e_{оп} + U_y}{\left(\frac{1}{R_н} + \frac{1}{R_0}\right) R_Д + 1}. \qquad (8.36)$$

If resistance $R_Д$ is small as compared with R_0 and R_H, then it is possible to consider that

$$e_a = +U_y.$$

Thus, the circuit will work more accurately, the less the value of $R_Д$. From this point of view for such circuits it is most rational to use semiconductor diodes.

In the considered circuits of multipliers the total input current always should be equal to

$$I_1 + I_2 = a[|U_x + U_y|^2 - |U_x - U_y|^2]. \qquad (8.37)$$

where a is the proportionality factor.

For an output integrating operational amplifier this equality is correct:

$$I_1 + I_2 = -I_{oc} = \frac{U_z}{R_{oc}}.$$

whence

$$U_z = -aR_{oc}[|U_x + U_y|^2 - |U_x - U_y|^2] = -4a R_{oc} U_x U_y \qquad (8.38)$$

So that when $U_x = U_y = 100$ v we receive $U_z = 100$ v, coefficient a on the basis of (8.38) should be

$$a = \frac{1}{400R_{oc}}.$$

Multipliers of this type may also be used to exectue dividing operations according to the diagram brought earlier in Fig. 119b. With this aim the multiplier is connected in a feedback circuit of an integrating amplifier (V. V. Gurov, B. Ya. Kogan, A. D. Talantsev, V. A. Trapeznikov [1]).

Input voltages of the multiplier now will be U_y and U_z, where U_z is removed from the output terminal of the integrating amplifier. To the integrating amplifier through resistor R_1 is fed voltage U_x, representing the dividend. As before, for the integrating amplifier the relationship $I_x = -I_z$ is correct and, since

$$I_x = \frac{U_x}{R_1}, \quad I_z = aU_z \cdot U_y,$$

then

$$U_z = -\frac{U_x}{aR_1U_y}. \qquad (8.39)$$

Voltage U_y changes the conductance of feedback circuit so that with growth of U_y conductance increases and output signal decreases. With a low U_y conductance becomes minute and integrating amplifier approaches even with a minute signal U_x saturation ("reproduction of ∞"). During change of sign of voltage U_z it is necessary for observance of conditions of static stability (presence of negative feedback) to change the sign of the current characteristic of the multiplier. Coefficient $\frac{1}{aR_1}$ due to selection of R_1 usually is selected equal to 10.

Error of fulfillment of operation of division does not exceed 1.5-2%.

Transition from a multiplication circuit to a division circuit usually is carried out by means of simple commutation by a toggle switch.

Peculiarities of calculation and construction of quadratic elements of multipliers. Calculation of square-law generators, made from diode elements with load resistor (see diagram of Fig. 138.) presents great difficulties.

We will show, how one calculates a circuit when using diode elements with potentially grounded diodes.

Current characteristics of separate square-law generators on the basis of (8.37) will be:

$$I_1 = \alpha|U_x + U_y|^2,$$
$$I_2 = -\alpha|U_x - U_y|^2,$$
$$\alpha = \frac{1}{400R_{oc}}.$$

Steepness of current characteristics will be

$$S_1 = 2\alpha|U_x + U_y|, \quad S_2 = -2\alpha|U_x - U_y|.$$

Maximum voltage of the argument will be

$$|U_x + U_y|_{max} = 200 \text{ v}.$$

Allowing an error of approximation ε - 0.25 v and decomposing in such a manner that the first segment of the straight line originates from the origin of coordinates, and points of switching lie on the ideal curve, we will receive directly from Fig. 141 all data necessary for calculation. Indeed, shaded area F expresses error in reporduction of current characteristic, and therefore

$$\frac{\Delta Y_1 e^*_{вх\,1}}{2} = \frac{\varepsilon}{R_{oc}}. \qquad (8.40)$$

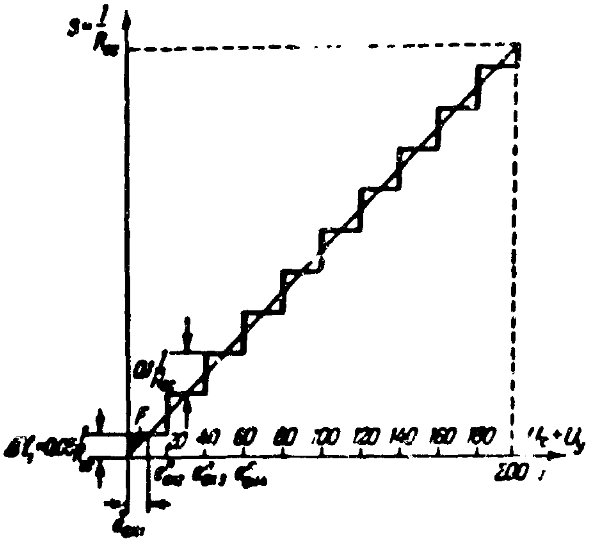

Fig. 141. Calculation of square-law generators of multipliers.

On the other hand, from expression for steepness of current characteristics it follows:

$$\Delta Y_1 = e^*_{вх\,1} 2\alpha. \qquad (8.41)$$

From expressions (8.40) and (8.41) we find:

$$\left.\begin{aligned} e^{*}_{\text{вх}\,1} &= \sqrt{\frac{s}{aR_{oc}}}\,, \\ \Delta Y_1 &= 2\sqrt{\frac{as}{R_{oc}}}\,. \end{aligned}\right\} \tag{8.42}$$

When $a = \frac{1}{400R_{oc}}$ and $s = 0.25$ v we will receive $\Delta Y_1 = \frac{1}{20R_{oc}}$ and $e^{*}_{\text{вх}\,1} = 10\text{v}$.

From Fig. 141 it follows that decomposition of argument will be uniform:

$$\begin{aligned} &e^{*}_{\text{вх}\,1} = 0, \\ &e^{*}_{\text{вх}\,2} = 2e^{*}_{\text{вх}\,1} = 20\text{ v}, \\ &e^{*}_{\text{вх}\,i} = 2e^{*}_{\text{вх}\,1}(i-1), \\ &e^{*}_{\text{вх}\,n} = 2e^{*}_{\text{вх}\,1}(n-1) = 180\text{v}, \end{aligned}$$

whence the number n of sections of decomposition is

$$n = 10.$$

Increase of conductance for first diode element will constitute

$$\Delta Y_1 = 0{,}05\,\frac{1}{R_{oc}} = \frac{1}{R_{1\text{э}}}.$$

For subsequent diode elements all increases turn out to be identical and equal

$$\Delta Y_2 = \Delta Y_3 = \ldots = \Delta Y_{10} = 2\,\Delta Y_1 = 0{,}1\,\frac{1}{R_{oc}} = \frac{2}{R_{1\text{э}}}.$$

Thus, immediately are determined values of all equivalent resistances:

$$\begin{aligned} &R_{1\text{э}} = 20R_{oc}, \\ &R_{2\text{э}} = R_{3\text{э}}, \ldots = R_{10\text{э}} = 10R_{oc}. \end{aligned}$$

By resulting values of $e^{*}_{\text{вх}\,i}$ we find:

$$\begin{aligned} &r_1 = R_{1\text{э}}\,\frac{e_0}{e^{*}_{\text{вх}\,1}} = \infty, \\ &r_2 = R_{2\text{э}}\,\frac{e_{oa}}{e^{*}_{\text{вх}\,2}} = 10R_{oc}\,\frac{30}{20} = 15R_{oc}, \\ &r_3 = R_{3\text{э}}\,\frac{e_{oa}}{e^{*}_{\text{вх}\,3}} = 10R_{oc}\,\frac{30}{40} = 7{,}5R_{oc}, \\ &r_4 = 5R_{oc}, \\ &r_5 = \frac{300}{80}\,R_{oc} = 3.75R_{oc}, \\ &r_6 = \frac{300}{100}\,R_{oc} = 3R_{oc}, \\ &r_7 = \frac{300}{120}\,R_{oc} = 2.5R_{oc}, \\ &r_8 = \frac{300}{140}\,R_{oc} = 2.18R_{oc}, \\ &r_9 = \frac{300}{160}\,R_{oc} = 1.88R_{oc}, \\ &r_{10} = \frac{300}{180}\,R_{oc} = 1.67R_{oc}. \end{aligned}$$

Using theorem of equivalent generator, we determine by found values of equivalent resistances of resistance R_{1i} , R_{2i} and R_{3i} in each i-th integrating diode element.

Thus, for example, for upper square-law generator of the circuit of Fig. 140b, we obtain

$$i_{\text{вых}} = 4\left(\frac{U_x + U_y}{2}\right) - U_x - U_y = \left(\frac{U_x - U_y}{2} \frac{1}{R_{1i}} - \frac{U_x}{R_{2i}} - \frac{U_y}{R_{3i}}\right) R_{0i}.$$

From this expression it follows that

$$\frac{R_{0i}}{R_{1i}} = 4, \qquad \frac{R_{0i}}{R_{2i}} = \frac{R_{0i}}{R_{3i}} = 1.$$

whence

$$R_{1i} = \frac{1}{4} R_{0i}, \quad R_{2i} = R_{3i} = R_{0i}.$$

Fundamental circuit of diode square-law generator is shown in Fig. 142.

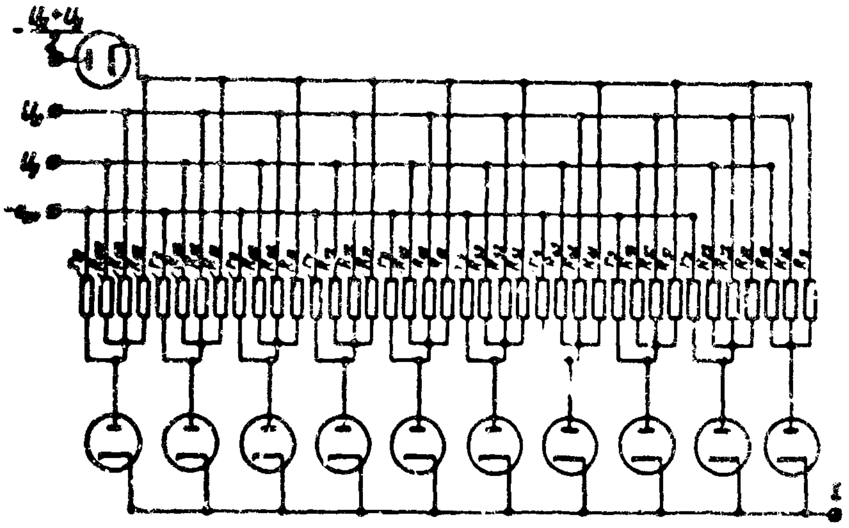

Fig. 142. Circuit of square-law generator of multiplier made by the circuit of Fig. 140b.

Application of diode square-law generators, based on piecewise-linear approximation, in considered multipliers along with merits (wide passband and sufficiently high accuracy (0.5-1%) has also a number of deficiencies. Among them one be mentioned the necessity of expenditure of power on heating diodes and stabilization of reference-voltage source, stepnature of reproduction steepness of characteristic and limited period of service.

Multipliers, based on application of square-law generators, using natural quadratic nonlinear resistances, in principle should be free from these deficiencies. However till now wide application of such multipliers has been limited by unstability of natural nonlinear characteristics and inaccuracy of approximation of their characteristics to quadratic ones. In connection with this such devices on the whole possessed a low operating accuracy. As an example there is the multiplier for a correlator (I. N. Holmes, M. A. Duken [1], K. W. Goff [1]), in which as quadratic nonlinear characteristic is used dependence of plate current of pentode on voltage of its grid, approximated by expression

$$i_a = a + be_g + ce_g^2. \qquad (8.43)$$

Error or such a multiplier is not less than 8%.

In recent years considerable attention was allotted to use for quadratic elements of certain carborundum resistors (thyrite, villaite), possessing stable volt-ampere characteristics (A. A. Maslov [1], M. A. Rozenblat and O. A. Sedykh [1], L. D. Kovach and W. Comley [1], G. N. Balasanov [1], V. L. Benin [1], L. M. Fitsner [2]).

In Fig. 143a is brought a typical volt-ampere characteristic of carborundum (thyrite) disk 50 mm in diameter and 10 mm in height. This characteristic differs from a quadratic one and usually is approximated (M. A. Rosenblat and O. A. Sedykh [1]) by expressions:

$$\left.\begin{aligned} I_{\text{т}} &= a_1 e_{\text{т}} + b_1 e_{\text{т}}^3 + c_1 e_{\text{т}}^5, \\ \text{or} \qquad I_{\text{т}} &= k e_{\text{т}}^n. \end{aligned}\right\} \qquad (8.44)$$

where a_1, b_1, c_1 and k, n are constants, determined from experimentally received characteristics.

In order to obtain a nonlinear dependence, close to a quadratic one, by the natural characteristic of thyrite, the latter should be deformed, constituting an electric circuit, which is a combination of linear resistances and the resistance of thyrite.

Circuit diagrams of thyrite, applied at present, are presented in Fig. 143b.

Coupling in a series resistor (L. D. Kovach and W. Comley [1]) as it were stretches and straightens the volt-ampere characteristic of tyrite, at the same time reducing its working section in the same limits of change of input signal.

Coupling in a parallel resistor (A. A. Maslov [1], L. N. Fitsner [2]) cause turn of the characteristic a certain angle to the left or to the right depending upon sign of conductance of parallel circuit. Therefore in general in parallel circuit there should be a sign-inverter.

Application of two adjusting resistors R_1 and R_2 allows us more accurately to move the characteristic of thyrite to the given characteristic, not complicating here considerably calculation of the circuit.

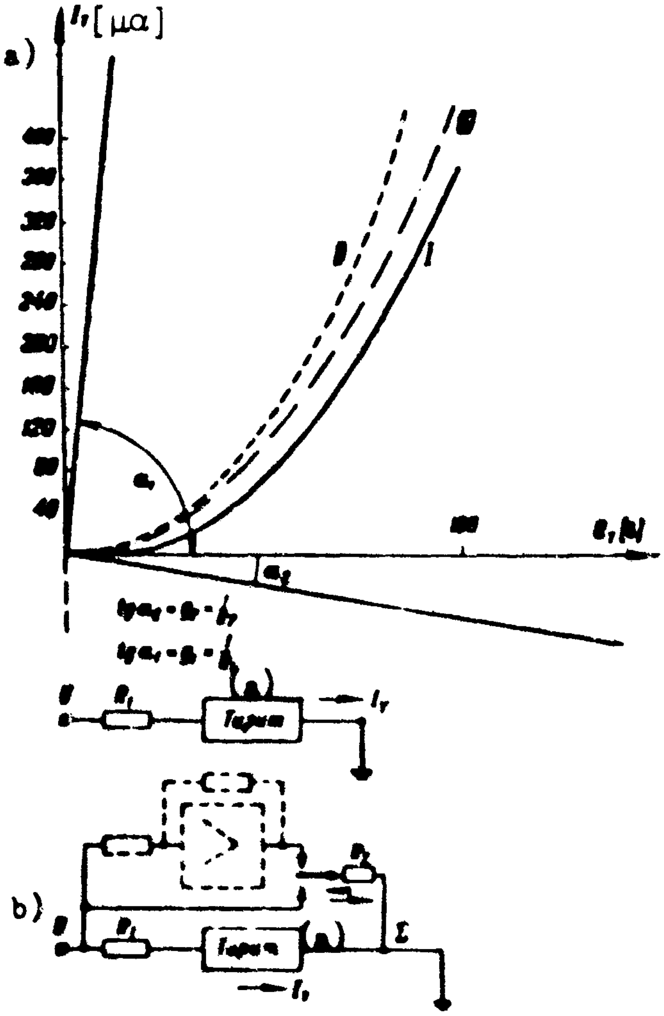

Fig. 143. Volt-ampore characteristic of thyrite and circuit diagram. I--ideal quadratic current characteristic; II--current characteristic of thyrite; III--current characteristic of thyrite with series connected resistor.
KEY: (a) Thyrite.

Determination of required values of resistors R_1 and R_2 in principle can be done analytically (V. L. Benin [1]), using one or another analytic expression for volt-ampere characteristic of thyrite and placing the condition of minimum integral quadratic error. However this does not lead to expressions in closed form and an answer can be obtained only by method of successive approximations.

More expedient turned out to be the method, consisting of the fact that the characteristic of thyrite is approximated by an expression, determining the ideal characteristic which thyrite ought to have in the considered circuit so that the circuit as a whole had a quadratic current characteristic.

For a circuit with parallel connection of resistances it turns out that ideal current characteristic of thyrite, determined through parameters of circuit, should have form

$$I_T = \frac{k g_1^2}{2m} - \sqrt{\frac{k^2 g_1^4}{4m^2} - \frac{k g_1^2 e_T}{m}} - g_1 e_T, \tag{8.45}$$

where k = $\frac{g_2 - g_1}{g_1}$, m--the scale factor of ideal quadratic current characteristic of circuit, $g_1 = \frac{1}{R_1} \cdot g_2 = \frac{1}{R_2}$ --linear conductances of circuit, e_T --voltage on thyrite resister.

Taking for approximation of volt-ampere characteristic of real thyrite the form of relationship (8.45), we obtain

$$I_T = a - \sqrt{a^2 - be_T} - ce_T. \tag{8.46}$$

From comparison of expressions (8.45) and (8.46) ensue relationships, connecting parameters of circuit with parameters of thyrite:

$$\left.\begin{aligned} R_1 &= \frac{1}{g_1} = \frac{1}{c}. \\ R_2 &= \frac{b}{(2ac + b)c}. \\ m &= \frac{c^2}{b}. \end{aligned}\right\} \tag{8.47}$$

Coefficients a, b and c are determined by coordinates of three points of experimentally received characteristic of thyrite $(e_{\text{т}1}, I_{\text{т}1})$; $(e_{\text{т}2}, I_{\text{т}2})$; $(e_{\text{т}3}, I_{\text{т}3})$ rom expressions:

$$c = \frac{\beta + \sqrt{\beta^2 + \alpha\gamma}}{\alpha},$$

$$a = \frac{e_{\text{т}3}(I_{\text{т}3} + ce_{\text{т}3})^2 - e_{\text{т}1}(I_{\text{т}1} + ce_{\text{т}1})^2}{2(e_{\text{т}3}I_{\text{т}1} - e_{\text{т}1}I_{\text{т}3})},$$

$$b = \frac{2a(I_{\text{т}3} + ce_{\text{т}3}) - (I_{\text{т}3} + ce_{\text{т}3})^2}{e_{\text{т}3}}.$$

Values of α , β and γ are found from expressions:

$$\alpha = [e_{\text{т}1}e_{\text{т}2}I_{\text{т}3}(e_{\text{т}2} - e_{\text{т}1}) - e_{\text{т}1}e_{\text{т}3}I_{\text{т}2}(e_{\text{т}3} - e_{\text{т}1}) + e_{\text{т}2}e_{\text{т}3}I_{\text{т}1}(e_{\text{т}3} - e_{\text{т}2})].$$

$$\beta = [I_{\text{т}1}I_{\text{т}2}e_{\text{т}3}(e_{\text{т}2} - e_{\text{т}1}) - I_{\text{т}1}I_{\text{т}3}e_{\text{т}2}(e_{\text{т}3} - e_{\text{т}1}) + e_{\text{т}1}I_{\text{т}2}I_{\text{т}3}(e_{\text{т}3} - e_{\text{т}2})].$$

$$\gamma = [I_{\text{т}1}I_{\text{т}2}e_{\text{т}3}(I_{\text{т}2} - I_{\text{т}1}) - I_{\text{т}1}I_{\text{т}3}e_{\text{т}2}(I_{\text{т}3} - I_{\text{т}1}) + e_{\text{т}1}I_{\text{т}2}I_{\text{т}3}(I_{\text{т}3} - I_{\text{т}2})].$$

These formulas* allow us to calculate parameters of circuit of square-law generator so that its characteristic will coincide with the given one at three points. So that at remaining points we receive the best approximation of characteristic of circuit to the given, selection of points on experimental volt-ampere characteristic of thyrite for finding parameters a, b and c one should execute graphically by the polygon of P. L. Chebyshev (M. Ye. Fobrinskiy [1]).

Since at the basis of analytic determination of values of linear conductances of circuit $g_1 = \frac{1}{R_1}$ and $g_2 = \frac{1}{R_2}$ lie experimentally received values of coordinates of three points on volt-ampere characteristic of thyrite and since the variance of characterisitcs of thyrite from sample to sample is very great, then the expreimental method of determination of these conductances also merits attention. Essence of method is easily perceived from Fig. 144. To input of integrating amplifier 2 are connected two input circuits. One circuit is composed of a standard square-law generator and sign-inverting scale unit 1 the other circuit will be formed from tested thyrite with series connected resistor R_1 and resistor R_2, connected to a source of reverse polarity. In case of complete coincidence of characteristics of ideal square-law generator and circuit with thyrite the sum of currents $I_1+I_2=0$ for all values of input voltage E. Coincidence of currents is fixed by a zero reading of a voltmeter, connected to output of adder. In practice selection of the resistance is conducted taking into account given allowances for noncoincidence of current characteristics. Selecting on the basis of calculation tentatively resistor R_1, then by change of scale m by setting of the transmission factor of amplifier 1 we seek straightening of the characteristic of error (see curve ΔI_{III} in Fig. 144b). After that by coupling R_2 we introduce error within the given allowance.

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*Derivation of these formulas and development of methods of calculation belong to A. A. Maslov [1].

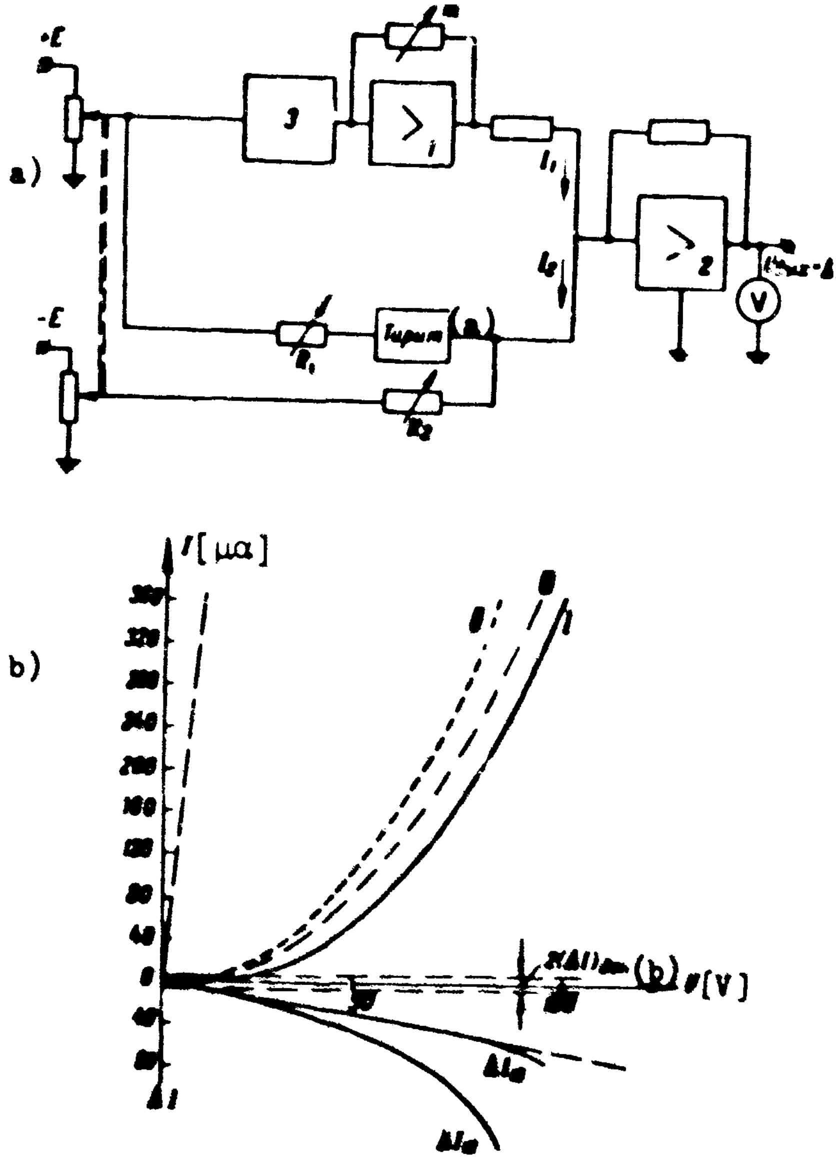

Fig. 144. Method of experimental determination of parameters of circuit diagram of thyrite for square-law generators. 1--scale unit; 2--integrating amplifier; 3--standard square-law generator. I--ideal quadratic current characteristic; II--current characteristic of thyrite; III--current characteristic of thyrite with series connected resistance.
KEY: (a) Thyrite; (b) Permissible.

Circuits of multipliers made from thyrite square-law generators (L. N. Fitsner [2], A. A. Maslov [1]) are shown in Figs. 145 and 146. The circuit shown in Fig. 146 is outstanding in its minimum number of operational amplifiers (two) and gives at low current levels great accuracy of approximation of characteristic of circuit with thyrite to a quadratic one. Switch ПП serves to change polarity of output signal, and switch ПР --for selection of operating regime (division or multiplication). In both circuits summation is carried out directly on thyrite resistors.

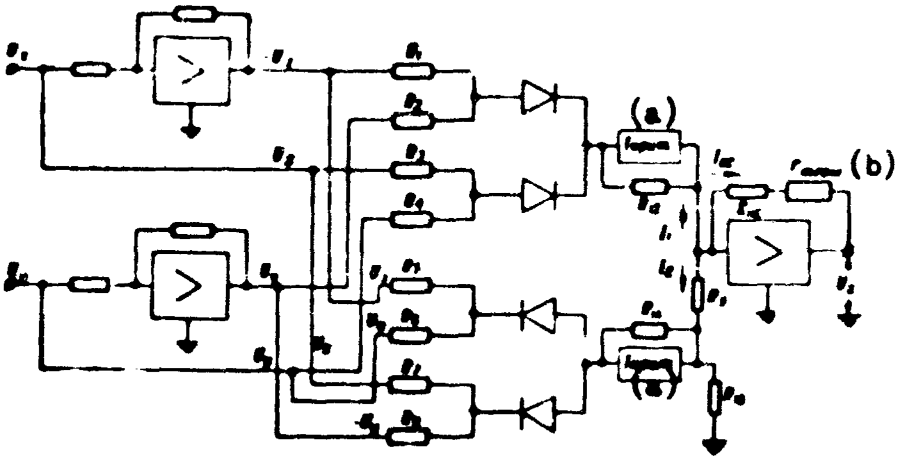

Fig. 145. Circuit of multiplier made of thyrites.
KEY: (a) Thyrite; (b) Thermistor.

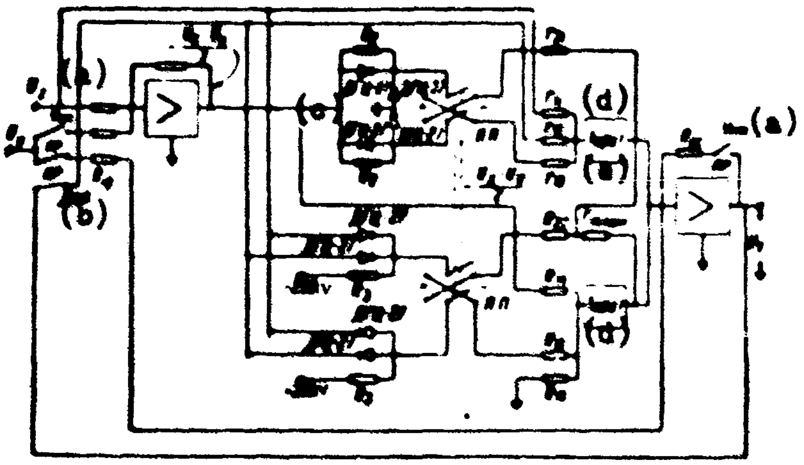

Fig. 146. Modification of circuit of multiplier with thyrites.
KEY: (a) Multiply; (b) Divide; (c) DCTs 21, etc; (d) Thyrite; (e) Thermistor.

For compensation of error, caused by dependence of resistance of thyrite on temperature, there are used thermistors with negative temperature coefficient. Resistors R_9, R_{10} (Fig. 145) and R_{16} (Fig. 146) serve for equalising transmission factor of lower square-law generator. Circuits of Fig. 145 and Fig. 146 differ in methods of singling out the modulo and method of approximation of characteristic of thyrite to the quadratic. In the first there are used for singling out the modulo commutating circuits; in the second--the method, presented on pages 268 and 270, in reference to a multiplier with diode square-law generators. One deficiency of the latter circuit as compared with a circuit with three operational amplifiers is increase of requirements for lowering phase distortions, introduced by inverting amplifier 1.

Error of work of such multipliers does not exceed 1%, and passband is 100 c.

4. Multipliers Based on Combining the Considered Principles

Multipliers of this type reproduce relationship (8.1) without use of special quadratic functional generators.

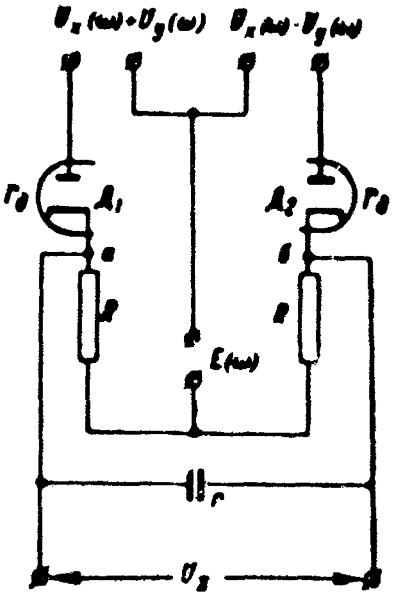

Fig. 147. Circuit of multiplier, based on obtaining the mean value of rectified sum and difference of two a-c voltages.

In Fig. 147 is brought fundamental circuit (offered by I. S. Bruk, see N. N. Lenov [1]) of a multipler, in which reproduction of shown relationship is based on obtaining mean value of rectified sum of two alternating voltages, shifted 90°. To input of device are fed voltages $U_x + U_y$ and $U_x - U_y$, modulated by frequency ω, and the voltage of carrier frequency E (ω), shifted with respect to voltages $(U_x + U_y)$ and $(U_x - U_y)$ 90°. Diodes $Д_1$ and $Д_2$ serve as halfwave rectifiers, providing simultaneous correct operation of the device with sign-alternating input signals. Mean value of rectified voltages $U_а$ and $U_б$ can be found from expressions:

$$\left.\begin{aligned}(U_а)_{ср} &= \frac{1}{\pi}\frac{R}{r_a+R}\sqrt{(U_x+U_y)^2+E^2},\\ (U_б)_{ср} &= \frac{1}{\pi}\frac{R}{r_a+R}\sqrt{(U_x-U_y)^2+E^2}.\end{aligned}\right\} \qquad (8.48)$$

which with sufficiently large E can approximately be presented in the form:

$$\left.\begin{aligned}(U_{a_0})_{cp} &\approx \frac{E}{\pi}\frac{R}{r_0+R}\left[1+\frac{1}{2}\left(\frac{U_x+U_y}{E}\right)^2\right],\\(U_{b_0})_{cp} &\approx \frac{E}{\pi}\frac{R}{r_0+R}\left[1+\frac{1}{2}\left(\frac{U_x-U_y}{E}\right)^2\right]\end{aligned}\right\} \quad (8.49)$$

Difference of potentials between points a and b of circuit will be

$$\begin{aligned}U_z = U_{ab} - (U_{a_0})_{cp} - (U_{b_0})_{cp} &= \frac{1}{2\pi E}\frac{R}{r_0+R}\left[(U_x+U_y)^2-(U_x-U_y)^2\right]\\ &= \frac{2}{\pi E}\frac{R}{r_0+R}U_xU_y\end{aligned} \quad (8.50)$$

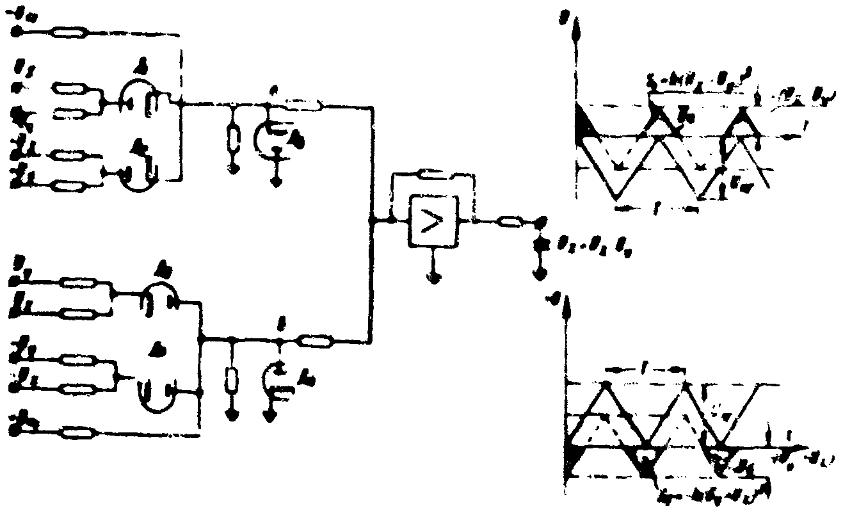

Fig. 148. Circuit of multiplier in which for obtaining quadratic dependences there is used the theorem of the area of similar triangles.

In the muliplier (I. S. Bruk [2]) presented in Fig. 148 for squaring there is used the theorem known from elementary geometry that the area of similar triangles vary as the squares of their altitudes. For conversion of voltage of input signals into pulses of triangular form with altitude, proportional to the resultant input signal $U_x + U_y$ or $U_x - U_y$, to the input of each branch is fed additionally a saw-toothed voltage. Voltages at points a and b of the circuit will differ from zero when $|U_x+U_y| > |U_m|$ and $|U_x-U_y| > |U_m|$ and diodes D_5 and D_6 are locked. Here voltages at points a and b will change as shown in Fig. 148, representing triangular pulses with altitude $(U_x + U_y)$ and $(U_x - U_y)$. Areas of these pulses are

$$S_1 = k(U_x - U_y)^2, \quad S_2 = -k(U_x + U_y)^2. \quad (8.51)$$

where

$$k = \frac{T}{2U_{\text{пг}}}$$

Mean value of output voltage is

$$U_{z\,\text{ср}} = -\frac{S_1 + S_3}{T} = \frac{-1}{2U_{\text{пг}}}[(U_x - U_y)^2 - (U_x + U_y)^2] = \frac{2}{U_{\text{пг}}} U_x \cdot U_y. \tag{8.52}$$

For correct operation of the device it is necessary that $|U_x + U_y|_{\max} < |U_{\text{пг}}|$, and the frequency of repetition of saw-toothed oscillations was, at least, two orders higher than the frequency of change of input signals. It is natural that accuracy of operation of the multiplier directly depends on linearity and stability of voltage of generator of saw-toothed voltages.

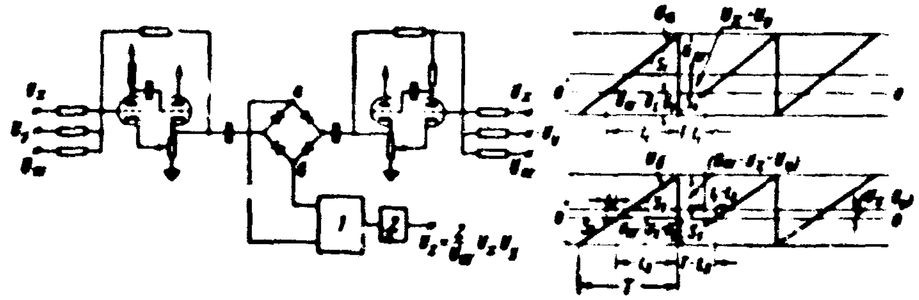

Fig. 149. One variant of realization of circuit of Fig. 148. 1--subtractor-1; 2--filter.

On this principle there was built a multiplier (R. L. Mills [1]) using cyclic connection of rectifiers for singling out triangular pulses of required polarity (Fig. 149). Potential diagram of voltages, applied in sections a and b of the annular circuit when $(U_x + U_y) > 0$ and $(U_x - U_y) > 0$, is shown in the same figure.

From analysis of the circuit it follows that mean value of output voltage of the subtractor for the period of change of $U_{\text{ПГ}}$ will be

$$U_{z\,\text{ср}} = \frac{S_1 - (S_2 + S_3) - (S_3 - S_4)}{T}. \tag{8.53}$$

Since triangles with areas S_1, $(S_2 + S_3)$, S_3, S_4 are similar, then:

$$S_1 = k(U_{\text{пг}} + U_x + U_y)^2, \quad S_2 + S_3 = k[U_{\text{пг}} - (U_x - U_y)]^2,$$
$$S_3 = k(U_{\text{пг}} + U_x - U_y)^2, \quad S_4 = k(U_{\text{пг}} - U_x - U_y)^2,$$

where $k = \frac{T}{4U_{\text{пг}}}$.

After substituting these values in (8.53) we will received finally:

$$U_{xcp} = \frac{1}{2U_{пг}}[(U_x + U_y)^2 - (U_x - U_y)^2] = \frac{2}{U_{пг}} U_x U_y \qquad (8.54)$$

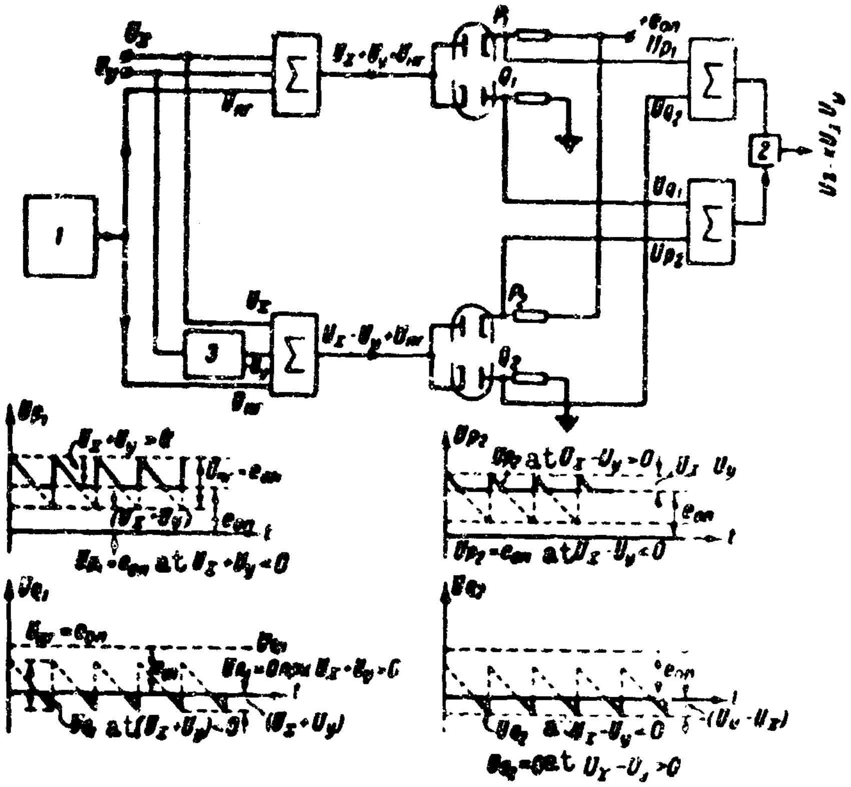

Fig. 150. Second variant of multiplier on the circuit of Fig. 148. 1--generator of saw-toothed voltage, Σ --adder; 2--subtractor; 3--unit for change of sign.

An interesting modification of method of physical realization of considered principle of construction of multipliers (K. H. Norsworthy [1]) is illustrated by skeleton diagram of Fig. 150. Instead of an annular circuit diagram of rectifiers here are used two identical commutating circuits. In each circuit the cathode of one diode is supported by a positive voltage, while plates of other diodes are connected through resistances to the ground. Amplitude of saw-toothed voltage $U_{пг}$ is selected equal to magnitude of reference voltage e_{on}. Potential diagrams of voltage at points P_1, Q_2 and P_2, Q_2 show that mean value of voltage at these points are:

when $$(U_x + U_y) > 0$$

$$(U_{P_1})_{cp} = e_{on} + \frac{1}{2e_{on}}(U_x + U_y)^2,$$

$$(U_{Q_1})_{cp} = 0;$$

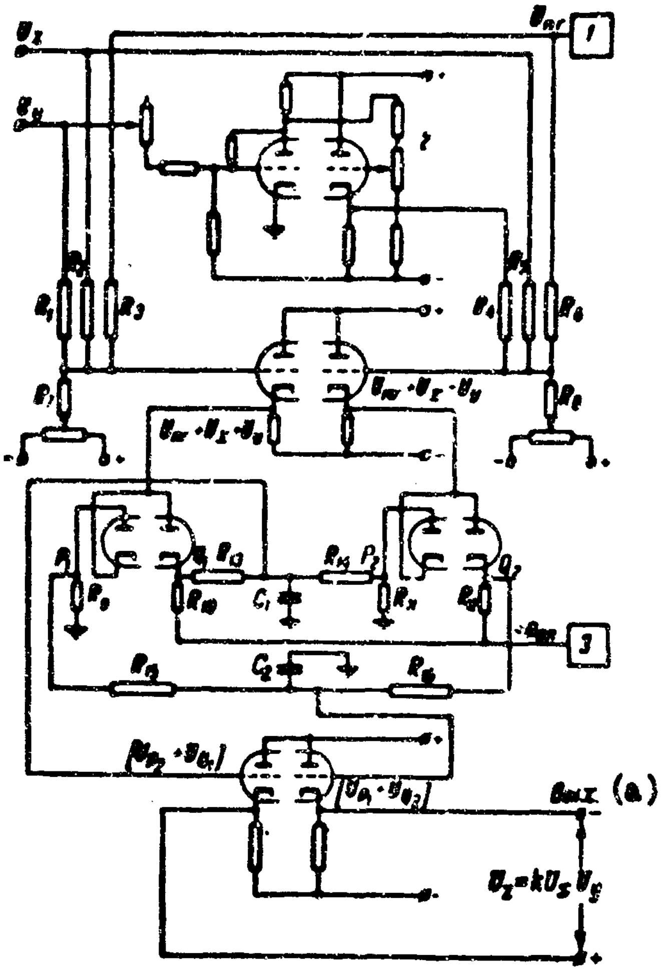

Fig. 151. Fundamental circuit of multiplier fulfilled according to Fig. 150. 1--saw-toothed generator; 2--unit of change of sign; 3--reference-voltage source; $R_1 = R_2 = R_3 = R_4 = R_5 = R_6 = 100$ kom, $R_7 = R_8 = 470$ kom, $R_9 = R_{10} = R_{11} = R_{12} = 68$ kom, $R_{13} = R_{14} = R_{15} = R_{16} = 820$ kom.
KEY: (a) Outlet.

when

$$(U_x + U_y) < 0$$

$$(U_{P_1})_{\text{ср}} = e_{\text{оп}},$$

$$(U_{Q_1})_{\text{ср}} = -\frac{1}{2e_{\text{оп}}}(U_x + U_y)^2;$$

when

$$U_x - U_y > 0$$

$$(U_{P_2})_{\text{ср}} = e_{\text{оп}} + \frac{1}{2e_{\text{оп}}}(U_x - U_y)^2,$$

$$(U_{Q_2})_{\text{ср}} = 0;$$

when

$$U_x - U_y < 0$$

$$(U_{P_2})_{\text{ср}} = e_{\text{оп}},$$

$$(U_{Q_2})_{\text{ср}} = -\frac{1}{2e_{\text{оп}}}(U_x - U_y)^2.$$

Mean value of output voltage of subtractor here is determined by sum

$$U_{z\,\text{ср}} = [(U_{P_1})_{\text{ср}} + (U_{Q_2})_{\text{ср}}] - [(U_{Q_1})_{\text{ср}} + (U_{P_2})_{\text{ср}}].$$

With any combinations of signs of input signals this expression leads to relationship

$$U_{z\,\text{ср}} = \frac{1}{2e_{\text{оп}}}[(U_x + U_y)^2 - (U_x - U_y)^2] = \frac{2}{e_{\text{оп}}} U_x U_y. \tag{8.55}$$

Fundamental circuit of such a multiplier is shown in Fig. 151.

As source of saw-toothed voltage there is used a special generator, made from three envelopes, providing a frequency of output voltage from 1 to 2 kilocycles.

Reference-voltage source in the circuit should have low internal resistance. Error of device constitutes 1%. Maximum frequency of input signal 50 c.

CHAPTER IX

PRINCIPLES OF CONSTRUCTION OF D-C ELECTRONIC ANALOG COMPUTERS

1. Composition of Computing Elements, Methods of Their Arrangement in Installation and Interconnection for Solving a Problem.

Composition of computing elements of an electronic analog is determined by assignment of computer. We distinguish general-purpose and specialised devices. General-purpose computers have as their main assignment solution of ordinary linear and nonlinear differential equations, occurring during investigation of dynamics of various technical devices. Specialised computers are intended for investigation of only one definite class of these devices and therefore must provide resolution of differential equations of fixed structure. As an example there can serve various link trainers (see G. B. Ringham and A. E. Culter [1]), installation of type "Typhoon"*) and "Tridac,"** adapted correspondingly for instruction of flying staff and solution of problems of dynamics of guided missiles.

In specialised devices depending upon their assignment there can be provided completely definite composition of computing elements.

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*New computer aids air defense (Project Typhoon), Electronics, 1951, Vol. 24, No. 2, page 132. (For short characteristic and photo of general appearance see appendix II, page 468.

**Three-dimensional analogue computer, Engineer, 1954, Oct. 15, Vol. 198, No. 5151, page 532. (For short characteristic and photo see appendix II, page 468.

In general-purpose devices it is impossible in principle to limit composition of computing elements since beforehand it is unknown what problems will be posed for solution.

Until now in world practice during construction of general-purpose analog computers they either went the way of limitation of the order of differential equations, solved by the computer (so-called stand installation): IPT-4, EDA (I. S. Bruk, N. N. Lenov [1]), MPT-9, MN-2, MN-7, EMU-1, EMU-2, EMU-3, EMU-5*, or the way of fulfillment of the computer in the form of separate identical units, assembled in needed quantities and types each time before beginning work (computers IPT-5, MPT-11, installation computer of the Philbrick firm)*. In first case most installations are limited to solution of linear differential equations up to the 6th order inclusively (IPT-4, EMU-3, OME-L2, installation of Short firm REAC, MN-2). For solution of nonlinear problems to them there was added or in their composition there was provided a definite quantity and assortment of nonlinear blocks.

Solution of problems, beyond the capabilities of one installation, was carried out by means of connection of several monotypic one (see, for example, MPT-9, MN-7, EMU-5, EMU-6, OME-L2).

During designing of such installations rational selection of relationship between number of linear and nonlinear computing elements presents great difficulties and has not yet found positive solution. Apparently, solution of this question can be found in designing simulators of block type. However in distinction from existing analogs of block type (IPT-5, MPT-11, the Philbrick firm) dimension of block should be increased so that with its help it was possible to reproduce equation of motion of system with one degree of freedom. Furthermore, separate blocks do not have to be connected by common power supplies and a common setting field, as takes place in the installations mentioned above.

It is useful to equip linear computing elements with nonlinear feedback circuits to execute nonlinear operations. Such construction of installation will allow us with the least number of standard sizes of blocks to satisfy various requirements, not fixing rigidly the general composition of computing elements of model. Furthermore, every separate block can also have idependent application (solution of differential equations of the second order, pickup of sinusoidal oscillations, instrument for measurement of resistance, etc.).

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*Short technical characterisation and general appearance of enumerated installations are brought in Appendix II.

Electronic analog EMU-8 (see Appendix II) can serve as an illustration of a first attempt at constructive embodiment of these ideas. Use in this installation of operational amplifiers, not requiring stabilized feed, and semiconductor elements (germanium diodes and thyrite resistors) in many respects promoted successful solution of the problems posed.

Linear and nonlinear computing elements considered in preceding chapters constitute the basis of both general-purpose and specialized installations. To these computing elements, executing operations of summation, integration, differentiation, multiplication, division, reproduction of given functional dependences, there usually are added devices for reproduction of typical nonlinear characteristics of CAP (Automatic Control Systems), for introduction of constant delay, factors variable in time, random disturbances, and also converting devices for connection with tested equipment.

All these additional devices are considered below during analysis of simulation of separate types of CAP.

In certain simulators (see T. N. Sokolov [1], L. V. Polonskaya [1], N. Klamka [1], J. T. Carleton [1], I. Obradovich [1]) designated for study of CAP, at present along with operational amplifiers they still provide passive electric circuits for reproduction of transfer functions of separate dynamic sections of the system (inertial link, link of the second order, etc.). If 8-10 years ago such a solution could be justified by absence of well-developed operational amplifiers, then at present it can be considered economically rational only for narrowly specialized devices.

By method of arrangement of separate linear computing elements in the installation we distinguish matrix and structural models. In matrix models separate computing elements are united beforehand in groups, each of which is intended for solution of a first-order differential equation (see, for example, ELI-6, ELI-14, IPT-4, OME-L2).

Set-up here consists in setting coefficients for the variables, introduction of signals corresponding to right side, and interconnection of such separate groups.

Structural models differ by the fact that in them all computing blocks are free and are coupled by the operator in an order, determined by the system of differential equations to be solved coupling of computing elements can be carried out on an operating field, formed by face panels of operational amplifiers (see, for example, computers EMU-3, of the Boeing firm, IPT-5, EDA) by flexible cords with plugs or by a special setting-up field, where leads from all inputs and outputs of operational amplifiers, their integrating points, units of input impedances, potentiometers, nonlinear computing elements and other additional equipment, added to the computer are assembled (see EMU-4, EMU-5 MPT-11, MN-7, REAC-400). Matrix models facilitate operation of computer, since they require a minimum number of switching operations during set-up of a problem. However as compared with structural models they require approximately twice as many operational amplifiers (see for more detail Ch. X). Furthermore, during solution of nonlinear problems we can not sustain completely the matrix principle of connection of blocks (see, for example, ELI-6 whose linear part is assembled by matrix principle, but the nonlinear part--by structural principle). In connection with this most simulators are constructed at present by the structural principle.

Connection of separate computing elements by flexible wires with plusgs has the advantage that connecting wires between separate computing blocks, and especially important, between their integrating points and input impedances can be made very short. During set-up of the problem on separate setting field these connecting wires, other conditions being equal, are significantly longer, which leads to increase of spurious capacitance between the integrating and common point of the amplifiers and impairment of their frequency responses. However, during solution of complicated problems the large number of wires, enmeshing the front panel of the simulator, leads to loss of graphicness and to subjective errors of the operator. Presence of a plug-in setting field (see computors of the Reeves firm, SEA firm, MN-14 and EMU-10) allows us to conduct preparation of problem beforehand and thereby considerably to reduce solution time.

Simulators are also divided into devices working in natural and unnatural time scales. In computers of first type processes are reproduced with the speed, which is determined by the initial differential equations given for solution. In computers of second type processes are reproduced at an accelerated rate.

During solution at an accelerated rate we often introduce artificial iteration of processes which allows us to observe the solution on cathode oscillograph and to build operational amplifiers in the form of a-c amplifiers (analogs ELI-14, ELI-12, Philbrick firm, and others), and also to adapt electronic analogs for solution of two-point boundary value problems in complete derivatives, certain variational problems, integral equations, etc.

Artificial iteration of solution is used in d-c electronic models for visual observation of the solution on electronic oscillograph, furnished with an electron-beam tube with a screen possessing afterglow. Frequency of iteration is established here from 0.5 to 8 c. Such computers can work both with a natural and unnatural time scale; here there do not appear excessive requirements as to width of the passband of separate computing elements.

2. Methods of Setting Initial Conditions and Transmission Factors of Computing Elements.

Before beginning solution of a problem on a computer it is necessary for every dependent variable (coordinate) to establish initial values. This is possible to carry out in principle two ways: charging the integrating capacitor and by connection to each integrating amplifier of an additional adder, to one of whose inputs there is fed constant voltage, corresponding to the constant of integration.

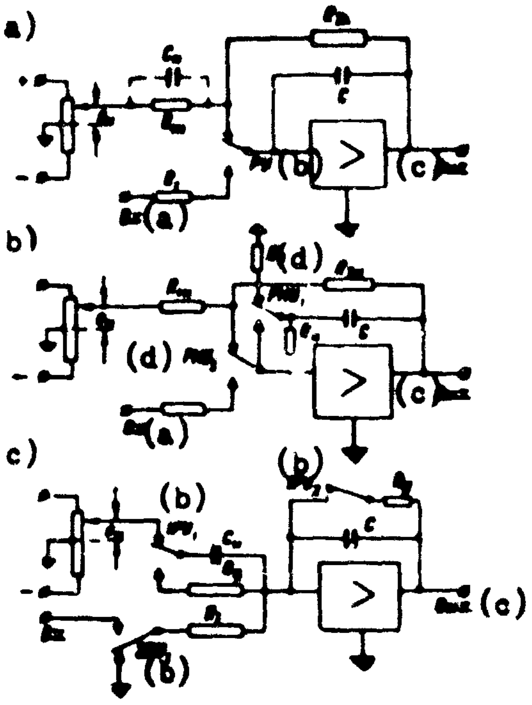

Fig. 152. Circuits of setting initial conditions.
KEY: (a) Input; (b) Control relay; (c) Output; (d) Initial conditions relay.

In Fig. 152a is the most wide-spread circuit of setting of initial conditions (computers IPT-4, IPT-5, MPT-9, REAC, EMU-1, EMU-2, EMU-3, EMU-4). Here each integrating unit before beginning work is shifts into the mode of a delay component. Here its output voltage will change in accordance with expression

$$\bar{e}_{\text{вых}} = -\frac{R_{2\text{н}}}{R_{1\text{н}}}\frac{1}{(1+R_{2\text{н}}Cp)}\bar{e}_{\text{н}}. \tag{9.1}$$

In steady-state regime

$$e_{\text{вых}} = -\frac{R_{2\text{н}}}{R_{1\text{н}}}e_{\text{н}}. \tag{9.2}$$

Thus, integrating capacitor before beginning work of the computer is forcibly charged to the required voltage.

The circuit does not requre limitation in duration of preoperational period, since in this interval of time the integrating capacitor is forcibly charged from a source of voltage of initial conditions. The main defieiency of the circuit is the fact that voltage, equivalent to initial conditions, is not established at output of amplifier at once, and therefore accurate adjustment is hampered and is linked with excessive waste of time. Acceleration of the setting-up process for initial conditions can be achieved in this circuit by coupling an additional capacitor $C_{н}$ in parallel to resistor $R_{1н}$ (see dotted line on Fig. 152a).

Here

$$\bar{e}_{\text{вых}} = -\frac{R_{2\text{н}}}{R_{1\text{н}}}\frac{(R_{1\text{н}}C_{\text{н}}p+1)}{(R_{2\text{н}}Cp+1)}\bar{e}_{\text{н}}. \tag{9.3}$$

If one were to select parameters in such a manner that $R_{1H} C_H = R_{2H} C$, then

$$e_{\text{вых}} = -\frac{R_{2\text{н}}}{R_{1\text{н}}} e_{\text{н}}$$

and output voltage of amplifier will be set practically instantly.

Inaccurate observance of equality of time constants of both RC-chains leads in the case, when $R_{1\text{н}}C_{\text{н}} < R_{2\text{н}}C.$ to delay in achievement of steady-state at output voltage, while when $R_{1\text{н}}C_{\text{н}} > R_{2\text{н}}C$ it leads to a jump of voltage at integrator output, exceeding steady-state voltage of initial conditions. The latter at $e_{\text{н уст}}$ near 100 v, may cause output of the block beyond the limits of linearity.

In the circuit in Fig. 152b* acceleration of process of setting the voltage of initial conditions is attained by switching the integrating capacitor to the output of the computing block, which in the period of setting of voltage of initial conditions is converted into the regime of a scale block. Since output resistance of the operational amplifier is low, the capacitor is charged practically instantly to the required voltage.

$$e_{\text{вых}}(0) = -e_{\text{н}} \frac{R_{2\text{н}}}{R_{1\text{н}}}. \tag{9.4}$$

In transition to operating regime the capacitor is switched to the feedback circuit, and resistors R_{1H} and R_{2H} are disconnected from the integrating point. Resistor R_H serves to preserve negative feedback of the block during transition of the armature of the relay of initial conditions *PHУ* from contact to contact. It is selected from 100 kilohms to 1 megohm.

In simulators with iteration of processes at a frequency above 10 c (computers ELI-14 and ELI-12 see L. I. Gutenmakher [2]) there is applied the circuit for setting initial conditions shown in Fig. 152 c. Due to low internal resistance of the source of e_H capacitors C_H and C are charged practically instantly:

$$e_{\text{вых}}(0) \approx -\frac{C_{\text{н}}}{C} e_{\text{н}}. \tag{9.5}$$

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*Applied in EMU-5 of Academy of Sciences of USSR.

Due to the fact that voltage at C_H equals voltage e_H, current does not flow in this circuit, and the source of e_H is disconnected from integrating amplifier. Resistances R_8 serve to limit current during discharge of capacitors C_H and C in the half-period of preparation of the circuit for solution of a problem.

A circuit, illustrating setting of initial conditions by adders, connected to output of integrating block, is shown in Fig. 153. Such method of setting initial conditions has the advantage that it does not require commutation of the circuit at the integrating point of the operational amplifier, it lowers requirement of high speed operation of control relay and allows us to open block in preparation for solution of a problem by short-circuiting integrating capacitors by contacts of PУ-1. Resistor R, serves to limit current through contacts of the relay at the moment of short-circuiting of the capacitor. Main deficiency of this means of setting initial conditions is the necessity of connecting the adder with every integrator.* Often as such an adder there can be used a unit in set-up circuit. However in general the number of operational amplifiers required here increases.

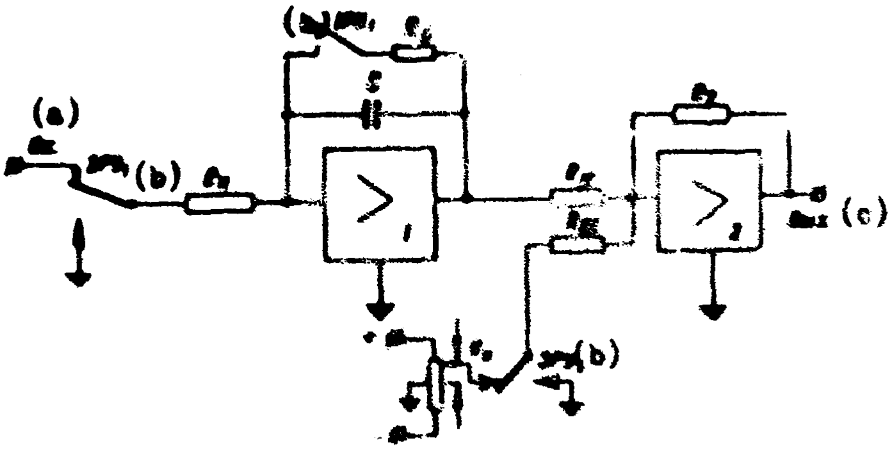

Fig. 153. Circuit of setting of voltage of initial conditions at output of integrating amplifier.
KEY: (a) Input; (b) Control relay; (c) Output.

Setting of transmission factors is carried out in most cases on linear computing elements. Nonlinear blocks usually are supplied with a fixed transmission factor, determined from the condition of obtaining at output a total voltage of the model scale (100 v) when supplying input with voltages of the argument, 100 v.

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*Besides this one should consider that application of this circuit requires having of scale of voltages of setting to eliminate possible output of amplifiers beyond the limits of linearity.

To solve differential equations it is necessary to provide possibility of change of transmission factors of every linear computing element in wide limits (from 100 and to 0.001). Setting of transmission factors greater than one usually is carried out continuously by change of resistance, connected to the input, and in steps by feeding the feedback circuit from a divider, connected to the amplifier output.

Setting of transmission factors less than one is carried out continuously by connection of voltage divider to output and in steps by transition do another magnitude of resistance of feedback. In case of work in integrator regime step-by-step change of transmission factor is carried out in the direction of decrease by parallel connection of an additional capacitor and in the direction of increase by transition to capacitors of lower capacitances, connected to the block from without. Smooth change of transmission factors greater than one is achieved by change of the resistance, connected to input, and smaller than one—by a divider at the output. In Fig. 154 is brought a circuit, in which are provided the above-mentioned methods of setting transmission factor of an operational amplifier.

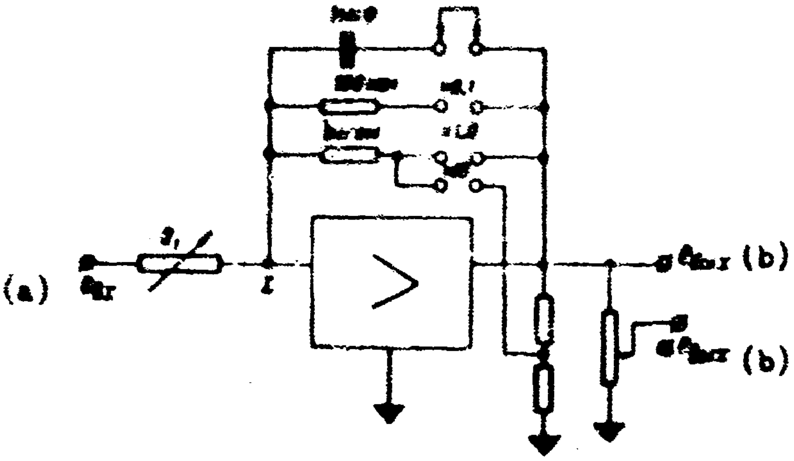

Fig. 154. Methods of setting transmission factor.
KEY: (a) Input; (b) Output.

In certain simulators the potentiometer, connected to the output of the block, is executed in the form of a ten-turn wire potentiometer, (for example, in EMU-10, PACE, REAC and others) or a three-decade divider according to the diagram in Fig. 155 (for example, IPT-4, IPT-5, MPT-9, etc.). Voltage dividers Д-1 and Д-2 have eleven sections of equal resistances each, and divider Д-3 has 10 sections. If resistance of sections of separate dividers are selected in such a manner that relationship $r_3 = 0.2r_2$ and $r_2 = 0.2r_1$ were fulfilled, then in the given diagram connections of dividers of resistance between cursors a and b, and also c and d will be constant and accordingly equal to r_1 and r_2.

Indeed,

$$r_{cd} = \frac{2r_2 \cdot 10r_3}{2r_2 + 10r_3},$$

whence when $r_3 = 0.2r_2$ we obtain $r_{cd} = r_2$;

$$r_{ab} = \frac{2r_1 \cdot 10r_2}{2r_1 + 10r_2},$$

whence when $r_2 = 0.2r_1$ we obtain $r_{ab} = r_1$.

Therefore independently of position of cursors ab and cd to divider Д-2 there will always be fed voltage $U_{ab} = \frac{e_{\text{вх}}}{10}$, and to divider Д-3 --voltage $U_{cd} = \frac{e_{\text{вх}}}{100}$.

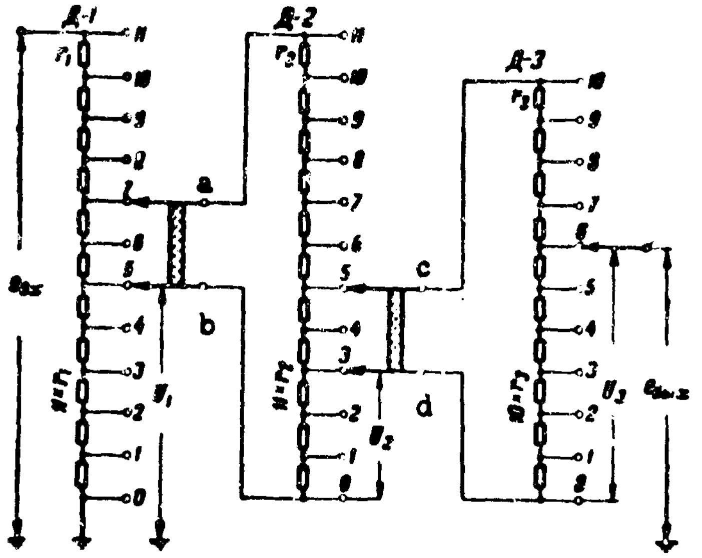

Fig. 155. Circuit of three-decade divider.

Total voltage at output of circuit will be

$$e_{\text{вых}} = U_1 + U_2 + U_3 = \frac{e_{\text{вх}}}{10} n_1 + \frac{e_{\text{вх}}}{100} n_2 + \frac{e_{\text{вх}}}{1000} n_3 = \frac{e_{\text{вх}}}{10}(n_1 + 0.1n_2 + 0.01n_3). \quad (9.6)$$

where n_1, n_2, n_3 are the numbers of lead-outs of a section of the dividers.

Thus, with the help of such a divider it is possible to set coeffient $x < 1$ with accuracy up to the third significant digit. Usually resistors of sections r_1, r_2 and r_3 are made wire resistor.

Setting transmissions factors greater than one is realized by change of input impedance. This resistance in certain analogs (EMU-3, EMU-4) is executed in the form of a two-decade box with smooth setting of the third significant figure by a series connected resistor (Fig. 156). Here there is necessary for each box to have two switchs of 10 positions each.

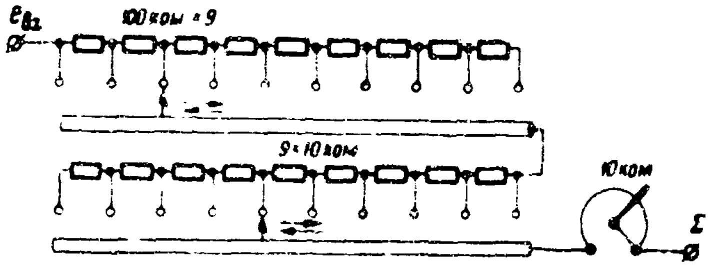

Fig. 156. Circuit of two-decade additional resistor.

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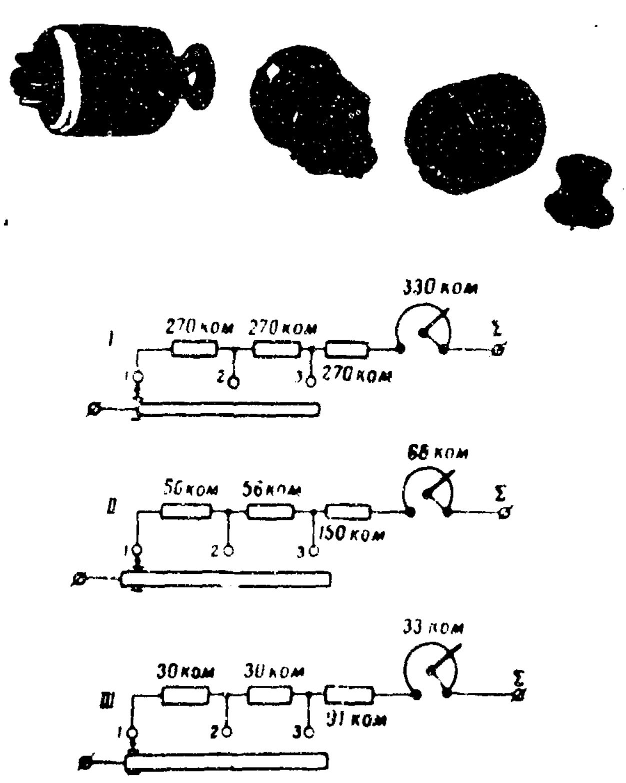
a)

b)

Fig. 157. Plug-in resistance box. a--General view; b--circuit.

Accuracy of setting of transmission factor with the help of smooth change of the third step of a 10 kilohm resistance will vary depending upon the magnitude of the series-connected resistance.

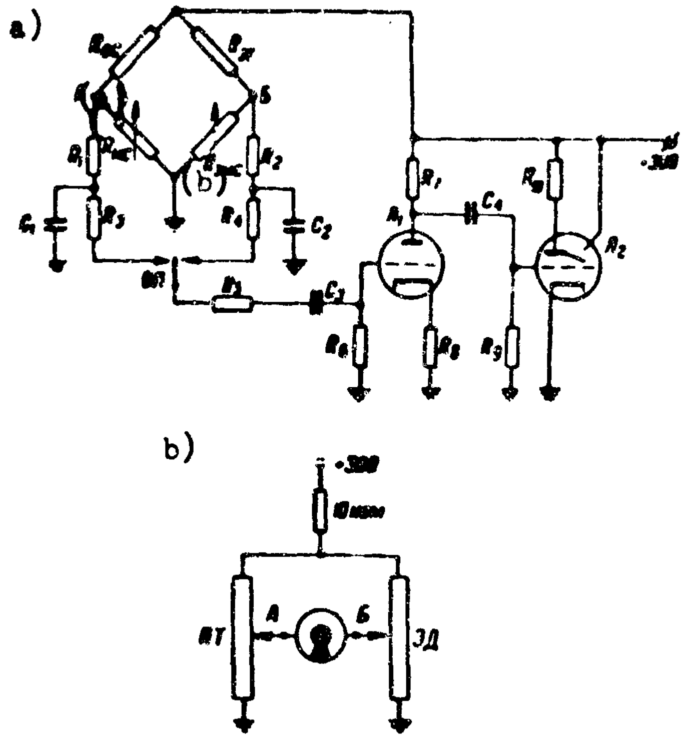

Fig. 158. Bridge circuit for measurement of resistances. ПТ --potentiometer; ЭД --standard divisor.
KEY: (a) Resistance box; (b) Standard resistance box.

In connection with this of interest is the method of change of input impedance, used in electronic analog EMU-5. Here input impedances are made in the form of separate plug-in blocks (Fig. 157). The blocks are made in three modifications, covering total range of change of transmission factor (from 1 to 10). Inside each block are mounted three constant resistances and one potentiometer for accurate adjustment of the coefficient. First modification ensures smooth adjustment in range from 1.14 megohm to 270 kilohm, the second--from 320 kilohm to 150 kilohm and the third--from 184 kilohm to 91 kilohm. In Fig. 157b are diagrams for connection of resistances in each modification. Switching of resistances in each block is carried out by setting the installation block in one of three positions, provided by an octal tube plug.

Application of resistors calibrated with great accuracy for setting of coefficients usually leads to sharp appreciation of the installation cost. Transmission factors of blocks can be fixed with accuracy up to 0.1% with application of ordinary carbon resistors of type BC, potentiometers with tolerance of ±10% and use of a bridge circuit for measurement of resistances (Fig. 158). Input impedance $R_{вх}$, subject to setting, resistor $R_{ос}$ in feedback circuit, and two standard resistances: $R_{эт}$ --unregulated and $R_{эмс}$ (standard three-decade wire resistance box ЭМС) form the bridge. To points A and B of this bridge is connected a zero-indicator. Zero-indicator is an a-c amplifier with contact modulator at the input and an output tube of type 6E5 ("magic eye"). During appearance of alternating voltage on grid of this tube the dimension of the shady sector decreases. Therefore, when bridge is in equilibrium, the whole shady sector is cleared.

When setting transmission factor of potentiometers, connected to output of operational amplifier, bridge will be formed by standard divider, whose diagram is shown in Fig. 155, and the most tested potentiometer ПТ (Fig. 158b).

Amplification factor of amplifier of zero-indicator is selected equal to 10 to 15 considering that it is possible reliably to distinguish one step of the third decade of the ЭМС and ЭД.

3. Individual and Group Setting of Zero Level of Operational Amplifiers, Methods of Combatting Leak and Methods of Indicating Overloading.

At the output of operational amplifier in the absence of input signal there can be a certain voltage. This voltage usually consists of a constant (or, more correct, slowly variable) component and higher harmonics. It appears as a result of inaccurate setting of voltages of power supplies, presence of grid current and change of emission of first cascade, leaks at integrating point from side of power supplies, etc (see for more detail Ch. III). In connection with this before beginning to solve a problem there always is carried out a check of zero level.

During use of operational amplifiers with a triode circuit of compensation for drift (see Ch. IV, page 102) setting of zero level is executed, as a rule, before each computation by a potentiometer, switched into the coupling network between the first and second cascades. With application of amplifiers with automatic stabilization of zero level the need for frequent check of zero level is gone. Here in the process of initial adjustment of the operational amplifier by a potentiometer of the interstage coupling they select an operating regime for the least amplitude of higher output harmonics, and displacement of zero under the influence of grid currents is eliminated by introduction of a certain additional emf at the input of the operational amplifier.

When the simulator consists of a large number of operational amplifiers with triode compensation (MN-2, EDA, etc.), for the purpose of facilitating the operation operation of setting of zero can be automated. Here all amplifiers of the computer are broken down into separate groups, and setting of zero of the amplifier of each group is produced in turn.

The system of group setting of zero is executed in two modifications: electromechanical and electronic. In both cases setting of zero is carried out by measurement and amplification of the signal of error e_i at integrating point Σ . In electromechanical variant (Fig. 159) an M-ДM amplifier (see Ch. IV, page 116), with polarized relay 2, connected to output, is periodically connected by stepping switch 3 to integrating point of amplifier. If voltage e_i exceeds 2—3 millivolts, output relay works and one half of electromagnetic clutch 4 is switched on, moving the cursor of potentiometer of zero setting 5.

Such a system ensures control of zero of amplifier not only before beginning work, but also during operation. According to the data of I. M. Vittenberg [1] with a periodicity of adjustment of 20-30 minute error due to inaccuracy of setting

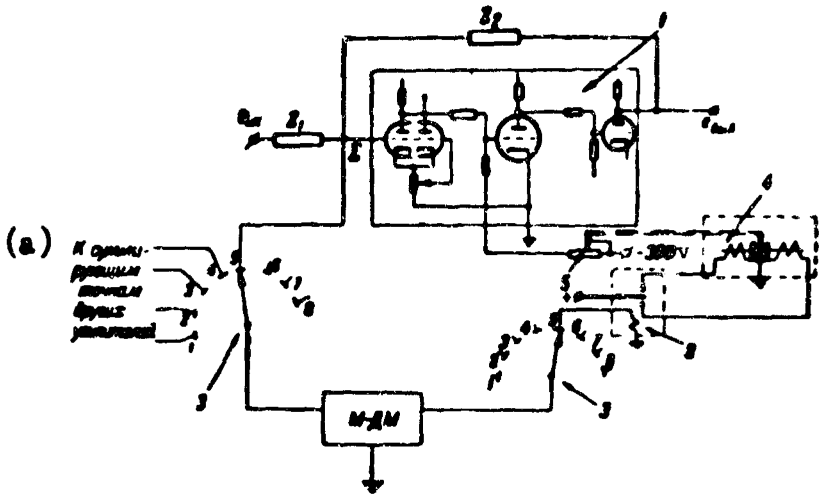

Fig. 159. Electromechanical variant of device for group correction of zero level of operational amplifiers. 1--main d-c amplifier; 2--control relay of clutch; 3--stepping switch; 4--electromagnetic clutch.
KEY: (a) To integrating points of other amplifiers.

of zero level does not exceed 2-3 millivolts. In electronic variant of the device (N. N. Lenov [1]), brought in Fig. 160, amplifier М-ДМ periodically is connected to integrating point of amplifier and charges capacitor, connected to grid of right half of first tube.

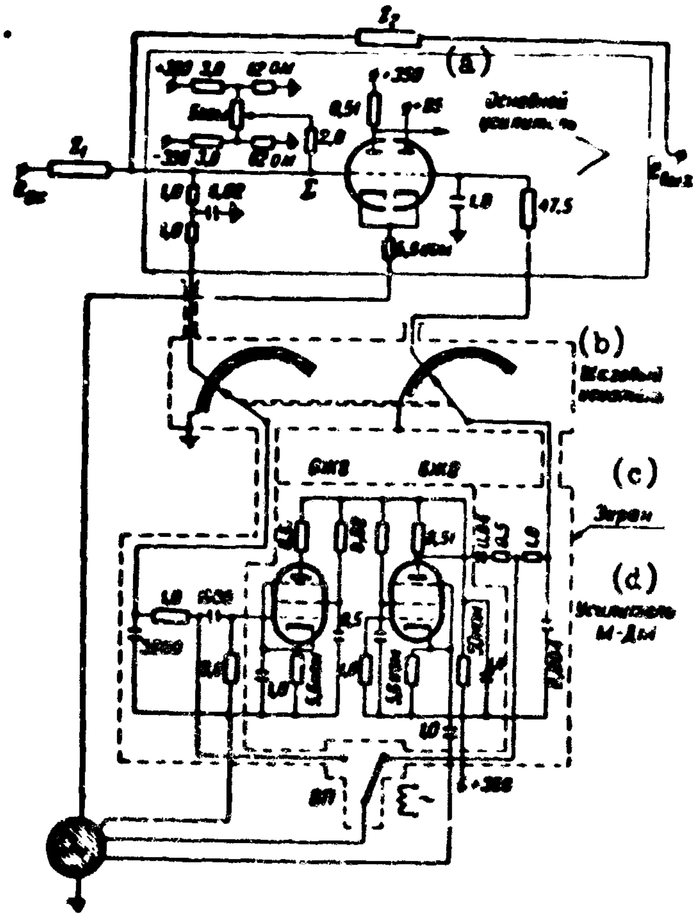

Fig. 160. Electronic variant of device for group adjustment of zero.
KEY: (a) Main amplifier; (b) Stepping switch; (c) Screen; (d) М-ДМ amplifier.

Since in the absence of correction the amplifier in this circuit remains without triode compensation, then it is necessary to switch on a device of periodic correction continuously and increase the frequency of control to 1 time each 10-15 sec. Error in setting of zero in such a circuit is not less than 2-3 millivolt (on grid of first tube).

Besides setting zero level, the described systems somewhat lower instability of zero level, mainly for an operational amplifiers, working in regimes of scale blocks and integrators. During work of amplifier in integrator regime these systems cannot in accuracy of stabilization of zero level complete with operational amplifiers equipped in individually with a system of automatic stabilization.

Experience shows that application of systems of group setting of zeroes in both variants in magnitude of zero drift does not give results better than in amplifiers with triode compensation, however it provides a complete process automation of setting zero.

Accuracy of work of integrating operational amplifiers depends both on the degree of stabilization of zero level and also on spontaneous discharge of integrating capacitor due to leaks through the dielectric and external insulation. In Fig. 161a are shown possible leaks in the circuit of an operational amplifier on the example of the amplifier of analog EMU-5. Besides these leaks, change of charge of integrating capacitor can be caused also by leaks between current-carrying wires of power supplies and the integrating point of the amplifier.

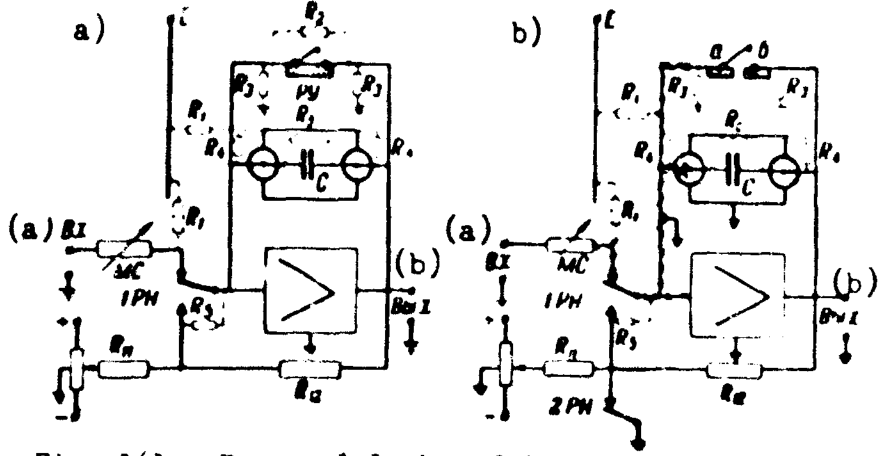

Fig. 161. External leaks of integrating capacitor and their removal. MC --resistance box; PY --control relay, PH --relay of initial conditions.
KEY: (a) Input; (b) Output.

Application of improved insolation between the indicated parts of the circuit has significantly less effect than introduction of so-called "ground insulation" (V. V. Gurov [1]). In Fig. 161b is brought a diagram of an operational amplifier, in which all elements, coupled with the integrating point and output, are separated from each other by metallic grounded parts. Here all connections to the integrating point should be by shielded wire with reliable grounding of the shield. The lead to the integrating point should be conducted not through a common plug, but to a separate jack, fixed in the metallic chassis. The metallic housing of the integrating capacitor should be reliably grounded.

Contacts of relays of discharge of integrating capacitor must not be on the general insulating plate. In order to use here ordinary telephone relays, it is expedient to connect contacts by the diagram brought in Fig. 162. Contacts a and b are connected correspondingly to the integrating point and the output of the operational amplifie..

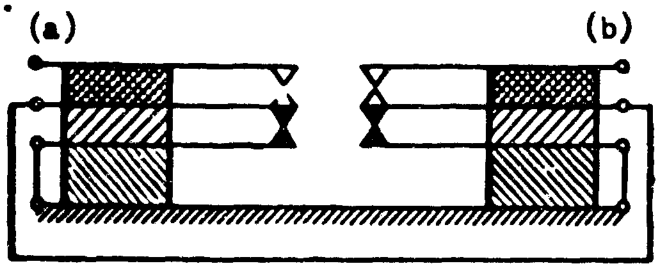

Fig. 162. Diagram of connecting of contacts of relays with "ground insulation."

Enumerated measures lead to replacement of leaks between integrating point and output of operational amplifier through surface of insulation by leaks R_{s1} between integrating point and the ground and leaks R_{s2} between output and the ground (Fig. 163). Increase of leaks between integrating point and the ground has not so essential an influence on error of work of the unit. Additionally appearing here, spurious capacitance between integrating point and the ground leads to certain reduction of passband of amplifier.

Effectiveness of described method "ground insulation" can be illustrated by example of solution of a differential equation of type

$$\frac{d^2y}{dt^2}+a^2y=0 \tag{9.7}$$

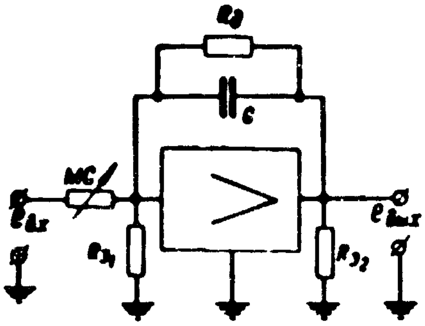

Fig. 163. Equivalent diagram of operational amplifier, with "ground insulation." MC --resistance box.

when $\varphi(0) = A$ and $\dot{\varphi}(0) = 0.$

In Fig. 164a is brought oscillogram of solution of this equation, when a = 0.314, by ordinary operational amplifiers, and in Fig. 164b, when a = 0.003, by amplifiers (electronic analog EMU-6), for which there is "ground insulation." As follows from these figures, lowering of amplitude after 30 min in first case constituted nearly 4.5%, while in the second it practically is inconspicuous. Lowering of amplitude of oscillation by 2% is observed in second case only after 120 min.

These data indicate possibility of use of such operational amplifiers for simulation of slowly proceeding processes of automatic control.

In simulators there are two accepted methods of signalling departure of operational amplifier beyond the limits of linearity. Usually for this on output of operational amplifier is connected a neon signal bulb, which burns upon achievement of a voltage on amplifier output of ±100 v. Often simultaneously there is connected a relay for supplying an audible signal and the command to cease computation.

When operational amplifiers are made with a system of automatic stabilization of zero level, it is possible for indication of overloading to use signal of error e_δ, amplified by an M-DM amplifier.

As is known,

$$e_\delta = e_{вх}\frac{R_2}{R_1(1+K_y)+R_2} \tag{9.8}$$

with departure of amplifier beyond limits of linearity, $K_y \to 0$, and $e_\delta \to e_{вх} \times \times \frac{R_2}{R_1+R_2}$ and will grow with increase of input signal.

In literature signs of this method of indication of overloading is met for the first time in the book of G. Korn and T. Korn [1] during description of operational

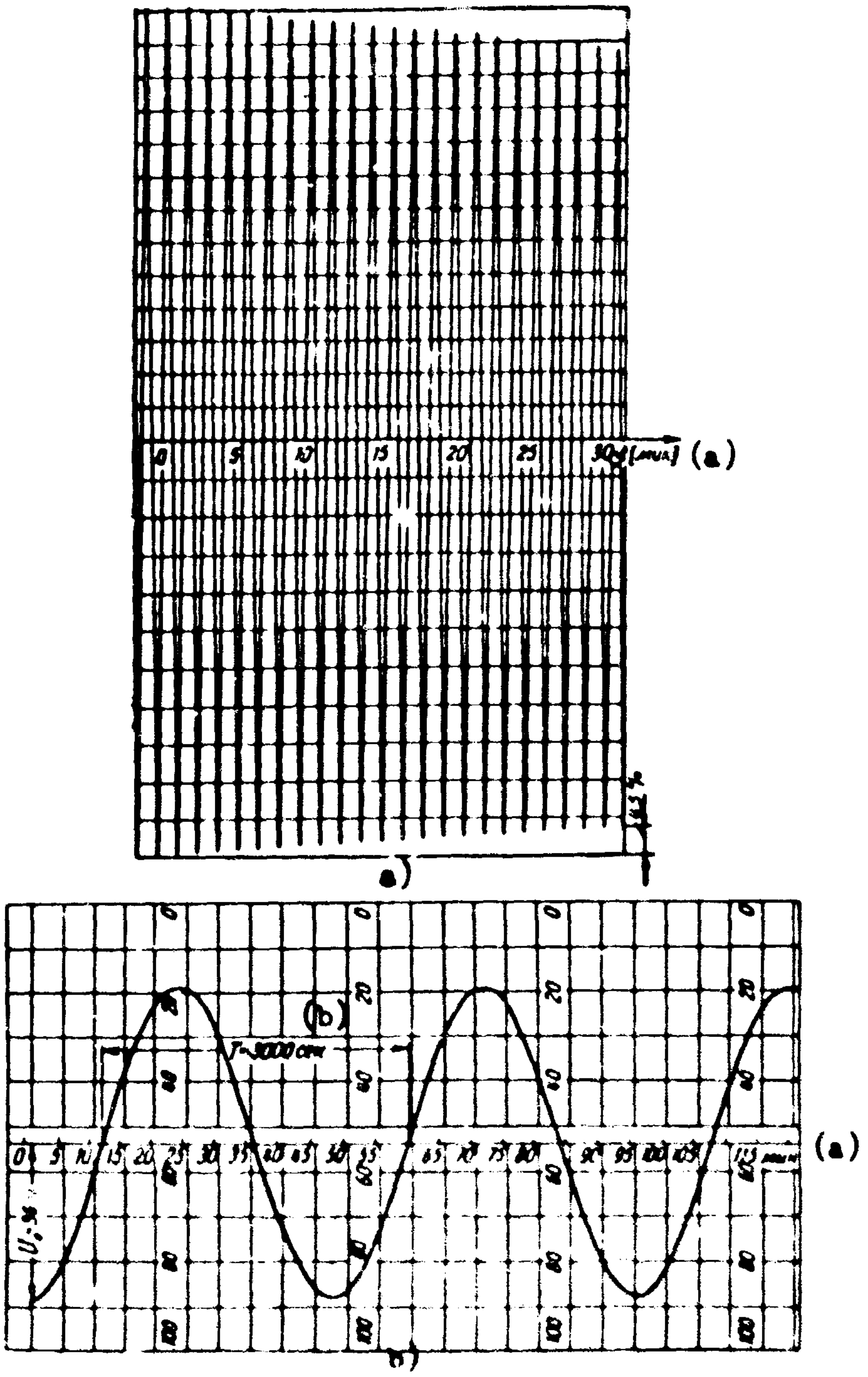

Fig. 164. Oscillogram of solution on analog of second order equation.
KEY: (a) Minutes; (b) Sec.

amplifier of simulator of the Rand firm.

In Fig. 165 is brought diagram of indication of overloading based on measurement of amplified signal e_δ and applied in electronic analogs EMU-5 and EMU-6 (Academy of Sciences of USSR).

Voltage of error e_δ in this circuit after amplification by М-ДМ amplifier moves from demodulating contact of vibrator ВП to one of the electrodes of neon tube НЛ. The second electrode is connected to a common terminal, to which are connected neon tubes of other operational amplifiers. The common terminal is connected to the ground through resistor R_3. With ignition of neon tube on resistor R_3 there will be formed alternating voltage, which is amplified by tube $Л_1$, rectified by diode $Л_2$ and after smoothing by filter R_6 C_3 moves to the control grids of tube $Л_3$. Relay P_1, connected in plate circuit of tube $Л_3$, here is triggered and closes contacts, $1P_1$ and $2P_1$. Contacts of the relay give signals to cease work. The system is tuned in such a manner that the relay works with a voltage at the integrating point, exceeding 2 millivolts.

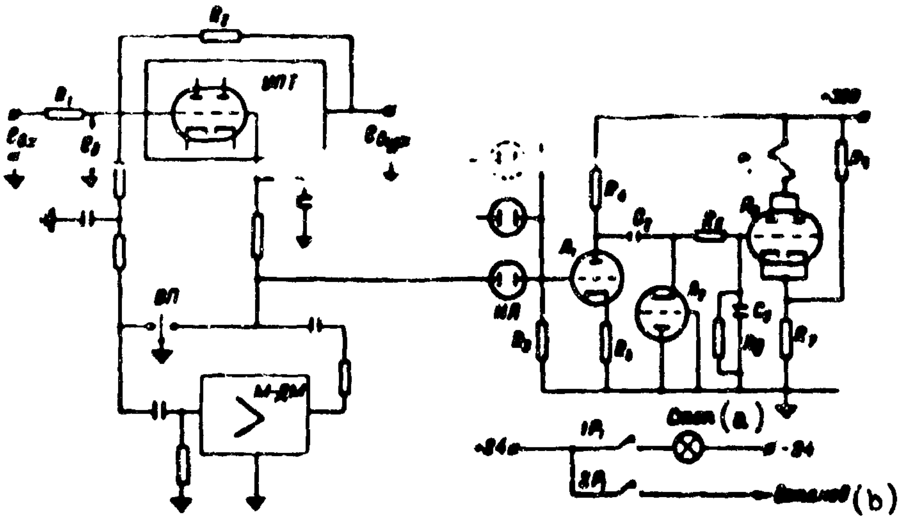

Fig. 165. Circuit for indication of overloading. УПТ --d-c amplifier, М-ДМ--amplifier with modulation and demodulation; НЛ --neon tube, ВП -- vibrapack.
KEY: (a) Stop (reset); (b) Stop.

Such a method of signalling departure from the limits of linearity has the advantage that it allows one to use in the absence of load on output of operational amplifier the full range of linear change of output voltage, considerably exceeding usually ±100 v. Furthermore, it indicates malfunction in the circuit when output voltage remains within limits of model scale ±100 v, but real limits of linearity are narrowed, for example, by too many potentiometers, connected in parallel to the output.

4. Main Functions and Principles of Construction of Control System of Simulators.

To control system of simulator usually are entrusted fulfillment of following operations: switching on process of solution ("Start"), ceasing process of solution ("Stop") and return to initial position before switching on process of solution ("Stop").*

Ceasing solution is necessary to carry out measurements of values of separate coordinates at corresponding moments of solution, and also during overloading or fault of computing elements. Furthermore, the control system should ensure necessary switching operations when setting initial conditions, transmission factors of separate computing units and supply of disturbances.

For possibility of visual observation of solution on screen of cathode-ray oscillograph with tube, possessing afterglow, system of control should ensure division of process into periods with period of 2 to 30 sec. It is very desirable here to carry out supplying of time marks and in turn connect to plates of electron-beam tube one of the two input quantities for their simultaneous observation on the screen.

Together with this in the installation there is often provided automatic ceasing of the process of solution and switching on or disconnecting input signal at a moment of time given beforehand.

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*Editor Note. Author apparently means "reset" for second version of "Stop." We will put (reset) after this varient.

The control system should be simple, reliable and convenient in operation. The less the switching contact equipment in the circuit, the greater its reliability.

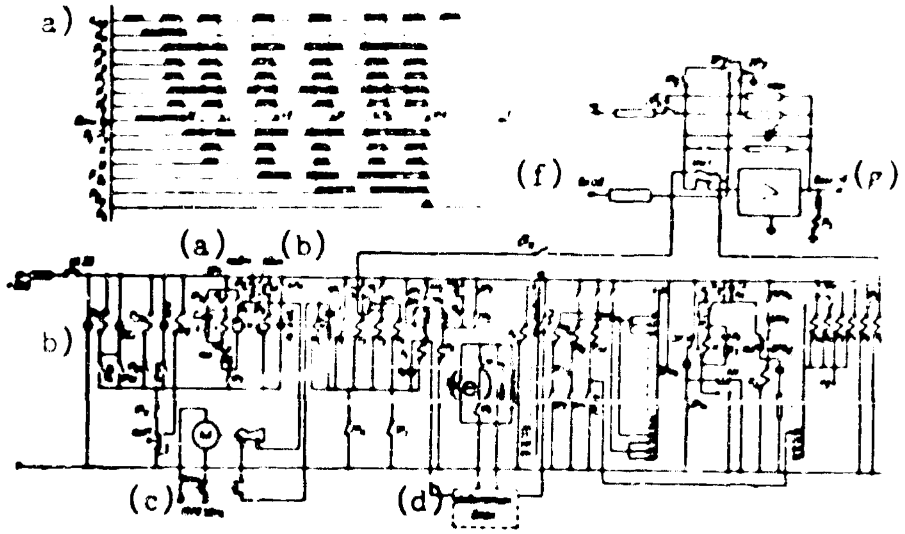

Fig. 166. Control circuit of electronic model, type IPT-5.
KEY: (a) Repeat; (b) Once; (c) Cycles; (d) Indicator unit; (e) Time mark; (f) Input; (g) Integrator; (h) Output.*

As example let us consider control circuit of electronic analogs IPT-5 and EMU-5. In Fig. 166a is brought the fundamental control circuit of simulator, IPT-5.** In addition of functions, mentioned above, in this circuit there is provided control of stepping switches of variable coefficients. In connection with the fact that power of contacts of the interrupter is insufficient for commutation of circuit of coils of stepping relays, in the circuit is provided control by contacts of multiple-throw switch K. Circuit of coil of multiple-throw switch K is interrupted by contacts of interrupter. To installation are added plug-in interrupters with pulse frequency 2/3, 1, 4/3, 2, 4 and 10 c.

With 100 steps of stepping selector this gives a time of duration of working cycle of 120 sec, 100 sec, 75 sec, 50 sec, 25 sec and 10.0 sec.

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*Editor Note: this is basically illegible diagram

**See description and operation instruction of electronic linear analog of type IPT-5, 1954.

Beginning of process of solution in this circuit is connected with beginning of first pause between pulses, issued by interrupter. This is achieved because the control relays in blocks of operational amplifiers are connected only after relays P_2 interlock P_2 and thereby feeds the coils of relay P_3 and relays P_y through normally closed contact $1P_1$.

Here relay P_3 interlocks by contact $2P_3$ and feeds of relays P_y in computing units for the duration of the whole time of solution.

To obtain time mark there is provided a circuit, consisting of series linked normally closed contact $2P_1$ and normally open contact $1P_4$, shunted normally by closed contact $3P_3$. At the beginning of each pause (starting with the second) between pulses of interrupter at a moment, when relay P_1 has already released its reed and, relay P_4 still holds, the indicated circuit is closed a short time. This is used for supplying time marks.

Duration of supplying time mark is determined by response time of relay P_4. In the circuit there is provided a pulse counter of type SI-1, by which it is possible to count these pulses.

Key Π_6 has two positions: work with constant and work with variable coefficients. In first position key Π_6 disconnects coil of contactor K. In second position pulses of interrupter through contacts $1P_0$ and $2P_0$ proceed to winding of contactor K, the lattor by its contact 1K gives pulse to coils of all selectors of blocks of variable coefficients of analog.

Control by work of blocks of variable coefficients is realized by stepping selector H_0, giving on the 49-th step the signal of switching of output circuits of blocks of variable coefficients from one field of lamellae of the selector to the other.

On the 90th step of the selector H_0 there moves a signal to relay P_a, by which is prepared a circuit for operation of relay P_k at the zero position of brush of selector H_0.

Operation P_k ensures return of installation to initial position.

In Fig. 166b is brought the timing circuit of the main switching elements of the circuit.

If it is required without interruption in work to iterate the solution, then for this toggle switch Π_4 must be placed in the "repeat" position. Here on the [illegible] step of selector $Ш_0$ excitation moves to windings of relay $\bar{P}_8$, fixed in blocks of operational amplifiers. Relay $\bar{P}_8$, disconnecting by means of contact $1P_8$ the circuit of relays P_y, which disconnect inputs of amplifiers and discharge, through a resistance of 1 kilohm, the integrating capacitors. On the 101st step of selector $Ш_0$ relay $\bar{P}_8$ is disconnected and to the operational amplifier there moves the voltage of initial conditions. With the help of contact $3P_6$ relay P_8 remains switched on and the whole circuit continues the following operating cycle.

The circuit allows us to stop solution and to fix the value of desired variables. For this we press button "Stop"--K_0, which excites relay P_5 through contact $1P_6$. Relay P_5 interlocks through contact $2P_5$ and excites through contact $1P_5$ relay P_9 in computing blocks which breaks the input circuit of integrating amplifiers. Simultaneously with this by opening contact $3P_5$ we disconnect contactor K which causes a halt of all stepping selectors of blocks of variable coefficients. Continuation of solution is carried out by pressing the push button $K\Pi$.

The control circuit of electronic analog computer EMU-5* is divided into two parts. One part, mounted directly in linear part of computer, ensures "Start" "Stop" and return of installation to initial position "Stop" (reset) and also switching operations during supplying of disturbances, setting of initial conditions and protection from overloading. Fundamental circuit of this part of the control system is shown in Fig. 167a.

In Fig. 167b is brought the circuit of an auxiliary control unit, intended for realization of repetitive operation, automatic ceasing of solution, supplying of time marks on screen of electron-beam tube, switching on and switching off of input signal at a predetermined moment of time. For bringing the circuit to a working state toggle switch of network T_c is set at the "On" position. Here there starts to revolve synchronous electric motor *СД* , which is coupled with three contacts of interrupter. These contacts produce pulses of direct current with frequency 1, 2, 5 c, which can be used for switching of stepping selector *ШИ* . Frequency of pulses is selected by switch *П*

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*See V. A. Trapeznikov, B. Ya. Kogan, V. V. Gurov, A. A. Maslov, [1].

Upon pushing the "Sta.t" button *K3П* relay *P3П* works, which locks itself by its own contact *1P3П* . Here the signal tube lights up and contact *2P3П* closes.

The first pulse coming from contacts of interrupter of synchronous electric motor *СД* , switches on the circuit of winding of relay *РП* . Contact 1PП is closed, which produces the operation "Start" of analog EMU-5. Simultaneously stepping selector switch makes the first step. During rotation of electric motor *СД* relay *РП* works with the frequency of the interrupter (1, 2 or 5 c) and stepping selector starts to count the number of pulses, coming from interrupter of electric motor *СД* . In the circuit there is applied a stepping selector with a maximum of steps, equal to 50.

To perform the operation "Stop" at a predetermined second to the "Stop" terminal they connect by a flexible cord with corresponding lead-out from lamellae of the stepping selector.

With connection of the brush of the selector with a lamella, to which is connected the cord "Stop" relay *PO* works and by contact 1*PO* breaks the circuit of blocking of relay *P3П* . Contact *2PO* produces in the analog EMU-5 the operation "Stop." With disconnection of relay *P3П* contact *2P3П* will be opened, the winding of relay *РП* is disconnected from the interrupter of the motor СД, and the stepping selector stops.

Period of iteration of the process is set by switch П and connection of cord

"Stop" with the corresponding lamella of the stepping selector. During switching on of toggle switch of division into periods T, relay *PIIII* works and by contact *3PIIП* produces the operation "Start," contact *1PIIП* closed, contact group *2PIIII* is switched and stepping selector begins to switch.

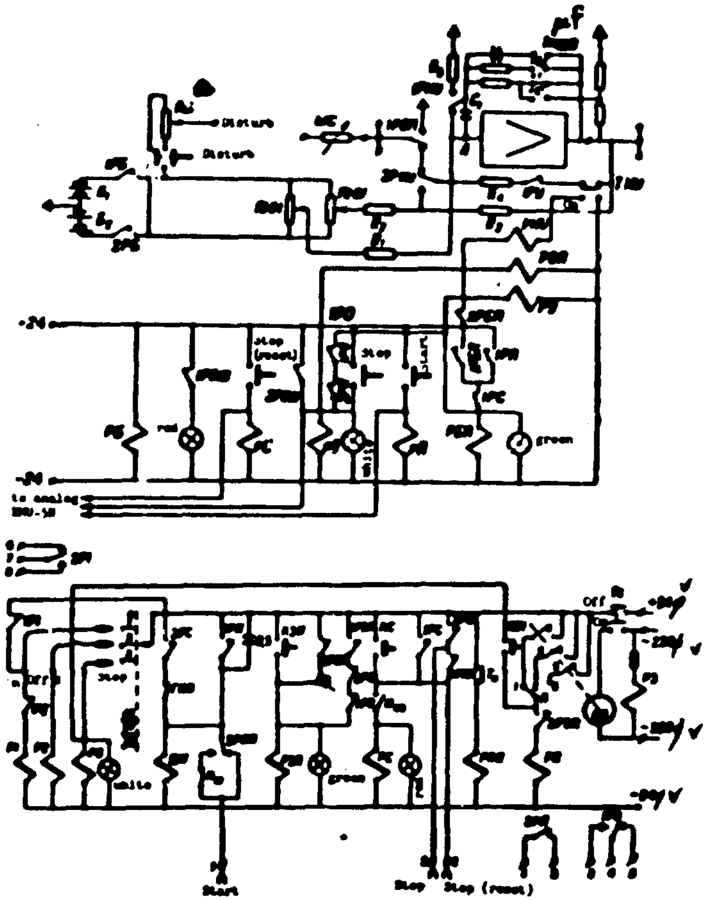

Fig. 167. Fundamental control circuit of electronic analog of type EMU-5.

Contact of the zero position of the selector H_{02} closes, and contact H_{01} opens.

With connection of the brush of the selector to the lamella, united by the flexible lead with socket "Stop", relay *PO* is triggered. Contact *2PO* closes and switches on relay *PC* ; simultaneously to analog EMU-5 moves the signal "Stop" (for discharge of capacitors of integrators and setting of initial conditions). Relay *PC* interlocks by its contact *1PC*.

Contact *3PC* closes and through the normally closed head contact of selector *ГКИ* switches on the winding of stepping selector *ШИ* ; here the brush of the selector automatically is set to zero position. With return of brushes to zero position contact И_{01} will open and relay *PC*. is disconnected, which will open contact 3 *PC* . Furthermore, contact И_{01} through contact *1PП* produces the operation "Start" of the model. The stepping selector again starts to switch and upon connection of the brush of the selector with the lamella, to which is connected the cord "Stop," the process is repeated.

In the additional controlling block there are two relays *P1* and *P2* . With the help of these relays and contact group *2P1* it is possible to execute the following operations:

1) switching on of input signal at a given moment of time;

2) disconnecting the input signal at a given moment of time;

3) switching of couplings in the set-up problem.

To execute the enumerated operations socket "On" is connected by cord with corresponding lamella of the stepping selector. When the brush of the selector is connected with this lamella, relay *P1* works and interlocks by contact *1P1* . Contact *2P1* is switched on. For switching off relay *P1* socket "On" is also connected with the corresponding lamella of the selector.

In the control circuit there is foreseen the possibility of parallel work of several simulators of type EMU-5 with control of the solution process from any installation. In distinction from installation IPT-5 here there is not anticipated work with blocks of variable coefficients, based on stepping selectors (see Ch. XI).

5. Observation and Recording of Solution.

Every simulator usually is supplied by means for observation and recording of the process of solution. These means are distinguished depending upon the speed of processes, which are to be recorded or examined. With slow processes with frequency, not exceeding 1-2 cycles, we can use self-recording instruments with drive of the stylus mechanism from servo systems (instruments of the type of recording self-balanced potentiometers, for example, EPP-9). At frequencies up to 100 cps for recording of the solution there usually are used loop oscillographs).* At higher frequencies they use oscillographs with electron-beam tubes.

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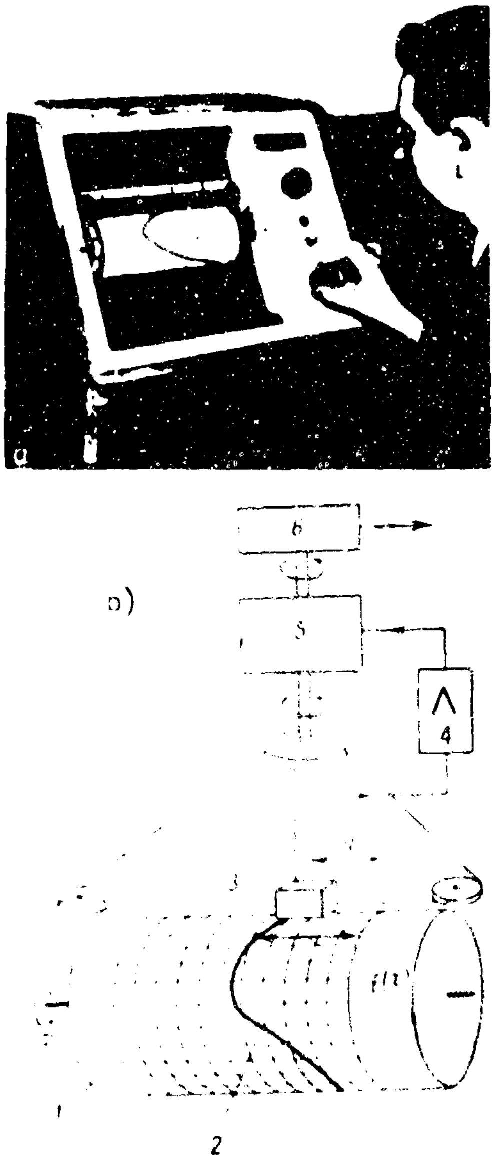

Fig. 168. General view and diagram of recording device of the Goodyear firm (United States). 1--drum; 2--conducting layer, 3--current-collector, 4--amplifier; 5--drive motor, 6--potentiometric pickup.

Spark recorders of type MIZ-2 (developed in industry), based on the principle of dynamic compensation, work in the same range as loop oscillographs, not requiring here comparatively expensive light sensitive material and time for its processing. However accuracy of recording here is 2-3 times lower. Heterogeneity of paper leads to puncture occurring with a large spread of points.

In Fig. 168 are brought the general view and diagram circuit of recording device of Goodyear firm (Goodyear Co, USA; see P. R. Vance and D. L. Haas [1]) with drive of the stylus and drum from servo systems. Analogous installations are also produced by the Reeves firm (G. Korn and T. Korn [1]). In case of rotation of drum with constant speed this device works as an ordinary recording instrument. Upon feeding the input of the servo system of the drum drive with another dependent variable there can be obtained recording of dependence between two coordinates in the process of solution; in particular, there can be obtained recording of trajectories of flying objects or phase portraits of studied dynamic systems. Accuracy of recording here is 0.2% at a maximum frequency of reproduced processes of 2 c. The device can also reproduce in the form of voltage an earlier registered dependence between two coordinates and, consequently, work as a functional generator.

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*It is necessary to indicate that with the help of loop oscillographs in principle it is also possible to record processes of higher frequency. However here the power consumed by loop and required speed of movement of paper sharply increase.

They also apply two-coordinate recording tables with drive from two servo systems. Thus, for example, recording device put out by the firm "Dobbie Mc. Innes Limited" (England; see S. A. Waso [1]) has a table 456 x 764 mm and ensures accuracy of 0.25% during recording of very slow processes. Error during reproduction of sinusoidal oscillations with frequency of 0.2 c and amplitude ±228 mm already constitutes 0.5%. With increase of frequency it is necessary to lower accordingly permissible amplitude of oscillations.

During operation of simulators it is expedient along with loop to use also a cathode-ray oscillograph. The latter serves here mainly for visual observation of the process and singling out from the whole set of received solutions those, which will be recorded. Here there can be used standard cathode-ray oscillographs of type EO-4 or EO-7, for which instead of tube 13Л037 there is installed a tube of type 13Л036 with long afterglow. Voltage of scanning of the beam is fed to the oscillograph from integrating amplifier of the analog and by this synchronizes beginning of the process with beginning of scanning. The time of scanning changes with change of the time constant of the integrator.

There are also constructed specialized cathode-ray indicators of type I-4, I-5 and ERU-1. The first two are equipped with tubes with long afterglow of type [illegible] and 13Л036 accordingly. The third type has an electron-beam tube with dark recording.

In Fig. 169 is brought the diagram of an indicator, made with small differences from the diagram of indicator I-4 Scanning of beam here is internal and is

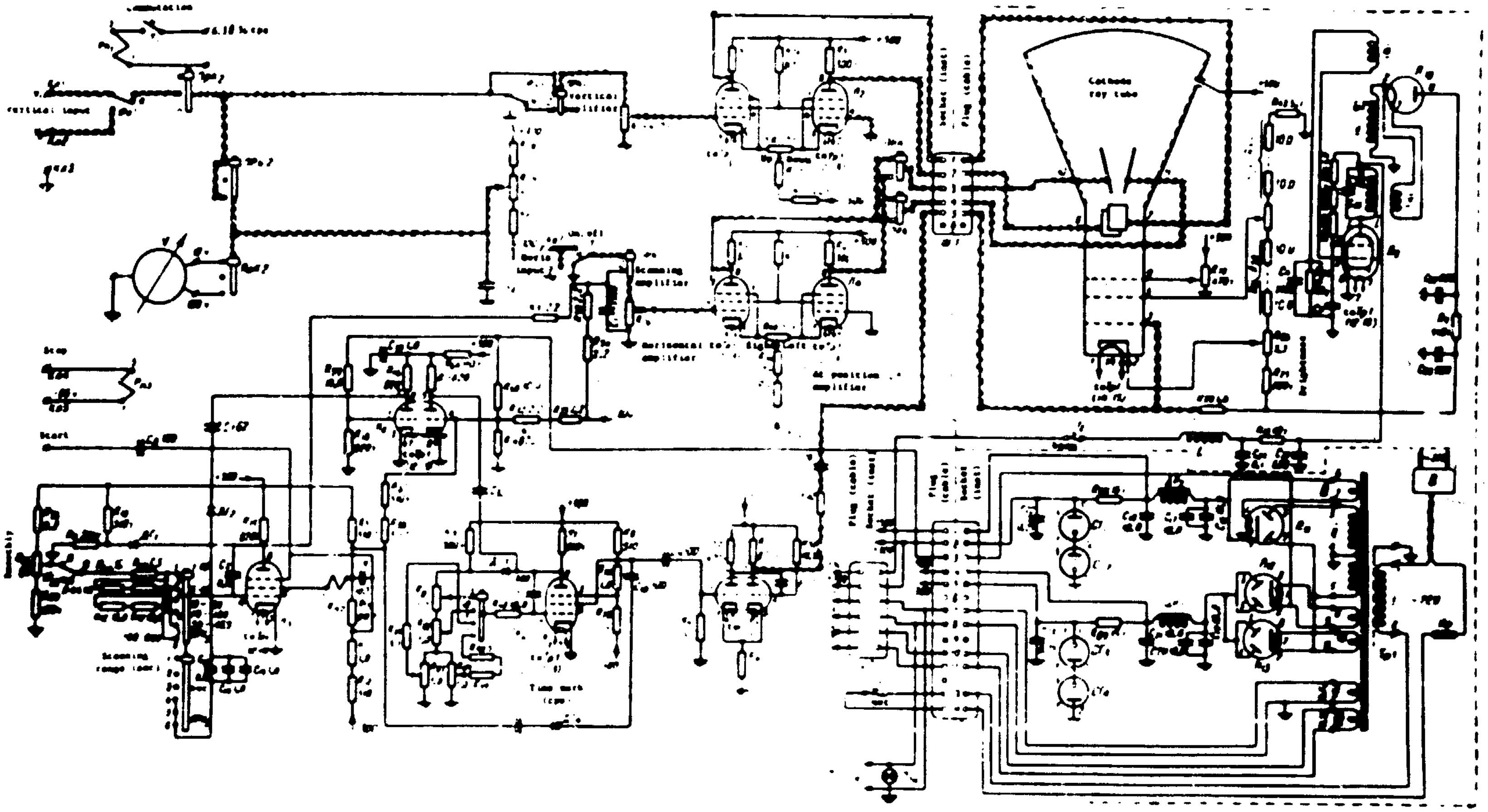

Fig. 169. Diagram of indicator I-4.

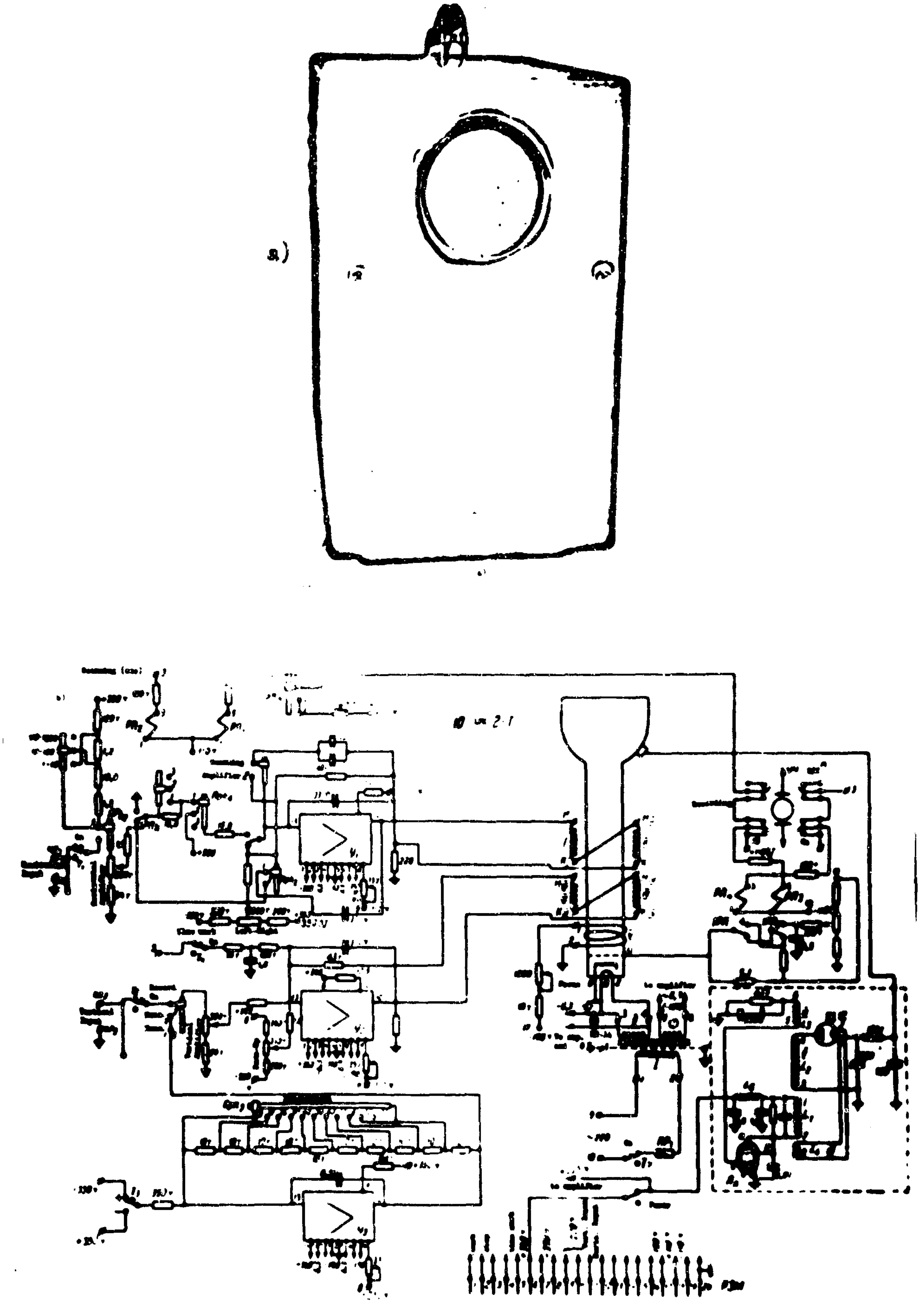

Fig. 170. General view and fundamental circuit of ERU-1.

carried out with the help of a phantastron oscillator circuit. Synchronization is carried out from a 50 cps network. There is anticipated possibility of setting duration of scanning of beam within 1-10 sec, 5-40 sec, 10-80 sec, 10-100 sec, 50-400 sec and 100-800 sec. The time mark is given also by oscillator circuit of phantastron type every 1, 5, 10 or 25 sec. The time mark acts on cutoff of the beam. In indicator is provided also relay Рл_1, which commutates plates with two inputs for obtaining on screen simultaneously the image of change of two coordinates.

Starting and stopping of scanning is carried out from the panel of the simulator. During starting of the analog to the phantastron there moves a starting pulse through capacitor c. When stopping the process, scanning is stopped by breaking of contact 1Рл_1 in grid circuit of tube Л5.

In cathode-ray recording device ERU-1 there is used a cathode-ray tube of type 10ЛРН2-T with recording by a dark line with magnetic deflection and focusing of beam.

Screen of this tube is covered by a layer of crystals of potassium chloride. When the electronic beam strikes the screen on it there will be formed dark violet-blue lines, caused by absorption of light by those crystals of potassium chloride, which experienced the electron bombardment of the beam.

Contrast of recording decreases in time and depends on illumination and temperature of screen. With ordinary temperature and illumination of screen the recording can be kept for several days.

Discoloration of screen--erasing of the recording is carried out by heating the screen by an external electric furnace. Therefore to the device is provided two sets of electron-beam tubes. While recording is conducted on one tube, the other is prepared for work. General view and fundamental circuit of the instrument is shown in Fig. 170. Coils, deflecting the beam, are fed through operational d-c amplifiers with negative feedback for coil current. This ensures linearity of dependence of current of coils on input voltage with accuracy up to 0.1%.

Sensitivity of circuit for both inputs is not less than 0.8 mm/v. For realization of scanning in time amplifier У-1 can be used as a generator of scanning, for which it is converted into integrator made. In the circuit by relays РП$_1$, РП$_2$, РП$_3$, and РП$_4$ there is ensured simultaneity of starting of analog and beginning of scanning of beam, ceasing of scanning upon stop of process of solution and return to initial position during switching off of simulator.

Table 6

No. in order	Type of indicator / Main characteristics	I-4	I-5	ERU-1
1	Diameter of tube screen	180 X 220	130 mm	100 mm
2	Type of tube	31LO33	13LO36	10LPK2-T
3	Sensitivity on both axes	8 mm/v	7 mm/v	0,8 mm/v
	at maximum amplification	8 mm/v	7 mm/v	0.8 mm/v
4	Duration of single scanning	from 10 to 400 sec	from 10 to 250 sec	from 1 to 1000 sec in three steps from 1 to 10, from 10 to 100 and from 100 to 1000 sec
	with repetition	from 1 to 80 sec	from 1 to 25 sec	
5	Frequency of internal time marks	1, 5, 10 and 25 c	0,1; 1; 10 c	Pulses of time marks move from without
6	Error of measurement	2%	—	
7	Feeding	from net 220 v	from net 220 v	From stabilizer rectifier ESV-1M

Basic data of the above-mentioned cathode-ray indicators are given in Table VI.

SECTION IV

APPLICATION OF ELECTRONIC MODELS FOR INVESTIGATION OF AUTOMATIC CONTROL SYSTEMS

CHAPTER X

METHOD OF SETTING AND SOLVING PROBLEMS

For successful use of simulators of great importance is correct fulfillment of a number of operations, connected with preparation of initial system of differential equations for setting on the installation. These preparatory operations include: composition of functional diagram of connection of computing elements in accordance with given system of differential equations, calculation of transmissions factors of separate computing elements with respect to coefficients of initial equations, selection of scales of representation of dependent variables and time, determination of initial conditions and disturbances in those physical quantities, which in the simulator are represented by initial variables of the problem.

1. Composition of Functional Diagram of Connection of Separate Computing Elements for Solution of Given Differential Equations

Differential equations, subject to solution by a simulator, can be given in the form of one equation of high order, in the form of a system of differential equations of different orders, and, finally, in the form of a system of differential equations of the first order. In principle set-up can be realized by increasing the order of the derivative or lowering the order of the derivative. In the first case the equation is solved for the desired function and separate computing elements are connected in such a way as to carry out sequential differentiation with subsequent summation of separate derivatives. In the second case equations are solved for the derivative of highest rank of the desired function. If one were to assume that the value of the derivative of highest rank is known, then to obtain the desired function it is necessary to execute consecutively many operations of integration as the order of the highest ranking derivative, and then sum all components, constituting the highest ranking derivative. One group of these components is the desired function and its lower rank derivatives are obtained by imposition of feedback from the output of the integrators to input of the adder; while the other is obtained by supplying from without (right side of equations, other dependent variables in case of solution of a system differential equations).

Thus, set-up by lowering the order of derivative requires that the basis of the simulator be integrating computing elements. Here there is attained essential decrease of influence of interferences, created, for example, by the steppea nature of potentiometers at input of separate units, the presence of higher harmonics in voltage of power supplies, and so forth. Therefore, as a rule, set-up on simulators is constructed by lowering the order of the derivative. Let us consider several practical examples. As the first example let us consider composition of functional diagram for a linear differential equation of the sixth order with constant coefficients

$$a_0\frac{d^6x}{dt^6}+a_1\frac{d^5x}{dt^5}+a_2\frac{d^4x}{dt^4}+a_3\frac{d^3x}{dt^3}+a_4\frac{d^2x}{dt^2}+a_5\frac{dx}{dt}+a_6x=y(t). \qquad (10.1)$$

Solving equation (10.1) for the highest ranking derivative, we will receive

$$\frac{d^6x}{dt^6}=-b_1\frac{d^5x}{dt^5}-b_2\frac{d^4x}{dt^4}-b_3\frac{d^3x}{dt^3}-b_4\frac{d^2x}{dt^2}-b_5\frac{dx}{dt}-b_6x+b_0y(t). \qquad (10.2)$$

where

$$b_1=\frac{a_1}{a_0},\quad b_2=\frac{a_2}{a_0},\quad b_3=\frac{a_3}{a_0},\quad b_4=\frac{a_4}{a_0},$$
$$b_5=\frac{a_5}{a_0},\quad b_6=\frac{a_6}{a_0},\quad b_0=\frac{1}{a_0}.$$

The functional diagram is shown in Fig. 171. During composition of functional diagram there was taken into account the property of computing elements, built on d-c amplifiers with negative feedback, to change the sign of the input signal.

It is possible to somewhat simplify the functional diagram by combination in the first unit of functions of summation and integration and unification in one auxiliary adder of the function of change of sign of input signals, which was executed in the above-mentioned circuit by three computing elements (units No. 8, 9 and 10). In Fig. 172 is brought a diagram of set-up taking into account these remarks.

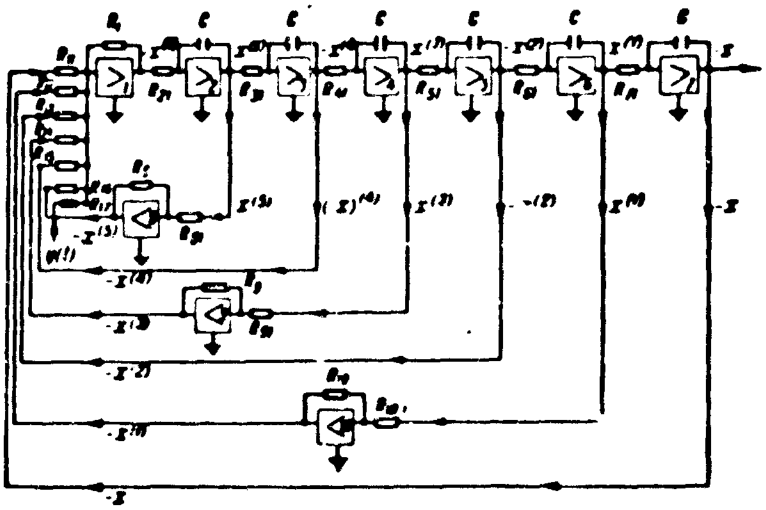

Fig. 171. Functional diagram of set-up of computing elements for solution of an inhomogeneous linear differential equation of the 6th order with constant coefficients.

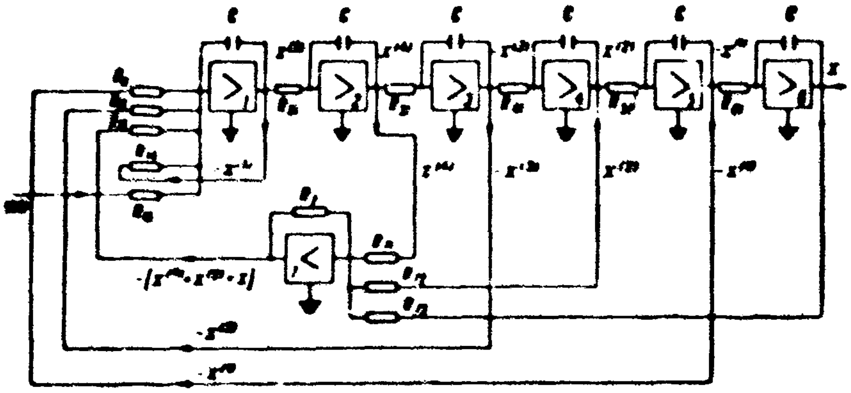

Fig. 172. Another variant of functional diagram.

As a second example let us consider composition of functional diagram for system of linearized differential equations, describing processes of course stabilization of an aircraft by automatic pilot. Not considering banking, with constant speed of aircraft v = const and an automatic pilot with proportional feedback and response to the first and second derivatives we will obtain

$$\left.\begin{array}{l}
\text{equation of moments with respect to } y \text{ axis}\\
T_1\ddot{\psi}+\dot{\psi}=-k_1\delta-k_2\beta+f_1(t).\\
\text{equation of projections of all forces on } z \text{ axis}\\
T_2\dot{\beta}+\beta=T_2\dot{\psi}+f_2(t).\\
\text{equation of rudder drive}\\
T_4\ddot{\delta}+\dot{\delta}=k_3 I.\\
\text{equation of amplifier and sensors}\\
T_3\dot{I}+I=k_{\psi}\psi+k_{\dot\psi}\dot{\psi}+k_{\ddot\psi}\ddot{\psi}-k_4\delta.
\end{array}\right\}\qquad(10.3)$$

In these equations there are designated:

$$\left.\begin{array}{r}
T_1=\dfrac{J}{M_y^{\dot\psi}},\\
T_2=\dfrac{mv}{Z^{\beta}}
\end{array}\right\}\text{—time constants of aircraft;}$$

$$\left.\begin{array}{r}
k_1=\dfrac{M_y^{\delta}}{M_y^{\dot\psi}},\\
k_2=\dfrac{M_y^{\beta}}{M_y^{\dot\psi}}
\end{array}\right\}\text{—amplification factors of aircraft;}$$

$$\left.\begin{array}{r}
f_1(t)=\dfrac{M_b(t)-M_b^{0}}{M_y^{\dot\psi}},\\
f_2(t)=\dfrac{Z_b(t)-Z_b^{0}}{Z^{\beta}}
\end{array}\right\}\text{—increments of disturbing moment and forces;}$$

T_4 — time constant of acceleration of rudder actuator,

T_3 — time constant of amplifier,

k_3 — static transmission factor of rudder drive,

k_4 — coefficient, determining rigidity of feedback,

k_{ψ}, $k_{\dot\psi}$, $k_{\ddot\psi}$ — coefficients, determining influences with respect to deflection of its first and second derivatives,

ψ — course angle increment,

β — increment of slip angle,

δ — increment of angle of rudder,

I — increment of total control signal.

Composition of functional diagram for system of differential equations assumes composition of a separate functional diagram for each equation, and then interconnection of these diagrams.

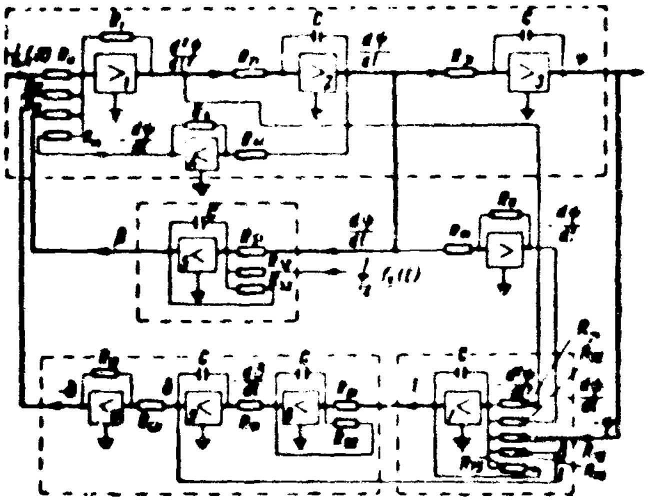

Fig. 173. Functional diagram of set-up of computing elements for investigation of processes of course stabilization of aircraft by an automatic pilot.

We construct the functional diagram of these equations, as before, preliminarily solving each equation of the system for the highest ranking derivative:

$$\frac{d^2\psi}{dt^2} = -\frac{1}{T_1}\frac{d\psi}{dt} - \frac{k_1}{T_1}\delta - \frac{k_2}{T_1}\beta + \frac{f_1(t)}{T_1}. \qquad (10.4)$$

$$\frac{d\beta}{dt} = -\frac{1}{T_2}\beta + \frac{d\psi}{dt} + \frac{f_2(t)}{T_2}. \qquad (10.5)$$

$$\frac{d^2\delta}{dt^2} = -\frac{1}{T_4}\frac{d\delta}{dt} + \frac{k_3}{T_4}l. \qquad (10.6)$$

$$\frac{dl}{dt} = -\frac{1}{T_5}l + \frac{k_4}{T_5}\psi + \frac{k_5}{T_5}\frac{d\psi}{dt} + \frac{k_6}{T_5}\frac{d^2\psi}{dt^2} - \frac{k_7}{T_5}\delta. \qquad (10.7)$$

The overall functional diagram is shown in Fig. 173. Dotted lines circle circuits, corresponding to separate equations of investigated system.

When there is given a system of first-order differential equations of the form

$$\frac{dx_i}{dt} = -\sum_{j=1}^{n} a_{ij}x_j + b_i f(t). \qquad (10.8)$$

for each equation there can be composed an identical functional diagram (Fig. 174).

This allows us once and for all to connect computing elements in simulator among themselves, and to set-up by setting coefficients a_{ij} and b_i, as this is shown

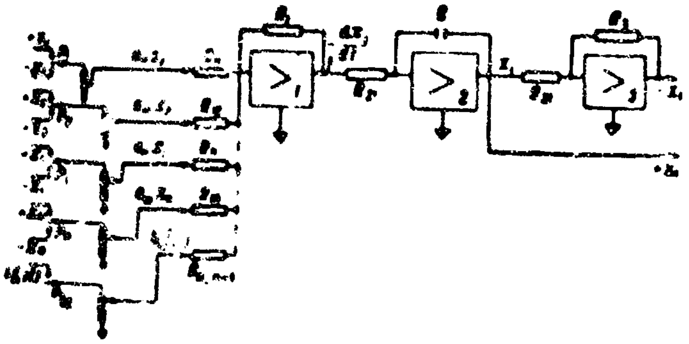

Fig. 174. Functional diagram of set-up of computing elements for solution of linear differential equation of first order in matrix models.

in Fig. 175 for solution of a linear differential equation of the third order

$$a_0 \frac{d^3x_1}{dt^3} + a_1 \frac{d^2x_1}{dt^2} + a_2 \frac{dx_1}{dt} + a_3x_1 = f_3(t). \qquad (10.9)$$

By preliminarily substitution $\frac{dx_1}{dt} = x_2$ and $\frac{dx_2}{dt} = x_3$ equation (10.9) is reduced to a system of three differential equations of the first order:

$$\begin{aligned}
\frac{dx_1}{dt} &= 0 \cdot x_1 + x_2 + 0 \cdot x_3 + 0 \cdot f_1(t).\\
\frac{dx_2}{dt} &= 0 \cdot x_1 + 0 \cdot x_2 + x_3 + 0 \cdot f_2(t).\\
\frac{dx_3}{dt} &= -\frac{a_3}{a_0} x_1 - \frac{a_2}{a_0} x_2 - \frac{a_1}{a_0} x_3 + \frac{1}{a_0} f_3(t).
\end{aligned}$$

Here:

$$\begin{array}{llll}
a_{11} = 0, & a_{12} = 1, & a_{13} = 0, & b_1 = 0,\\
a_{21} = 0, & a_{22} = 0, & a_{23} = 1, & b_2 = 0,\\
a_{31} = -\frac{a_3}{a_0}, & a_{32} = -\frac{a_2}{a_0}, & a_{33} = -\frac{a_1}{a_0}, & b_3 = \frac{1}{a_0}.
\end{array}$$

For set-up there is not required composition of a functional diagram; it is sufficient only to determine values of coefficients a_{ij} and b_i and set them by cursors of corresponding dividers.

The diagram of Fig. 175 is the totality of the three diagrams of Fig. 174, but with this distinction, that in each diagram, solving a first order equation, besides an adder, integrator and sign-change unit, there is introduced a unit for setting the scale of variable x_i (units No. 2, 6 and 10 on the diagram of Fig. 175).

Such a principle of set-up is called matrix. Simulators, for which computing

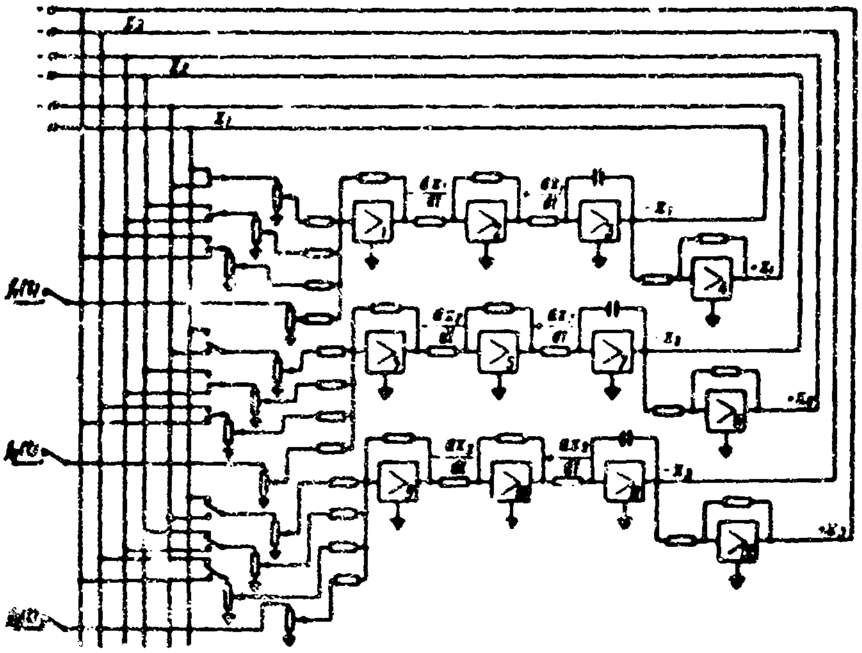

Fig. 175. Example of commutation of blocks in matrix models.

elements are united beforehand in circuits for solution of first order differential equations, also are called matrix.

Use of matrix principle of set-up ensures an automatically correct sequence of coupling separate computing elements, and also indicates the way of transforming equations to a form, with which natural parameters of computing elements do not disturb stability of work of the device as a whole (I. S. Gradshteyn [1]). However such a principle of set-up also requires increases of the quantity of computing elements in the installation. Indeed, in considered example of solution of a third order equation there were required 12 operational amplifiers. In general it is possible to consider that for solution of a third order differential equation in this way there will be required, at least, 4n operational amplifiers. Often it may be that a significant part of these computing elements either in general, will not be used, or may be absent during solution of the formulated problem with another method of connection of computing elements. Increase of the number of computing elements leads to the necessity of increasing the number of power units, and consequently, dimensions and cost of the installation. Besides this, the matrix method of set-up hampers investigation on models of systems of automatic control by equations of their dynamic sections which significantly lowers the graphicness of solution of problems of automatic control and hampers research on syntheses of their structures.

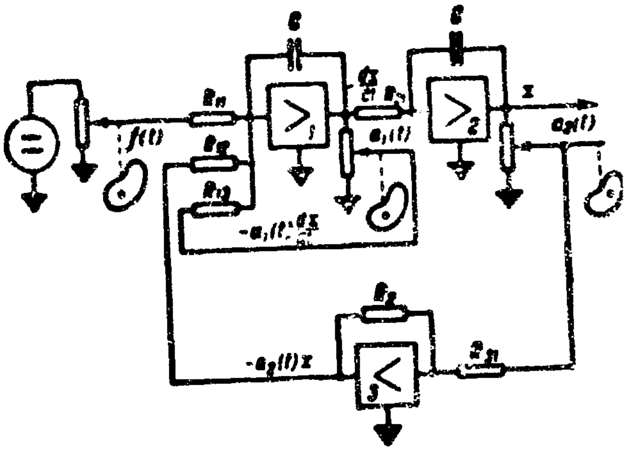

Fig. 176. Functional diagram of set-up of computing elements for solution of a linear differential equation with variable coefficients.

During composition of functional diagrams of connection of computing elements for solution of differential equations with variable coefficients one should provide possibility of connecting to the output corresponding units voltage dividers, controlled from special bunchers (see Ch. XI). In Fig. 176 for illustration there is brought a functional diagram for solution of equations

$$\frac{d^2x}{dt^2}+a_1(t)\frac{dx}{dt}+a_2(t)x=f(t). \qquad (10.10)$$

Equations with coefficients variable in time can also be set-up on installation by nonlinear computing elements, multipliers and functional generators.

In Fig. 177 is brought a functional diagram of set-up of a Mathieu equation

$$\frac{d^2y}{dx^2}+(a-2q\cos 2x)y=0.$$

Since the independent variable in electronic models is time, then it is useful to introduce in Mathieu's equation replacement of variables by relationship $x=\frac{\omega}{2}t$; $\omega=\frac{2\pi}{T}$ designates here angular frequency of change of parameter q.

After replacement of variables we will receive

$$\frac{d^2y}{dt^2}+\frac{\omega^2}{4}ay-\frac{\omega^2}{2}qy\cos\omega t=0.$$

Expression $z=\frac{\omega^2}{2}q\cos\omega t$ can be obtained in the form of solution of the corresponding differential equation

$$\frac{d^2z}{dt^2}+\omega^2z=0 \text{ when } \begin{cases} z(0)=\frac{\omega^2}{2}q, \\ \dot{z}(0)=0. \end{cases}$$

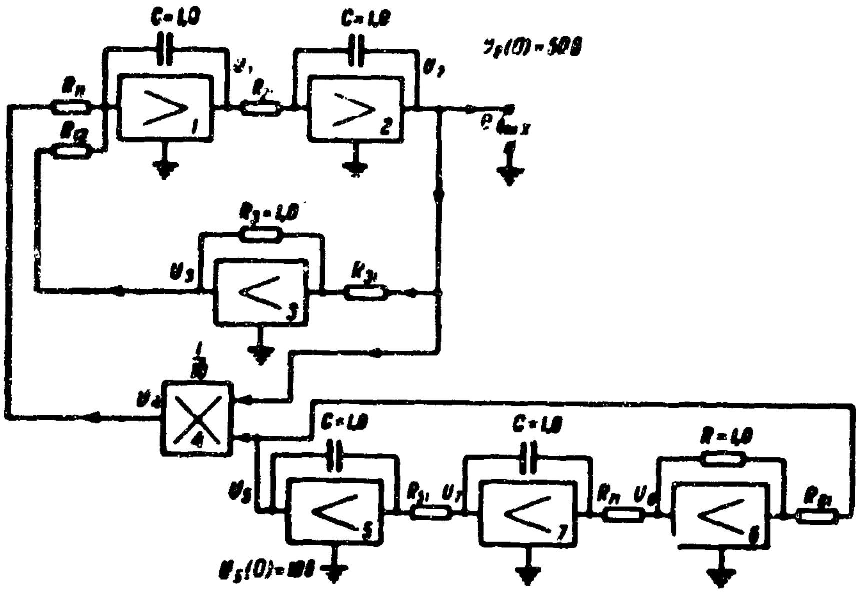

Fig. 177. Functional diagram of set-up of computing elements for solution of a Mathieu equation.

Therefore for composition of the functional diagram of Fig. 173 we use the system of equations:

$$\left.\begin{aligned} \frac{d^2y}{dt^2} + \frac{\omega^2}{4} ay &= zy, \\ \frac{d^2z}{dt^2} + \omega^2 z &= 0. \end{aligned}\right| \qquad (10.11)$$

The presented method of presentation of the right side of the set-up equation as the solution of a corresponding differential equation, introduced first by Bush for a mechanical integrator, turned out to be very fruitful also for electronic models.

Composition of functional set-up diagrams of nonlinear differential equations does not present any essential peculiarities.

A number of cases with typical nonlinear characteristics are examined in Chapter XIII.

2. Determination of Transmissions Factors of Separate Computing Elements by Coefficients of Initial Equations. Selection of Scales of Representation of Dependent Variables and Time

During determination of transmission factors of separate computing elements we will start from the position that processes of simulators must be described by differential equations the same in form as for the initial problem, and initial variables and their physical analogs and models can differ only by scale factors. Therefore in principle there can be two methods of determination of transmission factors of computing elements in a circuit. The first consists in replacing in initial equations variables by their physical analogs by the corresponding transformation. On the basis of the resulting equations we constitute the functional set-set-up diagram. By this diagram we write equations, connecting variables of the model, and these equations are compared with the transformed initial equations. From comparison of equations they determine relationships between coefficients of initial equations and scales of transformation of variables on the one hand, and transmission factors of separate computing elements on the other.

In the other method in equations, composed by functional set-up diagram, we effect by transformation of variables replacement of physical analogs by initial variables and compare resulting equations with the initial. Both systems of differential equations should be identical. From this condition there are obtained relationship between transmission factors and scale factors on the one hand, and coefficients of the initial system on the other. From this ensues complete equivalence of both methods of determination of transmission factors of computing elements in a circuit.

Both methods assume composition of differential equations by functional set-up diagram. For each computing element, participating in the set-up diagram, one should write an equation, connecting output magnitude with the input, renumbering preliminarily in succession all computing elements and their input and output circuits.

Transmission factor of a unit here acquires a number in the index, consisting of two or three figures: the first figure (or first two figures) indicates the number of the unit and the second, the number of the input circuit.

Thus, for example, in the general case for the i-th operational amplifier, working in adder regime with n components, it is possible to write

$$U_i = -\sum_{j=1}^{n} a_k K_{ij} U_k \qquad (k = 0,\ 1,\ 2,\ \ldots,\ n). \tag{10.12}$$

where a_k --is the coefficient, indicating what part of the output voltage U_k of the k-th unit is fed to the j-th input of the given unit from the potentiometer, utilized for setting the coefficient of the smaller unit.

For a unit, working in integrator-adder regime,

$$U_i = -\frac{1}{p}\sum_{j=1}^{n} a_k K_{ij} U_k \qquad (k = 0,\ 1,\ 2,\ \ldots,\ n). \tag{10.13}$$

When using operational amplifier as an adder-differentiator

$$U_i = -p\sum_{j=1}^{n} a_k \cdot K_{ij} U_k \qquad (k = 0,\ 1,\ 2,\ \ldots,\ n). \tag{10.14}$$

Equations of computing units, working in scale amplifier regime, and also utilized for change of sign or integration, can be obtained from the given equations as a particular case, when the number of components n = 1.

For nonlinear computing elements there is fixed only an ordinal number in the diagram, and this number is assigned to the corresponding transmission factor. Thus, for a multiplier-divider we have:

$$\left.\begin{aligned} U_i &= \beta_i U_m U_n, \\ U_i &= \gamma_i \frac{U_m}{U_n}. \end{aligned}\right\} \tag{10.15}$$

where β_i is the scale factor of the multiplier, γ_i —the scale factor of the divider, i—the ordinal number of the unit in the circuit.

Usually so that output voltage does not exceed the scale of 100 v, β_i is taken equal to 0.01 with permissible change of U_m and U_n within ± 100 v. Coefficient γ_i usually is taken equal to 10 in the interval of change of dividend U_m ± 100 v and divisor U_n from 10 to 100 v.

For functional generator of one argument the equation, connecting output magnitude with the input, will be

$$U_i = \nu_i f(\zeta_i U_k). \tag{10.16}$$

In particular, for units, reproducing given functions, in analog ЭМУ-5 there is taken:

$$\left.\begin{aligned} U_i &= 100 \sin \frac{\pi}{100} U_k. \\ U_i &= 100 \cos \frac{\pi}{100} U_k. \\ U_i &= 0.01\, U_k^2. \\ U_i &= 10 \sqrt{U_k}. \\ U_i &= 0.0001\, U_k^3. \\ U_i &= 21.54 \sqrt[3]{U_k}. \end{aligned}\right\} \tag{10.17}$$

In general when using general-purpose functional generators (for example, built on electron-beam tubes in Academy of Sciences of USSR) magnitude ν_i can change from 0 to 15, and ζ_i from 0.8 to 5.

If the problem is solved with rigidly fixed coefficients, greater than one, then all a_k, which can change within limits $0 \leq a_k \leq 1$, should be set equal to one. During calculation of transmission ratios one should also consider static transmission factors of generators, connected to input and output of simulator in case of simulation with elements of the control loop.

Values of resistances at the input of operational amplifier required for realization of given transmission factor are calculated from expressions:

for integrator $K_{ij} = \frac{1}{R_{ij} C_j}.$

for differentiator $K_{ij} = R_j C_{ij}.$

for scale amplifier $K_{ij} = \frac{R_j}{R_{ij}}.$

Capacitances C_{ij} and C_j here are expressed in microfarads, but resistance R in megohms.

For example we will constitute equations, connecting input and output magnitudes for separate computing units of the functional diagram considered in Fig. 172. Using relationships (10.12) and (10.13) we will receive:

$$\left.\begin{aligned}
U_1 &= -\frac{1}{p}(K_{11}U_5 + K_{12}U_3 + K_{13}U_7 + K_{14}U_1 + K_{15}U_0).\\
U_2 &= -\frac{1}{p}(K_{21}U_1).\\
U_3 &= -\frac{1}{p}(K_{31}U_2).\\
U_4 &= -\frac{1}{p}(K_{41}U_3).\\
U_5 &= -\frac{1}{p}(K_{51}U_4).\\
U_6 &= -\frac{1}{p}(K_{61}U_5).\\
U_7 &= -(K_{71}U_2 + K_{72}U_4 + K_{73}U_6).
\end{aligned}\right\} \qquad (10.18)$$

Here p = $\frac{d}{dt_m}$; t_m is an independent variable of the installation for time.

In the resulting equations voltage U_6 represents the sought for variable x. Solving this system for U_6, we receive

$$\begin{aligned}
U_6[p^6 + K_{14}p^5 + K_{13}K_{21}K_{71}p^4 + K_{12}K_{21}K_{31}p^3 + K_{13}K_{21}K_{31}K_{41}K_{72}p^2 +\\
+ K_{11}K_{21}K_{31}K_{41}K_{51}p + K_{13}K_{21}K_{31}K_{41}K_{51}K_{61}K_{73}] =\\
= K_{61}K_{51}K_{41}K_{31}K_{21}K_{15}U_0.
\end{aligned} \qquad (10.19)$$

Magnitude U_6 can represent the initial variable in a certain scale; analogously the independent variable, time, can differ from time of initial problem in the sense that processes on installation are reproduced in a somewhat delayed or accelerated rate.

Introducing equations of transformation of variables, we receive:

$$\left.\begin{aligned}
x &= M_x U_6.\\
y &= M_y U_0.\\
t &= M_t t_m.
\end{aligned}\right\} \qquad (10.20)$$

where M_x is the scale of representation of magnitude x in the installation in the form of voltage, M_y is the scale factor of representation of magnitude y, M_t is the time scale.

Substituting relationships (10.20) in equation (10.19), we receive equation of modelling circuit, written in transmission factors, scale factors and initial variables:

$$\frac{d^6x}{dt^6}+\frac{K_{16}}{M_t}\frac{d^5x}{dt^5}+\frac{K_{15}K_{21}K_{71}}{M_t^2}\frac{d^4x}{dt^4}+\frac{K_{15}K_{21}K_{31}}{M_t^3}\frac{d^3x}{dt^3}+$$
$$+\frac{K_{15}K_{21}K_{31}K_{41}K_{72}}{M_t^4}\frac{d^2x}{dt^2}+\frac{K_{11}K_{21}K_{31}K_{41}K_{51}}{M_t^5}\frac{dx}{dt}+ \qquad (10.21)$$
$$+\frac{K_{15}K_{21}K_{31}K_{41}K_{51}K_{61}K_{73}}{M_t^6}x=\frac{M_x}{M_yM_t^6}K_{15}K_{21}K_{31}K_{41}K_{51}K_{61}\cdot y(t).$$

It is obvious that coefficients of this equation should be equal to coefficients of initial equation (10.2).

Equating coefficients for corresponding derivatives and right sides in equations (10.2) and (10.21), we receive:

$$\left.\begin{array}{l}\frac{K_{16}}{M_t}=b_1,\quad \frac{K_{15}K_{21}K_{71}}{M_t^2}=b_2,\quad \frac{K_{15}K_{21}K_{31}}{M_t^3}=b_3,\\ \frac{K_{15}K_{21}K_{31}K_{41}K_{72}}{M_t^4}=b_4,\quad \frac{K_{11}K_{21}K_{31}K_{41}K_{51}}{M_t^5}=b_5,\\ \frac{K_{15}K_{21}K_{31}K_{41}K_{51}K_{61}K_{73}}{M_t^6}=b_6,\\ \frac{K_{15}K_{21}K_{31}K_{41}K_{51}K_{61}}{M_yM_t^6}M_x=b_0.\end{array}\right\} \qquad (10.22)$$

From these relationships it follows that during selection of transmission and scale factors there is some free play since the number of equations is less than the number of unknowns. Therefore for determination of magnitudes of separate transmission factors additional considerations are brought in, connected with peculiarities of work and properties of utilized computing elements. Among these considerations is the desire to limit error from zero drift and the finite value of the amplification factor, and also not to allow saturation of separate computing elements in process of solution of problems due to boundedness of linear range (± 100 v).

Experience shows that maximum transmission factor, established on computing unit (made with triode circuit for drift compensation in the first cascade) working as an integrator, must no exceed 5-10 and in scale block regime 20. For a computing element, made with automatic stabilization of zero level, the permissible transmission factor increases accordingly to 20 in the first case and to 100 in the second. Here the adder can be considered a scale unit with equivalent resistance at input, equal to resultant resistance of parallels connected input resistances.

If for given values of coefficients of initial equations transmission ratios required of operational amplifiers exceed the indicated boundaries, transmission ratios must be decreased by reconsideration of the functional diagram (changes of scale factors) or introduction of additional units.

During selection of scale factors one should see that solution occurs at the highest permissilbe level of voltages in the installation, since this will ensure best use of its possibilities in the sense of obtaining low error of the solution.

During calculation of transmissions factors for the case of factors of variable in time there are predetermined their maximum values and by these values, setting all $a_k = 1$, we select values of K_{ij}. During solution of a problem required values of transmission factors of separate units are obtained automatically by decrease of these coefficients by introduction of voltage dividers $a_k(t)$ in needed places in the circuit. Often the dividers are introduced also and when they wish to establish a transmission factor less than one or when it is necessary to pass investigation in a wide range of change of certain coefficients (parameters) of the initial system.

Expressions (10.22) also permit one to formulate rules of composition of relationships between transmission factors of computing units, scale factors and coefficients of initial linear differential equations, by which these relationships can be written only on the basis of a functional set-up diagram.

Indeed, each coefficient of the initial equation in a dependent variable or in its derivatives is expressed by the product of transmissions factors of separate computing elements which form a closed circuit, at whose output there is obtained the considered dependent variable. If investigation is conducted not in natural time scale ($M_t \neq 1$), then this product of transmission factors is divided by scale factor M_t in a degree, equal to the number of considered coefficient b_i. Coefficient in an independent variable is equal to the product of transmission factors of computing units, connected in series between the place of application of the independent variable and output of the sought for dependent variable, multiplied by the ratio of scale of representation of the dependent variable to the product of scales of the independent variable and time in a degree equal to the order of the highest ranking derivative.

It is known that accuracy of solution of differential equations on electronic analog computers is higher, the faster the solution converges. This means that error during investigation of stable systems of automatic control will be significantly smaller, than during investigation of unstable ones.

In the latter case error grows in time and can reach an impermissibly great magnitude. At the same time there exists a number of linear systems of automatic control with parameters variable in time, motion of which on separate time intervals for a number of reasons is characterized by instability, leading to build-up of deflections. The problem is to explain the magnitude of deflections and nature of motion of the system on a given time interval in order, by corresponding selection of parameters, to limit maximum deflection at the end of this time interval.

To solve such problems on simulators without elements of the control loop it is expedient to introduce λ-transformation for every variable of the initial problem:

$$x_i = e^{\lambda t} y_i. \qquad (10.23)$$

where λ is a sufficiently large positive number. Introduction of λ-transformation of variables* for solution of such problems on installations of type ELI leads to change of diagonal coefficients of initial equations by quantity λ. The resulting new system of differential equations will have a solution, characterized by diminishing deflections, which it is possible with sufficient accuracy to obtain by an electronic analog computer. For calculation of solutions of $x_i(t)$ from solution of $y_i(t)$ of the stable system we use again transformation equation

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*Offered by M. L. Brodskiy (see L. I. Gutenmakher, N. V. Korol'kov, I. A. Vissonov, L. S. Klabukov, and G. K. Kuz'minok [1]).

(10.23). For acceleration of conversion it is possible to apply nomographs.

When set-up is carried out an a matrix analog, selection of factor λ can be accomplished experimentally on the actual installation, for which to diagonal coefficients $a_{ii}(i = 1, 2, \ldots, n)$ there are added numbers, proportional to $b_{ii}(i = 1, 2, \ldots, n)$ until the process of solution on the installation converges. Indeed, if the initial system of equations is reduced to the form:

$$\begin{aligned} b_1\frac{dx_1}{dt} &= -(a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n) + f_1(t), \\ b_2\frac{dx_2}{dt} &= -(a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n) + f_2(t), \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ b_n\frac{dx_n}{dt} &= -(a_{n1}x_1 + a_{n2}x_2 + \ldots + a_{nn}x_n) + f_n(t) \end{aligned}$$

with initial conditions $x_1(0), x_2(0), \ldots, x_n(0)$, then after transition to variables $y_1 = e^{-\lambda t}x_1, \ldots, y_n = e^{-\lambda t}x_n$ we will receive

$$\begin{aligned} b_1\frac{dy_1}{dt} &= -[(a_{11} + b_1\lambda)y_1 + a_{12}y_2 + \ldots + a_{1n}y_n] + f_1(t)e^{-\lambda t}, \\ b_2\frac{dy_2}{dt} &= -[a_{21}y_1 + (a_{22} + b_2\lambda)y_2 + \ldots + a_{2n}y_n] + f_2(t)e^{-\lambda t}, \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ b_n\frac{dy_n}{dt} &= -[a_{n1}y_1 + a_{n2}y_2 + \ldots \\ &\quad \ldots + (a_{nn} + b_n\lambda)y_n] + f_n(t)e^{-\lambda t}. \end{aligned}$$

From comparison of transformed and initial systems there ensues the above-indicated rule of change of coefficients of an initial system of equations for finding the required value of λ.

When the initial system of equations is set up on an installation of structural type with respect to the general differential equation of a system with constant coefficients

$$a_n\frac{d^nx}{dt^n} + a_{n-1}\frac{d^{n-1}x}{dt^{n-1}} + \ldots + a_1\frac{dx}{dt} + a_0 = f(t),$$

for determination of the required value of λ it is possible to use known theorems of algebra (A. G. Kurosh [1]) about limits of roots of a polynomial F(x). For example, it is possible to use the calculation

$$M < 1 + \frac{|A|}{|a_n|}. \qquad (10.24)$$

where M is the upper limit of moduli of all real and complex roots of a characteristic equation, $|A|$ is the modulus of the largest coefficient, $|a_n|$ is the modulus of the leading coefficient.

If coefficients of the characteristic equation are positive, it is possible to use a generalization of Cockay's theorem according to which moduli of roots of characteristic equation λ_k are enclosed between numbers m and M, the least and greatest ratio of a subsequent coefficient of the considered characteristic equation to the preceding one. Knowing magnitude λ it is possible to calculate coefficients of a new equation, subject to solution on an installation by Horner formulas:

$$A_k = \frac{D^k(\lambda)}{k!},$$

where

$$D^{(k)}(\lambda) = \left(\frac{d^k[H(p)]}{dp^k}\right)_{p=\lambda}.$$

and H(p) is characteristic polynomial of initial equation. It is necessary to note that initial conditions for transformed equation and the initial remain with this identical.

In the transformed equation only coefficients of the left and the form of the right side change.

3. Determination of Initial Conditions and Disturbing Forces

To determine initial conditions and disturbing forces in those physical quantities, in which they are represented in the computer, one should use transformations (10.20). Thus, for example, if in solving differential equation (10.2) we are given initial conditions:

$$x(0) = C_0, \quad x^{(1)}(0) = C_1, \quad x^{(2)}(0) = C_2, \quad x^{(3)}(0) = C_3,$$
$$x^{(4)}(0) = C_4, \quad x^{(5)}(0) = C_5,$$

and disturbance y(t) = B = const, then on the basis of equation (10.20) we will receive

$$x = M_x U_0, \quad y(t) = B = M_y U_y,$$

whence

$$U_0(0) = \frac{x(0)}{M_x} = \frac{C_0}{M_x},$$

and

$$U_y = \frac{y(t)}{M_y} = \frac{B}{M_y} = \text{const.} \qquad (10.25)$$

In order to find expressions for initial conditions for remaining derivatives, we will use equations (10.18).

On the basis of these equations we obtain:

$$U_5(0) = -\frac{x^{(1)}(0)}{M_x K_{6i}} = -\frac{C_1}{M_x K_{6i}},$$

$$U_4(0) = \frac{x^{(2)}(0)}{M_x K_{5i} K_{6i}} = \frac{C_2}{M_x K_{6i} K_{5i}},$$

$$U_3(0) = -\frac{x^{(3)}(0)}{M_x K_{6i} K_{5i} K_{4i}} = -\frac{C_3}{M_x K_{6i} K_{5i} K_{4i}},$$

$$U_2(0) = \frac{x^{(4)}(0)}{M_x K_{6i} K_{5i} K_{4i} K_{3i}} = \frac{C_4}{M_x K_{6i} K_{5i} K_{4i} K_{3i}},$$

$$U_1(0) = -\frac{x^{(5)}(0)}{M_x K_{6i} K_{5i} K_{4i} K_{3i} K_{2i}} = -\frac{C_5}{M_x K_{6i} K_{5i} K_{4i} K_{3i} K_{2i}}.$$

If disturbance represents a given function of time, it can be reproduced on the installation by solution of a certain auxiliary equation (see page 331) or by introduction of a buncher of coefficients. In the latter case it is useful to present y(t) in the form

$$y(t) = y_{max}\,\alpha(t), \tag{10.26}$$

where y_{max} is the maximum value, taken by function y(t) on considered time interval. Here $U_{y\,max} = \frac{y_{max}}{M_y}$ = const, and change of $U_{y\,max}$ in time is carried out by the buncher, as shown in Fig. 178.

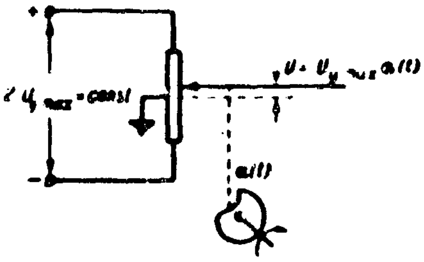

Fig. 178. Method of introduction in analog of given functions of time.

4. Simulating Systems of Automatic Control With Very Large Time Constants

During investigation of these systems of automatic control on analogs without elements of a regulator the problem boils down to selection of proper time scale. By introduction of a time scale they reduce speed of model processes to such a magnitude, there is no error due to leaks and output beyond the passband of separate computing elements.

In case of work with elements of a regulator, naturally, it is necessary to keep the time scale natural, and then in slowly flowing processes of adjustment it is necessary to solve differential equations with minute coefficients. Indeed, let us assume that on the analog there will be investigated a controlled member, described by differential equation

$$(p^2 + 2kp + \omega_0^2)\, x = ky. \qquad (10.27)$$

and let the period of natural oscillations of the object be T = 15 minutes, and their attenuation such that with zero initial conditions and a step-by-step change of y the x coordinate reaches 95% x_{yct} during the time t = 2T.

In these conditions

$$k = -\frac{\omega_0}{4\pi} \ln\frac{1}{20} \approx 1.6 \cdot 10^{-3}, \quad \omega_0^2 \approx 49 \cdot 10^{-6}.$$

Functional set-up diagram of differential equation (10.27) on installation is shown in Fig. 179.

On the basis of presented method coefficients of initial equation are connected with parameters of computing units by relationships:

$$2k = \frac{a_1}{R_{12}C_1}, \quad \omega_0^2 = \frac{a_2\alpha_3\alpha_4}{R_{13}C_1R_{21}C_2}, \quad k = \frac{M_x}{M_y}\,\frac{\alpha_0\alpha_2}{R_{11}C_1R_{21}C_2}.$$

Taking $R_{12}C_1 = R_{13}C_1 = R_{21}C_2 = R_{11}C_1 = 1$ and $K_{31} = 1$, we obtain

$$a_1 = 2k = 0.003, \quad a_2\alpha_3\alpha_4 = 49 \cdot 10^{-6}, \quad \alpha_0\alpha_2 = \frac{kM_y}{M_x};$$

when $a_3 = \alpha_4 = 0.007$, $k = 1$, $\frac{M_y}{M_x} = 1$ we obtain $\alpha_2 = 1$ and $\alpha_0 = 1$.

Thus during modeling of slow processes there is no necessity to have capacitors of very great capacity for integrators. Large time constants it is possible to reproduce by combination of the usual integrators and dividers. Here one should

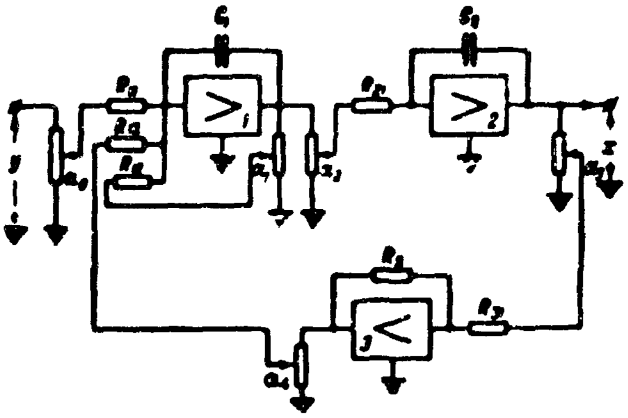

Fig. 179. Functional set-up diagram on simulator of equation of an object with slow processes.

apply operational amplifiers with automatic stabilization of zero level and turn special attention to eliminating possible leaks between their integrating point and output, especially during work as an integrator. A number of practical methods of decreasing these leaks is considered in Chapter IX.

5. Examples of Solution of Problems

Solution of linear differential equation with constant coefficients. Let us assume that it is required to find in full time scale solution of the differential equation

$$\frac{d^6x}{dt^6} + b_1\frac{d^5x}{dt^5} + b_2\frac{d^4x}{dt^4} + b_3\frac{d^3x}{dt^3} + b_4\frac{d^2x}{dt^2} + b_5\frac{dx}{dt} + b_6x = b_0y(t) \tag{10.28}$$

with given initial conditions: $x(0) = 1$, $x^{(1)}(0) = 0$, $x^{(2)}(0) = 0$, $x^{(3)}(0) = 0$, $x^{(4)}(0) = 0$, $x^{(5)}(0) = 0$ and the following numerical values of coefficients:

$$b_1 = 3,\ b_2 = 3.75,\ b_3 = 2.5,\ b_4 = 0{,}937,\ b_5 = 0{,}187,$$
$$b_6 = 0{,}0155,\ b_0 = 0.$$

The functional set-up diagram of the problem is shown in Fig. 180. It differs from the one considered earlier (see Fig. 172) by inclusion of two voltage dividers (z_1, z_2) for reproduction of coefficients less than one. Using the relationship received earlier (10.22) between transmission factors of separate units and coefficients of initial equation, we will receive when $M_t = 1$:

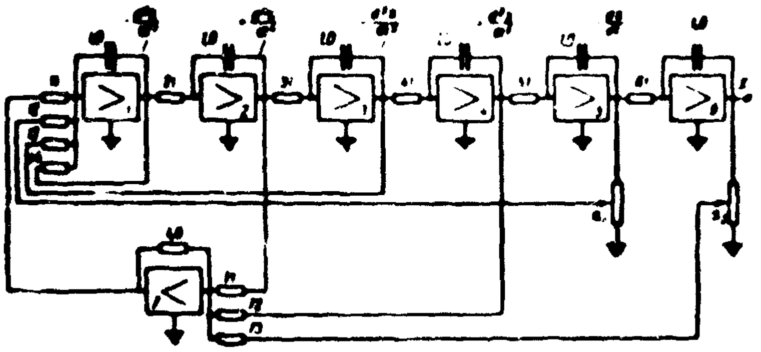

Fig. 180. Functional set-up diagram of differential equation of sixth order with wide range of magnitudes of coefficients.

$$K_{14} = 3,\ K_{11}K_{21}K_{71} = 3.75,\ K_{13}K_{21}K_{31} = 2.5,$$
$$K_{11}K_{21}K_{31}K_{41}K_{72} = 0.937,\ K_{12}K_{21}K_{31}K_{41}K_{51}a_1 = 0.187,$$
$$K_{11}K_{21}K_{31}K_{41}K_{51}K_{61}K_{73}a_2 = 0.0155.$$

Taking $K_{11} = K_{12} = K_{21} = K_{31} = K_{41} = K_{51} = K_{61} = K_{73} = 1$, we will obtain $K_{71} = 3.75$, $K_{13} = 2.5$, $K_{14} = 3.0$, $K_{72} = 0.937$ and

$$a_1 = 0.187,\ a_2 = 0.0155.$$

To guarantee work at the highest possible level of voltages in the installation we select scale M_x, proceeding from the fact that $x = 1$ corresponded to $U = 100$ v. Then

$$M_x = \frac{x}{U} = \frac{1}{100} = 0.01.$$

Voltage of initial conditions is

$$U_0(0) = \frac{x(0)}{M_x} = \frac{1}{0.01} = 100\ \text{v}.$$

On the oscillogram of Fig. 181 there is brought solution of considered equation, received on computer EMU-5. Comparison with calculating data shows that relative error does not exceed 1%.

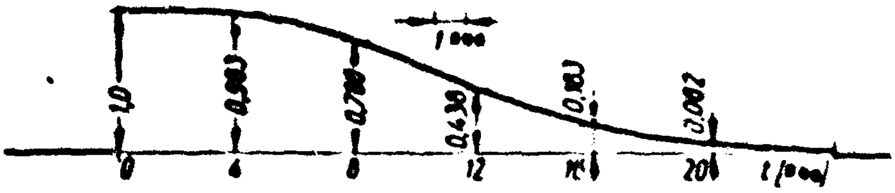

Fig. 181. Result of solution of linear differential equation of sixth order.

Solution of Mathieu equation. It is required to find periodic solution of the equation

$$\frac{d^2y}{dx^2} + (a - 2q\cos 2x)\,y = 0 \qquad (10.29)$$

when $y(0) = 1$, $\dot{y}(0) = 0$ and $q = 2$.

As it is known, periodic solutions for the given q take place not for all values of coefficient a.

If we want to obtain a periodic solution of the form

$$y = A_1 Ce_1(q,\ x), \qquad (10.30)$$

where

$$A_1 = \frac{y(0)}{Ce_1(q,\ x)}.$$

then, using known relationships or tables (see M. Strett [1]), we find that $a = a_1 = 2.38$. Replacing independent variable x by time t, we move to system of equations (10.11), whose set-up on a computer is shown in Fig. 177.

Equations of voltages for separate computing units of the circuit give:

$$\left.\begin{aligned} U_1 &= -\frac{1}{p}(K_{11}U_4 + K_{12}U_3), \\ U_2 &= -\frac{1}{p}K_{21}U_1, \\ U_3 &= -K_{31}U_2, \\ U_4 &= \beta_4 U_2 U_6, \end{aligned}\right\} \qquad (10.31)$$

$$\left.\begin{aligned} U_5 &= -\frac{K_{51}}{p}U_7, \\ U_6 &= -K_{61}U_5, \\ U_7 &= -\frac{K_{71}}{p}U_6 \end{aligned}\right\} \qquad (10.32)$$

Solving systems of equations (10.31) and (10.32) for U_2 and U_5, we will obtain

$$p^2U_2 + (K_{21}K_{12}K_{31} - \beta_4 K_{21}K_{11}U_6)\,U_2 = 0,$$
$$p^2U_5 + K_{51}K_{71}K_{61}U_5 = 0.$$

Introducing equations of transformation of variables $y = M_yU_2$ and $z = M_zU_5$, we will obtain

$$\left.\begin{aligned} p^2y + K_{21}K_{12}K_{31}y &= \frac{1}{M_z}\beta_4 \cdot K_{21}K_{11}zy, \\ p^2z + K_{51}K_{71}K_{61}z &= 0 \end{aligned}\right\} \qquad (10.33)$$

Comparison of coefficients in equations (10.33) and (10.11) gives

$$\frac{\omega^2}{q} a = K_{21}K_{12}K_{31}, \quad \frac{1}{M_z}\beta_4 K_{21}K_{11} = 1, \quad \omega^2 = K_{41}K_{71}K_{61}$$

Initial conditions are determined on the basis of transformation equations of variable relationships

$$U_2(0) = \frac{y(0)}{M_y}, \quad U_5(0) = \frac{z(0)}{M_z}.$$

when q = 2, a = 2.38, β_4 = 0.1, y(0) = 1, z(0) = $\frac{\omega^2}{2}q$ = 1, ω = 1 we will obtain

$$K_{12}K_{31} = 0.595, \quad K_{41}K_{71}K_{61} = 1, \quad \frac{1}{M_z}K_{21}K_{11} = 10.$$

Taking K_{12} = 1, K_{21} = 1, K_{11} = 1, we will obtain K_{31} = 0.595, M_z = 0.1. Knowing M_z, we find

$$U_5(0) = \frac{z(0)}{M_z} = \frac{1}{0.1} = 10 \text{ v}.$$

Scale M_y must be select such that units do not leave the bounds of linearity. Let M_y = 0.02; then

$$U_2(0) = \frac{y(0)}{M_y} = 50 \text{ v}.$$

Change of magnitude $U_2(0)$ influences only the scale of the obtained curve, and not on its shape. In Fig. 182a and b are brought results of solution of this problem on computer EMU-5 for a quarter period and for several periods of solution. Error of solution on the first quarter period does not exceed 2.5%

Solution of problem of oscillation of rotor of synchronous machine with step disturbance. Equation of machine without considering damping and electromagnetic processes in coils can be approximately presented in the form

$$\frac{d^2\delta}{dt^2} + A\sin\delta = F(t), \qquad (10.34)$$

where δ is the angle of displacement of axis of the rotor flux with respect to the axis resultant stator flux, F(t) is acceleration, equilivalent to applied disturbance, A is magnitude, proportional to synchronising moment. It is required to find the nature of change of $\delta(t)$ for various values of F(t), given in the form of a step function with A = 2 and zero initial conditions

$$\delta(0) = 0, \quad \dot{\delta}(0) = 0.$$

Functional set-up diagram in accordance with equation (10.34) is shown in Fig. 183. Equations of voltages for separate computing units will be:

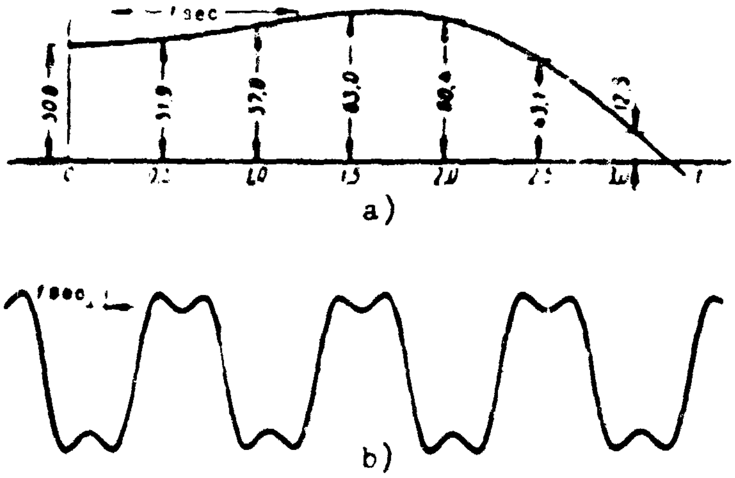

Fig. 182. Results of solution of Mathieu equation.

$$\left.\begin{aligned}&U_1 = -\frac{1}{p}(K_{11}U_0 + K_{12}U_4), \quad U_2 = -\frac{1}{p}K_{21}U_1,\\ &U_3 = -a_3\sin(\omega_3U_2), \quad U_4 = a_4U_3.\end{aligned}\right\} \tag{10.35}$$

Solving these equations for voltage U_2, we will receive

$$p^2U_2 + K_{21}K_{12}a_4a_3\sin(\omega_3U_2) = K_{21}K_{11}U_0. \tag{10.36}$$

Introducing equations of transformation of variables

$$\theta = M_\theta U_2, \quad F(t) = M_pU_0, \quad M_t = 1$$

and changing to initial variables, we will receive

$$\frac{d^2\theta}{dt^2} + K_{21}K_{12}a_4a_3M_\theta\left(\sin\omega_3\frac{\theta}{M_\theta}\right) = \frac{K_{21}K_{11}M_\theta}{M_p}F(t). \tag{10.37}$$

Comparing equations (10.34) and (10.37), we obtain

$$K_{21}K_{12}a_4a_3M_\theta = A, \quad \frac{\omega_3}{M_\theta} = 1, \quad \frac{K_{21}K_{11}M_\theta}{M_p} = 1.$$

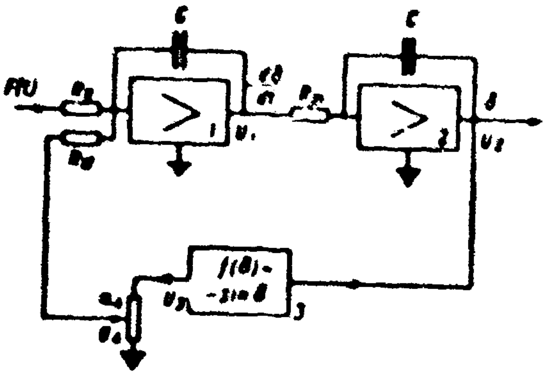

Fig. 183. Functional set-up diagram of computing elements for solution of problem of oscillation of rotor of synchronous machine.

In functional generators of computer EMU-5 for trigonometric functions we take $v_3 = 100$ and $\zeta_3 = \frac{\pi}{100}$. Therefore $M_4 = \zeta_3 = \frac{\pi}{100}$. When $A = 2$ and $K_{12} = K_{11} = K_{21} = 1$, $F(t) = 1$, we obtain:

$$a_4 = \frac{2}{\pi}, \quad M_p = M_4 = \frac{\pi}{100}, \quad U = \frac{F(t)}{M_p} = \frac{100}{\pi}.$$

In Fig. 184 is presented the oscillogram of the solution. Comparison of the given oscillogram with results, recieved on a mechanical integrator (see I. M. Markovich 1), shows that error of solution does not exceed 2%.

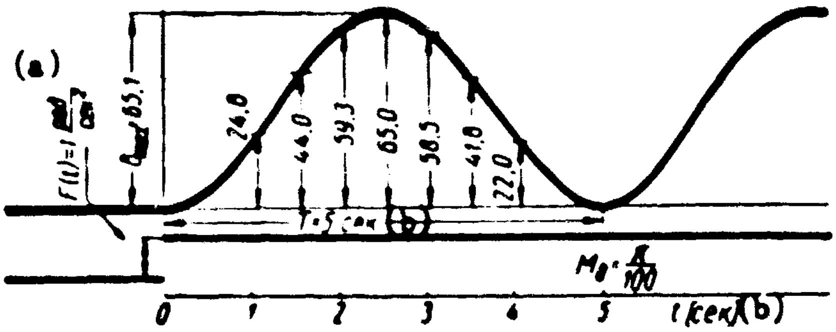

Fig. 184. Results of solution of problem of oscillation of rotor of synchronous machine.
KEY: (a) $\frac{\text{rad}}{\text{sec}^2}$; (b) sec.

CHAPTER XI

SIMULATING LINEARIZED SYSTEMS OF AUTOMATIC CONTROL

In principle every system of automatic control during strict calculation of various acting factors is described by a nonlinear system of differential equations. Linearized systems therefore are only coarse models, as it was a first approximation of the studied or recreated system of automatic control.

If in a number of cases the stability of the initial system can be judged by the stability of its linear model*, with respect to the quality of the transient this can be done, apparently, only for a very limited class of systems.

One can create the impression that in these circumstances investigation of transients of linear models of CAP loses its meaning in general, and, in particular, there is no sense for solving these problems in applying electronic simulators. However the matter is not so. As we know, development of systems of automatic control is conducted in separate stages. On the first stage, when the initial structure of the system is explained and there are established technical requirements on the object and regulator, analysis of a linearized system gives necessary initial data. Furthermore, analysis of a linearized model serves during further investigation for comparison and explanation of influence of separate nonlinear dependences, accompanying physical realization of the system, lying at the basis of the principle of action of the object or regulator, or introduced artificially for the purpose of improvement of dynamic properties of the system.

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*See, for example, M. A. Ayzerman [1].

Application of electronic analog computers allows us to accelerate and facilitate investigation of stability and transients in linearized sustems, especially when they are described by differential equations of a high order with variable coefficients and constant delay.

Equally with this combination of linear model of an object in the form of an electronic integrator with elements of control equipment gives the possibility to consider the influence of deflections of technological order and earlier ignored nonlinear dependences in this equipment.

Solution of linearized systems appears also of great help during checking of correctness of set-up and solution of nonlinear problems on electronic integrators. This check is carried out by means of maximum transition to a linear system, for which it is easy to analytically estimate stability and certain characteristics of transients. It is necessary to note that obtaining of correct results with the help of mathematical computers of any types requires from the researcher a clear idea of the physical processes in the system, skill to estimate a number of partial solutions, obtained as the result of such investigation.

1. Simulating Ordinary Linearized Systems of Automatic Control with Constant Parameters

Let us consider as an example simulation of the processes of automatic control of speed of d-c motor, fed from a generator with regulated voltage. A skeleton diagram of such a system of automatic control is shown in Fig. 185. Equations of motion of considered system of automatic control after linearization and disregard of reaction of armature and inductance of armature circuits of generator and motor can be presented in the form of the following system:

equation of indicator of error

$$U_0 - U_т = \Delta U. \qquad (11.1)$$

equation of tachogenerator

$$U_т = k_3 \Omega. \qquad (11.2)$$

equation of amplifier

$$e = K_y \Delta U. \qquad (11.3)$$

equation of exciter

$$T_1 \frac{dE_в}{dt} + E_в = k_1 e. \qquad (11.4)$$

equation of generator

$$T_2 \frac{dE_г}{dt} + E_г = k_2 E_в. \qquad (11.5)$$

equation of motion of motor

$$T_3 \frac{d\Omega}{dt} + \Omega = \frac{E_г}{c_e} - \frac{m_н}{r_0}. \qquad (11.6)$$

where U_0 is the master signal, Ω is angular velocity of motor, T_1 is time constant of excitation circuit of exciter, T_2 is time constant of excitation circuit of generator, T_3 is electromechanical time constant of acceleration of motor, $m_н$ is the load moment, k_1, k_2, k_3 are proportionality factors, K_y is amplification factor of amplifier

$$r_0 = \frac{c_e c_м}{R_0}.$$

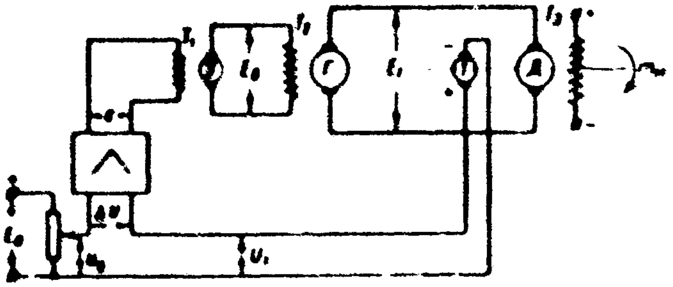

Fig. 185. Fundamental circuit of simulation of processes of automatic control of speed of d-c motor. В—exciter, Г—generator, Д—motor, T—tachogenerator.

It is necessary to explain stability of such a system of automatic control and the influence on stability and nature of transients of imposition of internal feedback with respect to acceleration of regulated motor with the following numerical values of parameters of the system:

$$T_0 = \frac{c_f c_e}{K_u} = 8\ k\ \frac{\text{sec}}{\text{rdn}}$$

$$c_e = 4\ \frac{\text{v sec}}{\text{rdn}} \qquad m_u = 48\ \text{kg-m} \qquad k_3 = 1\ \frac{\text{v sec}}{\text{rdn}}$$

$$T_1 = 0.1\ \text{sec} \qquad T_2 = 0.5\ \text{sec} \quad T_3 = \frac{IR_s}{c_f c_e} = 1\ \text{sec}$$

$$k_1 = 5,\ k_2 = 5,\ K_y = 10,\ \Omega_0 = 110\ \frac{\text{rdn}}{\text{sec}}$$

Before investigation on an electronic model it is expedient preliminarily to explain whether the system is stable for the given numerical values of parameters. With this aim it is possible to use known criteria of stability of linear systems. The quickest is the modification of Routh's criterion little-known in automatic control practice, offered by A. M. Kats, useful for systems of any order.

In order to explain stability of a system, it is sufficient according to this criterion to constitute a table from coefficients of the characteristic equation and prove that diagonal members of this table are all greater than zero. Below is described a sample of composition of Kats' table for the characteristic sixth order equation

$$p^6 + a_5 p^5 + a_4 p^4 + a_3 p^3 + a_2 p^2 + a_1 p + a_0 = 0. \tag{11.7}$$

System is stable when $a_5 > 0,\ b_4 > 0,\ c_3 > 0,\ d_2 > 0,\ e_1 > 0,\ f_0 > 0.$

In the considered case the characteristic equation will be of the third order:

$$p^3 + a_2 p^2 + a_1 p + a_0 = 0. \tag{11.8}$$

where

$$a_2 = \frac{1}{T_1} + \frac{1}{T_2} + \frac{1}{T_3} = 13,\ a_1 = \frac{1}{T_1 T_2} + \frac{1}{T_1 T_3} + \frac{1}{T_2 T_3} = 32.$$

$$a_0 = \frac{1 + \frac{k_1 k_2 k_3 K_y}{c_e}}{T_1 T_2 T_3} = 1270.$$

Table VII

a_5	a_4	a_3	a_2	a_1	a_0
	$b_4 = a_4 - b_3$	$b_3 = \frac{a_3}{a_5}$	$b_2 = a_2 - b_1$	$b_1 = \frac{a_1}{a_3}$	$b_0 = a_0$
		$c_3 = b_3 - c_2$	$c_2 = \frac{b_2}{b_4}$	$c_1 = b_1 - c_0$	$c_0 = \frac{b_0}{b_4}$
			$d_3 = c_3 - d_1$	$d_1 = \frac{c_1}{c_3}$	$d_0 = c_0$
				$e_1 = d_1 - e_0$	$e_0 = \frac{d_0}{d_3}$
					$f_0 = e_0$

Kats' Table for that case has the form

Table VIII

a_2	a_1	a_0
	$b_1 = a_1 - b_0$	$b_0 = \frac{a_0}{a_2}$
		$c_0 = b_0$

and leads for positive a_2, a_1, and a_0 to the known Hurwitz criterion of stability

$$a_1 a_2 > a_0. \tag{11.9}$$

For given numerical values of parameters this criterion is not satisfied, and therefore, if by considerations of accuracy it is undesirable to lower magnitude a_0 by a decrease of the total amplification factor of the system, then it is necessary to introduce means of stabilization.

Introduction of a signal, proportional to acceleration of motor, leads to increase of coefficient a_1:

$$a_1 = \frac{T_1 + T_2 + T_3 + k_4}{T_1 T_2 T_3}. \tag{11.10}$$

and, consequently, satisfaction of criterion of stability (11.9) with proper selection of coefficient k_4 in the acceleration signal.

Using the method brought in chapter X, we will constitute a diagram of connection of operational amplifiers and calculate their transmission factors for solution

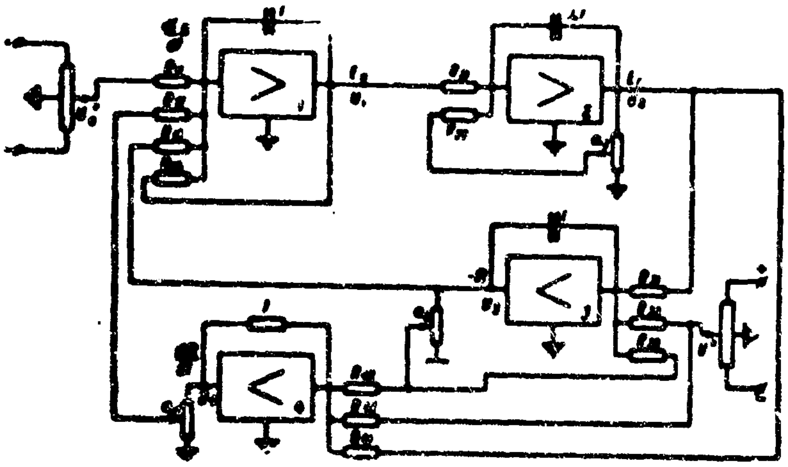

Fig. 186. Functional diagram of set-up of equations, describing processes of automatic control of speed of d-c motor.

of the system differential equations (11.1 - 11.6) in the absence and in the presence of the acceleration signal.

With introduction of acceleration signal after simple transformations we will obtain:

$$\left.\begin{aligned}\frac{dE_0}{dt} &= -\frac{1}{T_1}E_0+\frac{k_1K_y}{T_1}U_0-\frac{k_1K_yk_3}{T_1}\Omega-\frac{k_1K_yk_4}{T_1}\frac{d\Omega}{dt},\\ \frac{dE_r}{dt} &= -\frac{1}{T_2}E_r+\frac{k_2}{T_2}E_0,\\ \frac{d\Omega}{dt} &= -\frac{1}{T_3}\Omega+\frac{E_r}{c_eT_3}-\frac{m_c}{T_3c_e}.\end{aligned}\right\}\qquad(11.11)$$

The functional set-up diagram of these equations is shown in Fig. 186.

Equations of separate units of the circuit of Fig. 186 will be:

$$U_1 = -\frac{1}{p}(K_{11}U_0^*+K_{12}a_4U_4+K_{13}U_3+K_{14}U),$$

$$U_2 = -\frac{1}{p}(K_{21}U_1+K_{22}a_2U_2),$$

$$U_3 = -\frac{1}{p}(K_{31}U_2+K_{32}U^*+K_{33}a_3U_3).$$

$$U_4 = -(K_{41}U_2+K_{42}U^*+K_{43}a_3U_3).$$

Intr ducing equations of transformatuon of variable:

$$E_0 = M_{E_0}U_1,\quad \Omega = M_\Omega U_3,\quad m_c = M_mU^*,$$

$$E_r = M_{E_r}U_2,\quad U_0 = M_{U_0}U_0^*,\quad t = M_tt_м.$$

we will obtain:

$$\left.\begin{aligned}
\frac{dE_3}{dt} &= -\frac{M_{E_0}\cdot K_{11}}{M_t M_{u_0}}U_0-\frac{M_{E_0}}{M_u}K_{12}a_4\frac{d\Omega}{dt}-\\
&\quad-\frac{K_{13}M_{E_0}}{M_u M_t}\Omega-\frac{K_{14}}{M_t}E_0.\\
\frac{dE_r}{dt} &= \frac{M_{E_r}\ K_{21}}{M_t M_{E_0}}E_0-\frac{K_{22}a_2}{M_t}E_r.\\
\frac{d\Omega}{dt} &= -\frac{M_u K_{31}}{M_t M_{E_r}}E_r-\frac{K_{32}M_{\Omega}}{M_t M_m}m_0-\frac{K_{33}}{M_t}a_3\Omega.
\end{aligned}\right|\qquad(11.12)$$

Comparison of the received system of equations (11.12) with the initial (11.11) gives

$$\frac{1}{T_1}=\frac{K_{14}}{M_t}.\qquad \frac{k_1K_y}{T_1}=-\frac{M_{E_0}K_{11}}{M_tM_{u_0}}.$$

$$\frac{k_1K_yk_3}{T_1}=\frac{K_{13}}{M_u}\frac{M_{E_0}}{M_t}.\qquad \frac{k_1K_yk_4}{T_1}=\frac{K_{12}a_4M_{E_0}}{M_u}.$$

$$\frac{1}{T_2}=\frac{K_{22}a_2}{M_t}.\qquad \frac{K_2}{T_2}=-\frac{M_{E_r}K_{21}}{M_tM_{E_0}}.$$

$$\frac{1}{T_3}=\frac{K_{33}a_3}{M_t},\quad \frac{1}{c_eT_3}=-\frac{M_uK_{31}}{M\ M_{E_r}}.\quad \frac{1}{T_3r_0}=\frac{K_{32}M_{\Omega}}{M_tM_m}.$$

We select the time scale, proceeding from the condition that frequency of processes does not exceed the passband of the computing elements of the installation.

Estimating coarsely duration of process of adjustment by magnitude of the largest time constant $T_3 = 1$ sec, we will take $M_t = 1$. Here duration of processes in the simulator will be equal to duration of processes in initial system. Remaining scale factors, coefficients K_{ij} and a_k, are selected from condition of work of computing elements within limits of permissible error and the linear range of change of output voltage.

For the given numerical values of parameters we obtain:

$$\frac{K_{14}}{M_t}=10.\quad \frac{M_{E_0}}{M_t}\frac{K_{11}}{M_{u_0}}=-500.\quad \frac{K_{13}M_{E_0}}{M_tM_{\Omega}}=500.$$

$$\frac{K_{12}a_4M_{E_0}}{M_u}=500\lambda_4.\quad \frac{K_{22}a_2}{M_t}=2.\quad \frac{M_{E_r}K_{21}}{M_tM_{E_0}}=-10.$$

$$\frac{K_{33}a_3}{M_t}=1.\quad \frac{M_uK_{31}}{M_tM_{E_r}}=-0.25.\quad \frac{K_{32}M_{\Omega}}{M_tM_m}=\frac{1}{8}.$$

When $M_t = 1$ we take $M_{E_B} = -100$, $M_\Omega = -1$, $M_m = -10$, $M_{u_0} = 1$, $M_{E_T} = 10$.

As a result when $a_3 = 1$ we find:

$$K_{14} = 10. \quad K_{11} = 5. \quad K_{13} = 5. \quad K_{12}a_4 = 5k_4.$$
$$K_{22} = 2. \quad K_{21} = 100. \quad K_{32} = 1. \quad K_{31} = 2.5.$$
$$K_{33} = \frac{10}{3}. \quad U^0 = -4.8\,v. \quad U_0^0 = 110\,v. \quad a_2 = 1.$$

Transmission factor $K_{21} = 100$ is carried out by connecting to the feedback circuit of operational amplifier No. 2 a capacitor with capacitance $C = 0.1$ microfarad, and at input of resistor $R_{21} = 100$ kilohm, $K_{22} = 2$ is here carried by feeding input impedance $R_{22} = 1$ megohm from divider $a_2 = 0.2$.

In Fig. 187 are brought oscillograms of transients in the investigated system, received with the help of electronic model EMU-5 accordingly in the absence of feedback with respect to acceleration and with introduction of this feedback with coefficient $k_4 = 0.159$ and application to the shaft of the motor of a constant moment of load $m_H = 48$ kg-m.

As follows from the data, brought in the oscillograms, coincidence with calculated values is sufficiently near.

Indeed:

$$\Delta\Omega_{\text{уст. расч}} = -\frac{\frac{m_н}{r_0}}{1 + \frac{k_1 k_2 k_3 K_v}{c_e}} = -0.096 \text{ rdn/sec}$$

$$\Delta\Omega_{\text{уст. эк}} = -0.1 \text{ rdn/sec}$$

Frequency of process it is possible tentatively to estimate by equation (11.8), using formula

$$\left.\begin{aligned} a_2 &= -(\zeta - 2\gamma), \\ a_0 &= (\omega^2 + \gamma^2)\zeta \end{aligned}\right\} \qquad (11.13)$$

where ζ is a real root, and $-\gamma \pm j\omega$ — are complex roots of equation (11.8).

For process with small attenuation it is possible in expressions (11.13) to disregard magnitude 2γ as compared with ζ and γ^2 as compared with ω^2.

Then we will obtain:

$$\omega_{\text{расч}} \approx \sqrt{\frac{a_0}{a_2}} \approx 9.9 \text{ rdn/sec} \qquad \omega_{\text{эк}} = 9 \text{ rdn/sec}.$$

With the help of electronic models it is possible also to determine in the plane of parameters of adjustment of the boundary of the region of stability, boundaries

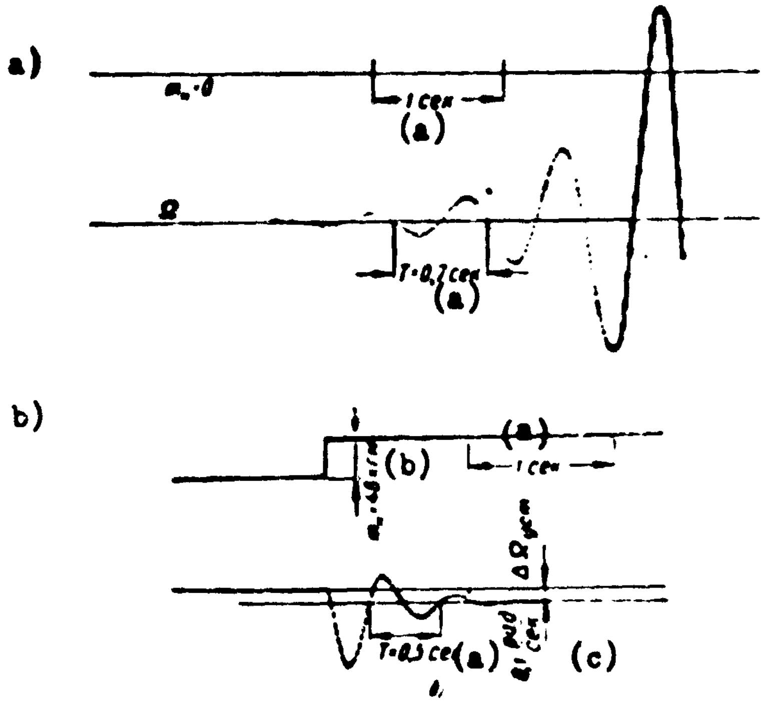

Fig. 187. Oscillograms of transients of automatic control of speed of d-c motor.
KEY: (a) Sec; (b) Kg-m; (c) $\frac{\text{rdn}}{\text{sec}}$.

of the region with given qualities of the transient, and execute rapid selection of parameters of various means of stabilization. This allows to accumulate factual material about properties of separate classes of systems of automatic control and thereby to fill the gap existing at present in theories of automatic control.

2. Simulating Systems of Automatic Control with Parameters Variable in Time

To differential equations with coefficients variable in time are brought problems of investigation dynamics of processes of stabilization of motion of flying objects about the center of gravity with variable flight speed. As an example let us consider simulating isolated motions of banking and course with elements of stabilization equipment.

Simulation of connected motion does not present any principal peculiarities, with the exception of the appearance of additional cross connections in the set-up diagram.

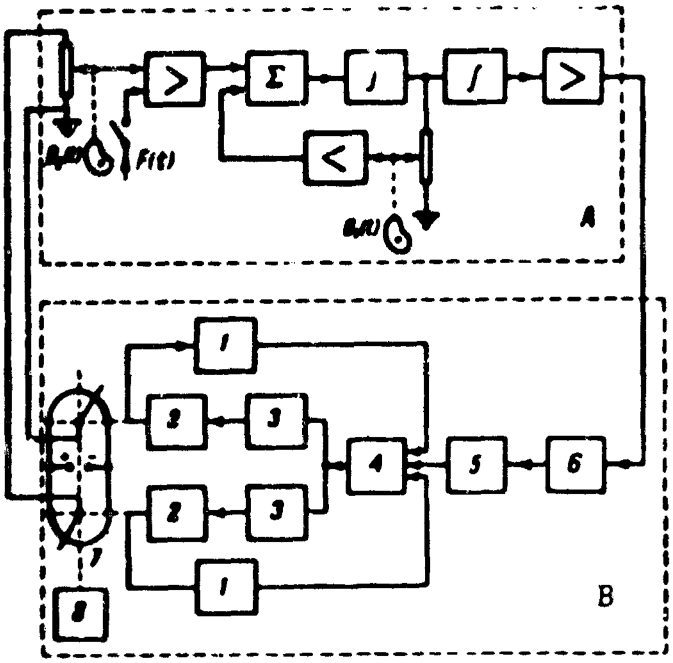

Fig. 188. Diagram of union of model with stabilization equipment.
1—feedback potentiometers, 2—steering motors, 3—relay units, 4—amplifier, 5—modulator, 6—differentiating cell, 7—potentiometers for receiving signal, proportional to deflection of steering motor, 8—load stand, A—analog, B—stabilization equipment.

Equation of motion of object with respect to banking in this consideration will differ from equation (1.16) by the fact that coefficients B_1 and B_2 are no longer constant, and functions of time are given:

$$\ddot{\varphi} + B_1(t)\dot{\varphi} = -B_2(t)\delta \pm F(t). \qquad (11.14)$$

where $F(t) = B_2(t)\delta_в$ is the disturbance acting on the object.

The diagram of coupling the model with stabilization equipment is shown in Fig. 188. In the general case at places of union with real equipment there must be established converters, whose transmission factors dictate selection of scale factors of conversion of corresponding variables.

In the considered problem tension, representing the regulated coordinate in the model, is converted by a special device into the angle of rotation either of all the stabilization equipment, or only of its sensory—a gyroscopic instrument. In those cases, where dynamics of the gyroscopic instrument can be disregarded, the need for a converter drops.

Output of stabilization equipment is connected with input of simulator by potentiometers. Voltage from cursors of potentiometers, proportional to angle of rotation of steering motors, moves to input of model. Here on shaft of the steering device there should be reproduced load (hinged moments) by the help of a special load stand. This load can change in time and depend on the angle of displacement of the steering wheel and the coordinates of the object (angle of incidence or angle of slipping). As such a load device in the simplest cases we apply stands equipped with springs (Fig. 189).

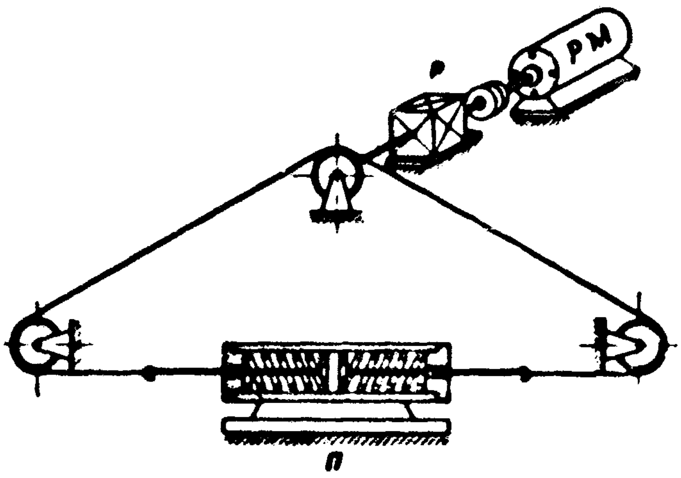

Fig. 189. Fundamental circuit of load stand. PM—steering machine, P—reductor, П—spring mechanism.

Set-up of the equation of motion of the object is conducted by maximum values of coefficients for the considered time interval, and to output of the corresponding units of the circuit are connected voltage dividers, bringing at every moment of time the voltage to the output of the units to the required value.

Let us assume that in the considered problem $B_{1\,max} = 4.45$, $B_{2\,max} = 309$, the gyroscope is taken as a zero-moment element with transmission factor $k_r = 2.3$ v/deg, and potentiometers on steering machines will convert the angle of rotation

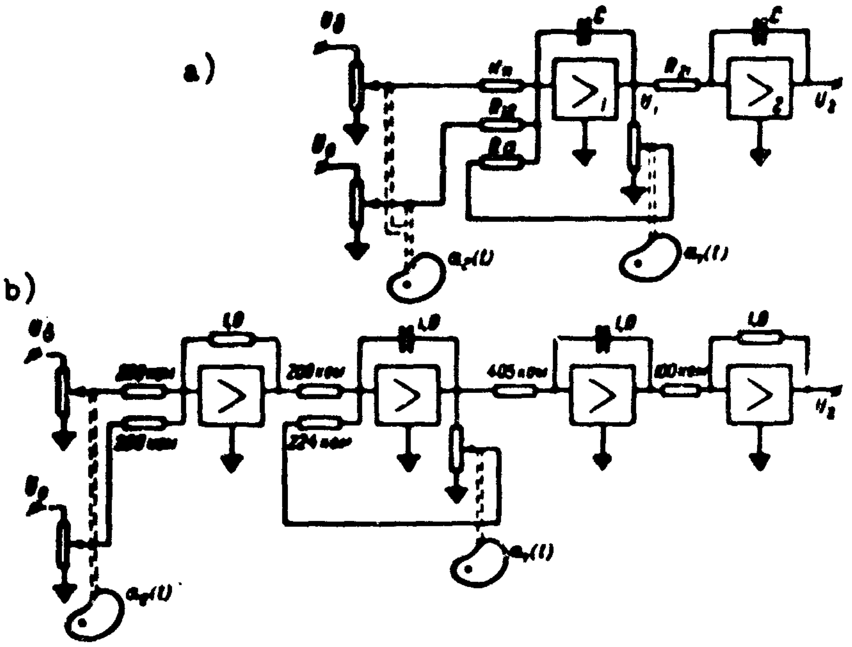

Fig. 190. Functional diagram of modeling of equations of banking with variable parameters.

of steering wheels δ into voltage U_δ with transmission factor $k_\delta = 1$.

The set-up diagram is shown in Fig. 190 a.

Equations of separate computing elements give

$$U_2[p^2 + K_{12}\alpha_1(t)p] = -K_{11}K_{21}\alpha_2(t)U_\delta - K_{12}K_{21}\alpha_1(t)U_\varphi.$$

After transformation of variables $U_r = M_r U_2$, $\delta = k_\delta U_\delta$. $T = M_t t_m$, we will receive

$$\frac{d^2U_r}{dT^2} + \frac{K_{12}\alpha_1(t)}{M_t}\frac{dU_r}{dT} = -\frac{M_rK_{11}K_{21}\alpha_2(t)}{k_\delta M_t^2}\delta - \frac{K_{12}K_{21}M_r\alpha_2(t)}{k_\delta M_t^2}\dot{\varphi}. \qquad (11.15)$$

Comparing equation (11.15) with initial equation, in which coordinate γ is replaced by U_r by substitution $U_r = k_r\gamma$, we will receive

$$\frac{K_{12}\alpha_1(t)}{M_t} = B_1(t). \quad k_rB_2(t) = \frac{K_{11}K_{21}M_r\alpha_2(t)}{k_\delta M_t^2}.$$

During investigation with elements of control loop $M_t = 1$. Furthermore, it is necessary to take $M_r = 1$ so that it is possible directly to feed output voltage of installation to the differentiating circuit of the equipment.

Therefore finally we obtain when $\alpha_1(t) = \alpha_2(t) = 1$ and $k_\delta = 1$:

$$K_{12} = B_{1\,max} = 4.45,$$

$$K_{11}K_{21} = B_{2\,max}\,k_r = 711.$$

These transmission factors could have been realized on two units of this circuit, if coefficients B_1 and B_2 were constant. In this case in the first unit it would have been possible to realize a transmisiion faction of the order of 50 by selection of a capacitor with 0.1 microfarad capacity and resistors $R_{11} = R_{12} = 200$ kilohm, and $R_{13} = 2.24$ megohm. Transmission factor of the second unit here should have been $K_{21} = 14.2$ which for operational amplifiers with automatic stabilization of zero level can be considered permissible.

Considering that coefficient B_1 can also take zero values, it is necessary to supplement the circuit with amplifiers at input and output (Fig. 190b).

Before solving a problem with variable coefficients and real equipment, it is useful to prove the correctness of set-up and solution of equations on models for several fixed values of coefficients.

Analytic solution of equation (11.14) with $\varphi(0) = 0$, $\dot{\varphi}(0) = 0$, $F(t) = -B_2\dot{\psi}_0$ and constant coefficients gives

$$\varphi = \frac{B_2\dot{\psi}_0}{B_1^2}(1 - B_1t - e^{-B_1t}). \qquad (11.16)$$

In Fig. 191 is brought the oscillogram of solution of this equation on a model for $B_1 = 0.98$, $B_2 = 19.8$ and $\dot{\psi}_0 = 1.9$, and in the table are brought results of comparison of this solution with the analytic one. The table indicates the near coincidence of calculated and experimental data.

In Fig. 192a are brought oscillograms of change of coordinate φ during solution of a problem with variable coefficients, given by graphs in Fig. 192b. From the oscillogram one may see characteristic change of period of oscillations , which follows a change of coefficient B_2 in time. Transients during application and removal of disturbance converge to natural oscillations which bears witness to the presence of nonlinearities in the stabilization equipment.

In the case of stabilization of course motion it is necessary to solve on the model system of differential equations (2.5) with the distinction that coefficients A_1, A_4, A_5 and A_6 are given functions of time.

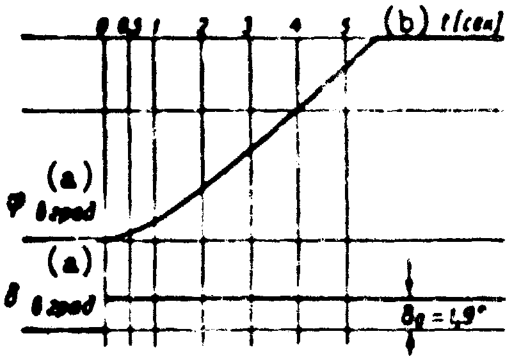

Fig. 191. On checking accuracy of solution.
KEY: (a) In deg; (b) Sec.

(a) Время t сек	0,5	1	2	3	4	5
(b) φэксп	[illegible]	[illegible]	[illegible]	75	112,[illegible]	[illegible],5
(c) φрасч	[illegible]	[illegible]	[illegible]	77	[illegible]	[illegible]2,0
Δφ	0	1	1,5	2,0	2,2	2,5

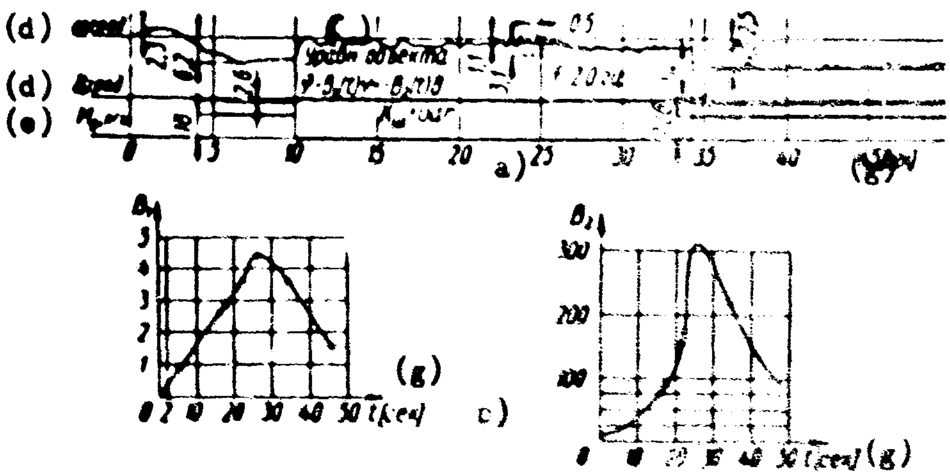

Fig. 192. Processes of stabilization of banking with variable parameters and supplying and removal of disturbing moment.
KEY: (a) Time T sec; (b) Experimental; (c) Calculated; (d) Deg; (e) Kg-m; (f) Equation of object; (g) Sec.

Proceeding by the above-stated method, we arrive at the set-up diagram brought in Fig. 193. As in the preceding case, it is useful before beginning a solution with coefficients variable in time to check accuracy of solution of equations on the model.

Solution of system (2.5) for coordinate ψ under the condition that $A_2 = 0$, disturbances $F_b = M_b = 0$, coefficients are constant and steering wheels at moment

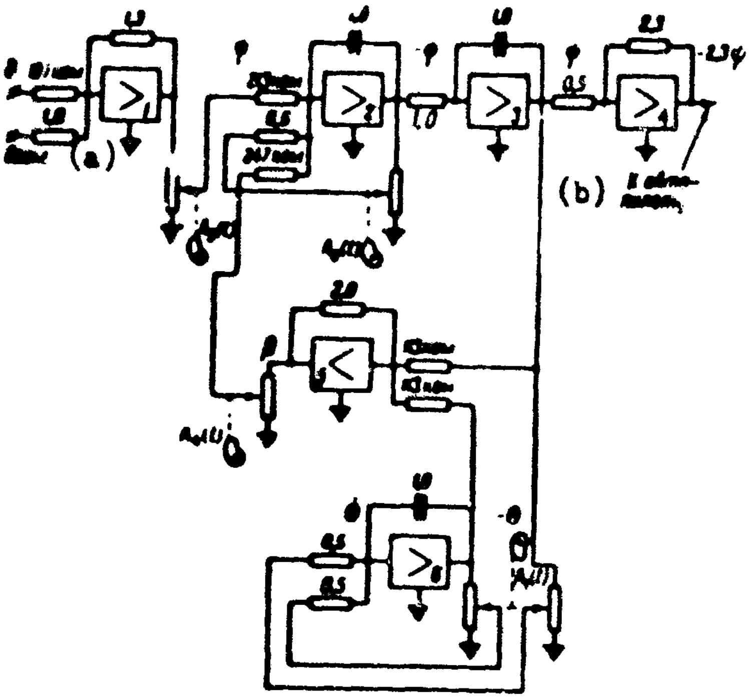

Fig. 193. Functional diagram of modeling of equation of course motion with variable parameters.
KEY: (a) Disturbance; (b) To autopilot.

t = 0 are instantly shifted an angle ψ_0, leads to equation

$$\frac{d^2\psi}{dt^2}+(A_1+A_6)\frac{d\psi}{dt}+(A_6A_1+A_4)\psi=-A_5A_1\psi_0 t+C_0.$$

The integral of this differential equation in the case of complex roots of the characteristic equation will be

$$\psi=C_1e^{-\gamma t}\sin\omega t+C_2e^{-\gamma t}\cos\omega t-\frac{A_5A_1}{A_6A_1+A_4}\psi_0 t+ \\ +\frac{C_0}{A_6A_1+A_4}+\frac{A_5A_1\psi_0(A_6+A_1)}{(A_6A_1+A_4)^2}. \tag{11.17}$$

where C_1, C_2, C_0 are constants of integration, determined by initial conditions $\psi(0)=0$, $\dot\psi(0)=0$, $\ddot\psi(0)=-A_5\psi_0$ in the form of expressions:

$$\left.\begin{aligned}C_1&=A_5\psi_0\left[\frac{1}{\omega}\,\frac{1-\frac{2\gamma A_1}{A_6A_1+A_4}}{\omega^2+\gamma^2}+\frac{1}{\omega}\,\frac{A_1}{A_6A_1+A_4}\right].\\ C_2&=A_5\psi_0\,\frac{1-\frac{2\gamma A_1}{A_6A_1+A_4}}{\omega^2+\gamma^2}.\\ C_0&=A_5\psi_0\left[\frac{A_6+A_1A_6-2\gamma A_1}{\omega^2+\gamma^2}+\frac{A_1(A_6+A_1)}{A_6A_1+A_4}\right].\end{aligned}\right\} \tag{11.18}$$

where γ - is the attenuation factor, and ω is the frequency of oscillations, determined by relationships:

$$\left.\begin{aligned} \gamma &= -\frac{A_6 + A_1}{2}, \\ \omega &= \sqrt{A_6 A_1 + A_4 - \left(\frac{A_6 + A_1}{2}\right)^2}. \end{aligned}\right\} \qquad (11.19)$$

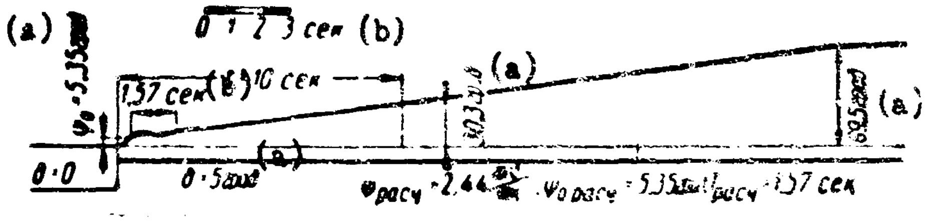

Fig. 194. Checking accuracy of solution.
KEY: (a) Deg; (b) Sec.

In Fig. 194 is brought the oscillogram of solution of the considered system of equations (2.5) in case of constant coefficients, equal to $A_1 = 0.435$, $A_2 = 0$, $A_4 = 17.67$, $A_5 = 20.65$, and $A_6 = 1.59$. Comparison of received solution with the analytic solution in distinction from the preceding case can be made not for separate points of the solution, but by comparing characteristic parameters of the solution:

$$\left.\begin{aligned} \psi_0 &= -\frac{C_0}{A_4 + A_6 A_1} + \frac{A_5 A_1 (A_6 + A_1)\delta_0}{(A_6 A_1 + A_4)^2}, \\ \dot{\psi}_{\text{уст}} &= -\frac{A_5 A_1 \delta_0}{A_6 A_1 + A_4}, \\ T &= \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{A_6 A_1 + A_4 - \left(\frac{A_6 + A_1}{2}\right)^2}}. \end{aligned}\right\} \qquad (11.20)$$

Results of comparison, shown in the oscillogram, bear witness to satisfactory accuracy of obtained solution. It is necessary to note that error of solution for parameter $\dot{\psi}_{\text{уст}}$ in significant measure is determined by how much transmission factors for both inputs of the operational amplifier, producing coordinate β (angle of slipping) differ. Accuracy of setting of these transmission factors should be not lower than the third significant digit.

As follows from this example, for estimating accuracy of a solution, received

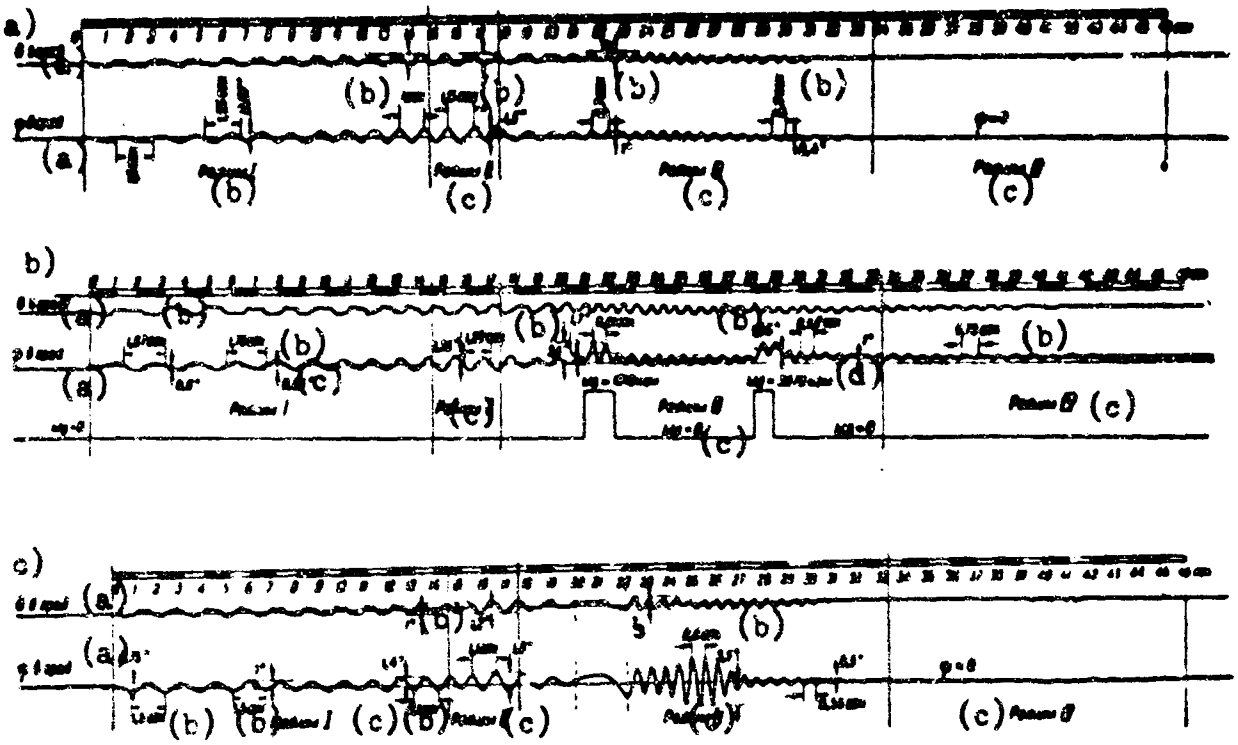

Fig. 195. Processes of stabilization of course with variable parameters.
KEY: (a) In degrees; (b) Sec; (c) Regime; (d) Kg-m.

on the model, in a number of cases it is more correct to conduct comparison not for separate points, but of characteristic parameters of the solution. This is sensible to do especially when the solution has an oscillatory character or sharply expressed extrema.

In Fig. 195 are brought three oscillograms of processes of course stabilization with coefficients variable in time, received during work of electric automatic pilot with proportional feedback, controlled by the signal and its first derivative. Parameters of the differentiating circuit changed as a function of time in steps. From oscillograms one may see characteristic change of amplitudes of natural oscillations, connected basically with change of coefficients A_4 and A_5. On the oscillogram of Fig. 195b is shown the influence of disturbances, applied in the form of rectangular pulses of various duration, and on the oscillogram of Fig. 191c is shown influence of brief change of sign of coefficient A_4 (statistically unstable object).

2, Variable Speed Drive of Coefficients

Devices, intended for introduction of coefficients variable in time, are called variators variable speed drives of coefficients.

We distinguish electromechanical and electronic variable speed drives of coefficients. In those cases, when solution of a differential equation is produced in full time scale and change of coefficients of differential equations occurs slowly, we apply electromechanical devices. With iteration of the solution with a frequency greater than 10 cps electromechanical devices cannot be applied due to great inertness. In these cases for introduction of variable coefficients we use diode functional generators, in which voltage of the argument changes linearly in time according to saw-tooth law with the frequency of iteration, and corresponding multipliers.

The simplest electromechanical variable speed drive of coefficients (Fig. 196) consists of a certain number of linear potentiometers, cursors of which are moved in time by a draw line and profiled cams, made in accordance with the function of time given for reproduction. A tension spring serves to guarantee constant contact of the roller and cam. All cams are planted on a common shaft, moved by an electric motor with constant speed.

One wide-spread modification of cam variator is shown in Fig. 197. Pulse step-by-step motor, moving the cam shaft, can be fed from a pulse generator of stable frequency, provided in electronic models (for example, in IPT-4, IPT-5, MPT-9) or from any other interrupter of current. This ensures possibility and simplicity of change of speed of rotation of cams in comparatively wide limits. Considered cam variators with accuracy of profile of cam ± 0.1-0.5 mm and maximum cam radius 60mm introduce error of ± 0.16-0.33%. Among deficiencies of these devices one should mention comparatively high labor-consumption of manufacture of cams.

In the design of variator of coefficients offered by engineer Ivanovskiy cams are replaced by slip-rings, made of wire, bent by a given law and fastened to fixed

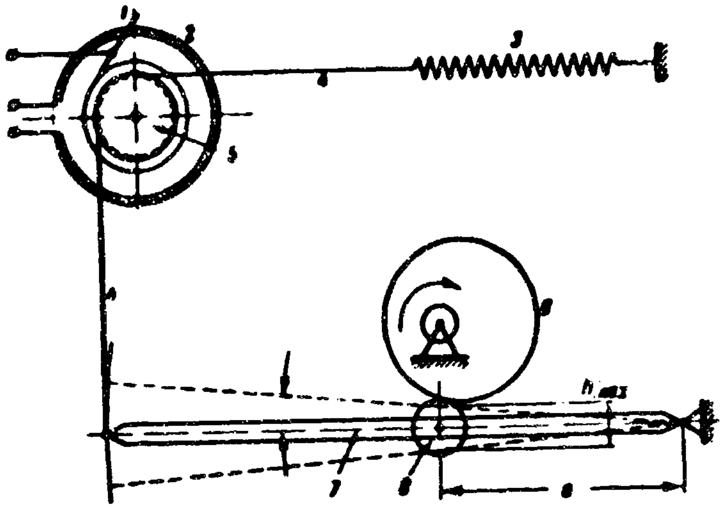

Fig. 196. Fundamental circuit of cam variable speed drive of coefficients. 1—slip-ring, 2—potentiometer, 3—tension spring, 4—string, 5—drum, 6—cam, 7—lever, 8—roller.

insulating plates 1 (Fig 198a). Along these plates moves a trolley with linear wire potentiometers 3 of cylindrical form. Potentiometers by springs 4 are pressed to the wire. Drive of the trolley is carried out from an asynchronous motor through reductor with a changeable transmission ratio (movement of trolley at full speed is provided after 440, 220, 110 and 55 sec). For stabilization of speed, on the shaft of the drive motor is put in a flywheel. The general appearance of such a variator of coefficients as made by the Academy of Sciences of USSR is shown in Fig. 198b.

Examples of variable speed drives of coefficients with step-by-step approximation are the units of variable coefficients of electronic models IPT-4 and IPT-5 (Fig. 199). The basis of each unit is a 100-lamellar stepping selector. Step-by-step motor is controlled by pulses from an interrupter, located in the control panel. Time of switching from lamella to lamella can be fixed equal to 1.5 sec, 1 sec, 0.75 sec, 0.25 sec and 0.1 sec. Consequently, to time of cycle of work of variator can have 6 values from 150 sec to 10 sec. Values of voltages, brought to lammellae of variator, can be set in steps by any law from the voltage divider, where on the divider there are 100 divisions of positive voltage and 100 divisions of negative voltage. Commutation from the divisor to lamellae of the selector is carried out

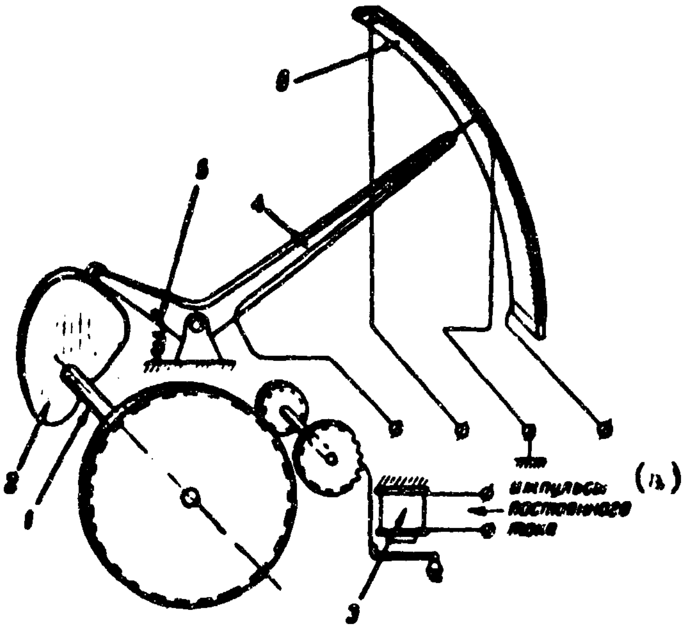

Fig. 197. Kinematic diagram of modification of cam variable speed drive of coefficients. 1—can shaft, 2—cam, 3—step-by-step motor, 4—lever, 5—tension spring, 6—winding of potentiometer.
KEY: (a) d-c pulses.

on a special setting field. Besides, the level of a coefficient can be lowered by an additional voltage divider with a coefficient of division $0 \leqslant a \leqslant 1$ at an interval 0.001 of the step.

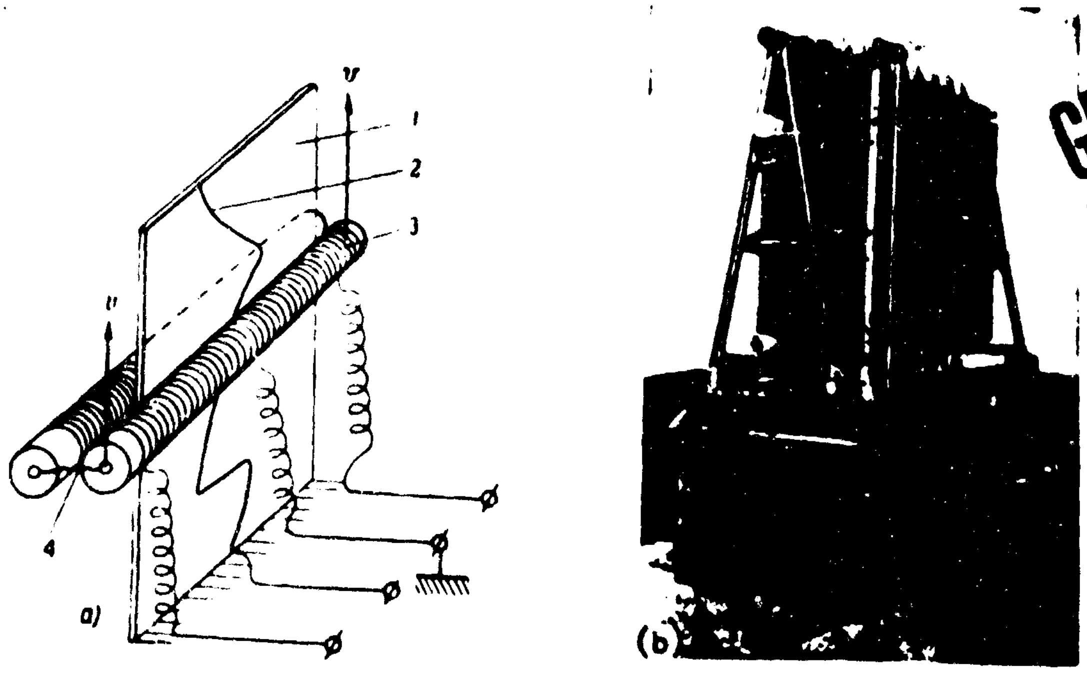

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Fig. 198. Variable speed drive of coefficients with wire-wand shaped slip-rings.

To obtain pulses, controlling the step-by-step motor, in analogs IPT-4 and IPT-5 there is a special device, consisting of a quartz oscillator with a frequency divider (with division of frequency from 1000 cps to 50 cps) and an auxiliary pulse generator, consisting of 6 synchronous motors, working from a frequency of 50 c and interrupting contacts with six different frequences. If permissible error of scanning of coefficients in time is 2%, then motors can be fed directly from an a-c net.

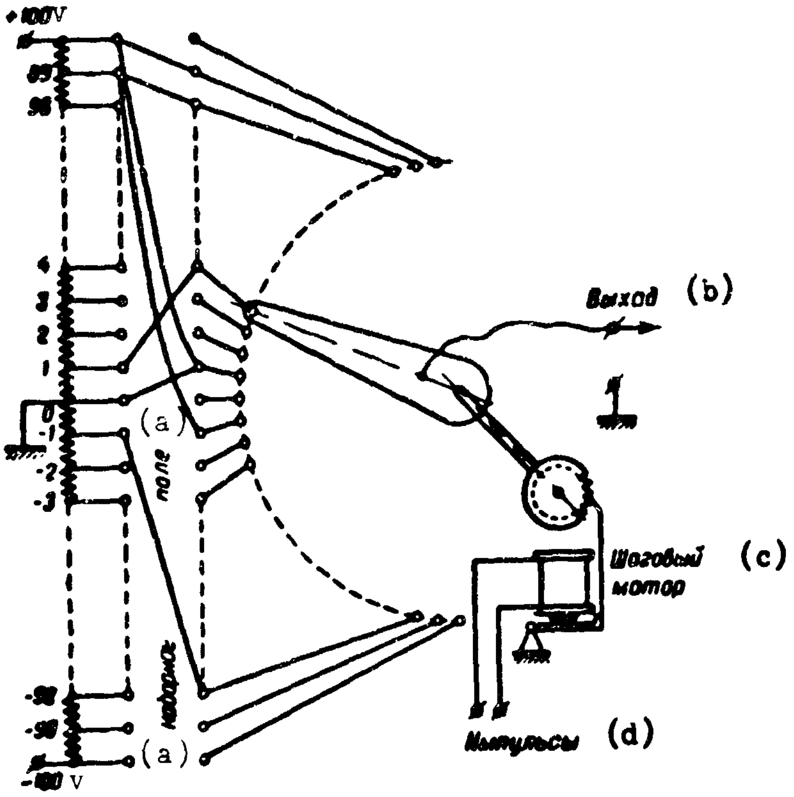

Fig. 199. Variable speed drive of coefficients with stepping selector.
KEY: (a) Setting field; (b) output; (c) Step-by-step motor; (d) Pulses.

Among the merits of the considered variable speed drive one should include the fact that in separate moments of time there can be received coefficients with great accuracy, since separate sections of the voltage divider can be made very accurately. Essential deficiencies of these devices are complexity of construction, large dimensions, appearance of additional interferences due to step-by-step introduction of the variable coefficient.

Variable speed drive of coefficients of analog MPT-9 is a modernization of the considered device. On the setting field of the unit of the variable coefficient there is no divider with negative values of voltages. Negative values of coefficients

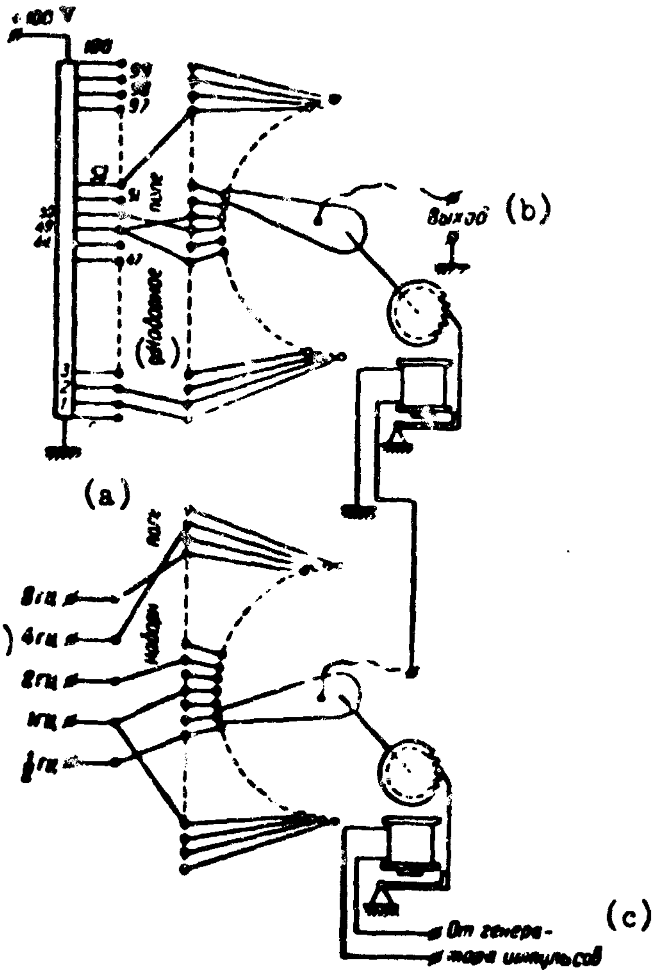

Fig. 200. Modification of variable speed drive of coefficients of Fig. 199.
KEY: (a) Setting field; (b) Output; (c) From pulse **generator**; (d) cps.

result from general displacement of level of the graph of the variable coefficient by a special device, introducing negative input voltage with a constant coefficient into the model. Furthermore, to the stepping selector of the variable coefficent unit there can be fed pulses with one of five frequencies by program in time which allows us to approximate the curve of the law of change of the coefficient more accurately, i.e., on steeper sections of the curve to increase the frequency of switching.

To obtain programmed introduction of frequencies of pulses on the stepping selector the variable coefficient unit, there is an additional device, including still another stepping selector with 100 lamellae and a setting field, where to every lamella there can be passed pulses with one of the following five frequencies: 8 cps, 4 cps, 2 cps, 1 cps, 0.5 cps. Lamella can also remain unconnected, and in this interval of time not one pulse will pass to the stepping selector of the variable coefficient unit. All main elements of variator of coefficients and auxiliary devices (generator of stable frequency and others) remain the same as in variator of analog IPT-4. The basic circuit is shown in Fig. 200.

Shown designs of variators of coefficients cannot be recognized as totally perfect. Cam variators require comparatively great expenditure of time on manufacture and replacement of cams; variators with wire on the insulating plate, do not ensure smooth change of the time—speed scale of potentiometers and have complicated drive mechanism. Variators with stepping selectors also are not very simple and frequently serve as a source of undesirable interferences, caused by the step nature of approximation of the variable coefficient.

Apparently, improvement of variators with wire, glued on insulating plate or drum, can lead to creation of devices which are simple, accurate and at the same time convenient to use. Significant simplification here can be attained by transition to smoothly regulated drive, in the form of a servo system. Supplying this servo system with feedbacks with respect to speed and position, it is possible to use the variator of coefficients also as a functional generator or multiplier-divider during investigation of processes with frequency up to 1 cps. By this it is possible considerably to supplement the possibilities of the electronic model without increasing the number of additional units.

4. Simulation of Systems of Automatic Control with Constant Delay

Constant delay is met primarily in industrial process systems.*

An essential role in formation of constant delay in industrial systems is played by the terminal velocity of transfer of the substance. An example is the system of automatic control of thickness of a sheet on a rolling mill, where the meter of thickness of the sheet 1 (Fig. 201a) and pressure device 2 are located a certain distance L from each other, due to which with a terminal velocity v of motion of the rolled sheets there appears delay τ between influence of pressure device and measurement of its result by the sensor. Another characteristic case of the presence of constant time delay takes place in system of automatic control of concentration of solution by change of input of one of the components (Fig. 201b).

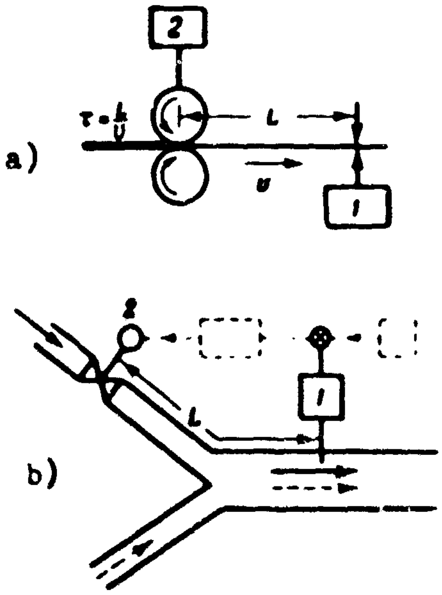

Fig. 201. Examples of systems of automatic control with delay. a) control circuit of thickness of sheet on rolling mill. 1—meter of thickness of sheet, 2—pressure device; b) control circuit of concentration of solution; 1—meter of concentration, 2—regulating element.

Furthermore, often it is convenient in studying multicapacity industrial processes to replace the equation of motion of high order by equations of the first or second order with constant delay. Finally, during investigation of control systems with very fast processes it is often necessary to explain the influence of small constant delays of the order of 10-100 millisecond in the controller, or delay of such an order, to which it is possible to reduce small parameters of system.

The general range of constant delay, encountered in systems of automatic control, embraces intervals of time from 5 milliseconds to 100 minutes. However during simulation, apparently, it is possible to limit maximum delay to 5 minute, since with greater delays processes of adjustment proceed already so slowly that the dynamics of equipment of adjustment, in essence, can be disregarded and one can conduct investigation on an electronic model of the total system of differential equations of the system of automatic control in an unnatural time scale.

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*See, for example, V. L. Lossiyevskiy [2].

For reproduction of delay in electronic models there are applied special devices, named delay blocks. In ideal case the delay block should be characterized by the following dependence between input $e_{вх}$ and output $e_{вых}$ magnitudes:

$$e_{вых}(t) = e_{вх}(t-\tau), \tag{11.21}$$

where τ = const is the constant delay. In operator form equation (11.21) will have the form

$$\bar{e}_{вых}(p) = e^{-p\tau}\bar{e}_{вх}(p). \tag{11.22}$$

Thus, output magnitude of delay block should copy accurately the input magnitude, but with a shift in time τ. Transfer function of block should be

$$W_{\tau}(p) = e^{-p\tau}.$$

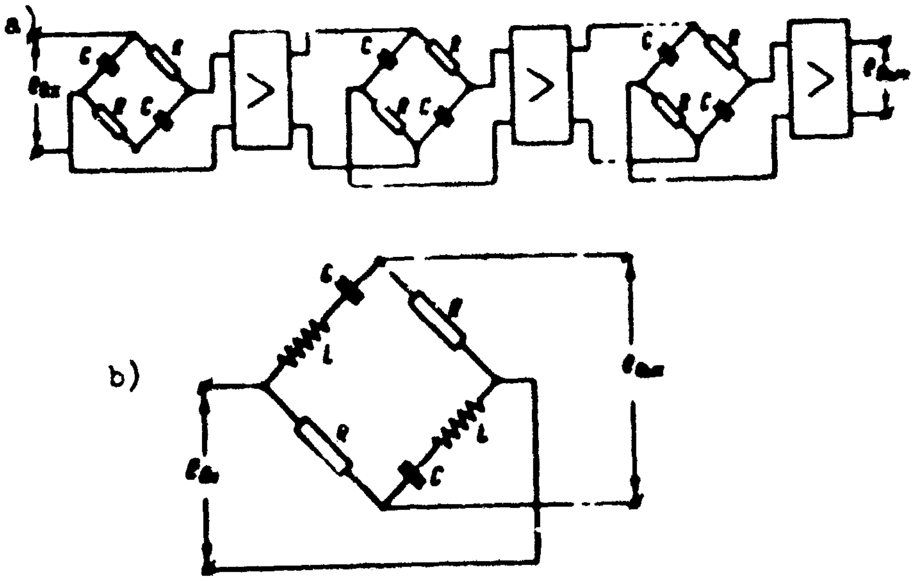

Fig. 202. Passive networks, utilized for obtaining delay.

Comparison of separate constructions of delay blocks and their appraisal can be carried out on the basis of comparison of frequency responses. For an ideal delay block the gain-frequency response is $|W_\tau(j\omega)| = 1$, and the phase-frequency is $\varphi_\tau = -\omega\tau$. Known constructions of delay blocks can be divided into two groups depending upon how the given gain and phases frequency responses are realized. In devices of the first group the gain-frequency response is reproduced in principle accurately, and that of phase, approximately; in devices of the second group, vice versa, the phase response is reproduced in principle accurately, and gain, approximately. Among devices of the first type are, in particular, RC circuits of passive and active quadripoles, simulating long lines. In Fig. 202a is shown one circuit, consisting of passive RC networks, isolated from one another by separating amplifiers (for example, cathode followers).

Transfer function of each such network is

$$W_i(p) = \frac{1 - RCp}{1 + RCp}. \tag{11.23}$$

Transfer function of circuit of such identical sections will be

$$W_\Sigma(p) = \prod_{i=1}^{n} W_i(p) = \left(\frac{1 - RCp}{1 + RCp}\right)^n. \tag{11.24}$$

We will estimate error, introduced by replacement of $W_\tau(p)$ by $W_\Sigma(p)$. This estimate can be made by comparison of frequency responses. For considered the bridge circuit we have:

$$\left.\begin{array}{c} \varphi_\Sigma = -2n \operatorname{arctg} RC\omega, \\ |W_i(j\omega)| = 1. \end{array}\right\} \tag{11.25}$$

From this, considering $2nRC = \tau$ and comparing with ideal values of $|W_\tau(j\omega)|$ and φ_τ, we will find error in phase response

$$\Delta\varphi = -\omega\tau + 2n \operatorname{arctg} \frac{\omega\tau}{2n}$$

or when $\frac{\omega\tau}{2n} < 1$ approximately

$$\Delta\varphi \cong \frac{(\omega\tau)^3}{12n^2}. \tag{11.26}$$

Error in gain response is

$$\Delta M = 0.$$

where

$$F_{\mu\nu}(x) = 1 + \frac{\nu x}{(\mu+\nu)\,1!} + \frac{\nu(\nu-1)\,x^2}{(\mu+\nu)(\mu+\nu-1)\,2!} + \dots + \frac{\nu(\nu-1)\dots 2\cdot 1 x^\nu}{(\mu+\nu)(\mu+\nu-1)\dots(\mu+1)\,\nu!},$$

$$O_{\mu\nu}(x) = 1 - \frac{\mu x}{(\mu+\nu)\,1!} + \frac{\mu(\mu-1)\,x^2}{(\mu+\nu)(\mu+\nu-1)\,2!} - \dots + (-1)^\mu \frac{\mu(\mu-1)\dots 2\cdot 1 x^\mu}{(\mu+\nu)(\mu+\nu-1)\dots(\nu+1)\,\mu!}.$$

When $\mu = \nu = 2$, $x = -pT$ we obtain

$$e^{-pT} \cong W(p) = \frac{T^2p^2 - 6Tp + 12}{T^2p^2 + 6Tp + 12}. \qquad (11.28)$$

Conducting, as and before, comparison of the frequency response, received from (11.28) with the ideal, we find that

$$|W(j\omega)| = |W_\tau(j\omega)| = 1,$$

and

$$\Delta\varphi = -\omega\tau + 2\,\mathrm{arctg}\,\frac{6\omega\tau}{12-(\omega\tau)^2}, \qquad (11.29)$$

where $\tau = T$.

If we take maximum permissible error $\Delta\varphi = 4°$, then from expression (11.29) it follows that $\omega\tau_{\text{пред}} \leq 2.3$. Connection in series of identical cells with transfer function (11.28) leads to increase of permissible magnitude of $\omega\tau$:

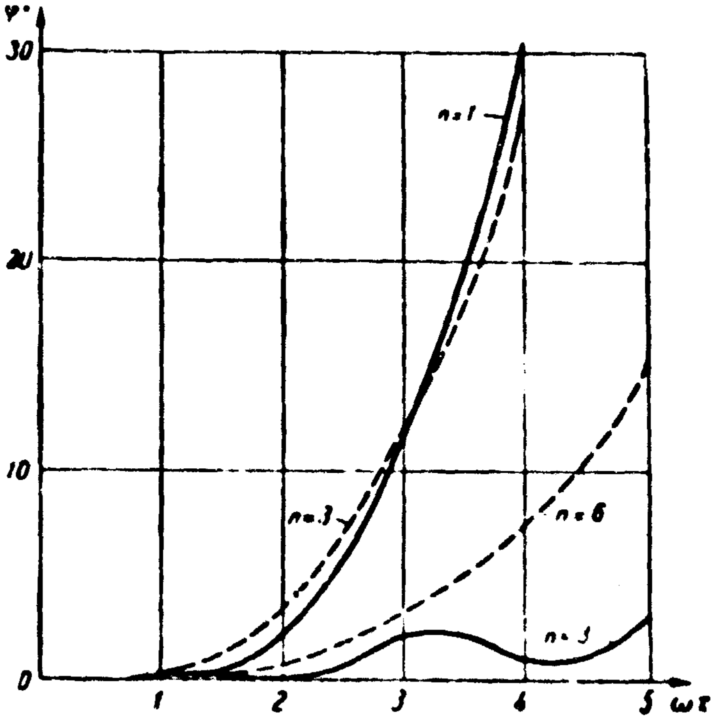

Fig. 204. Dependence of error on $\omega\tau$ for various types of sections.

In Fig. 203 is brought graph of dependence of number of sections n on $\omega\tau$ for various $\Delta\varphi$. From expression (11.26) and graphs of Fig. 203, calculated by this expression, it follows that when the number of cells $n = 6$ and permissible error $\Delta\varphi = 0.04$ rdn maximum permissible value of $(\omega\tau)_{np} = 2.5$. If one were to limit magnitude of resistances R and capacitances C of capacitors of the circuit to 1 megohm and 0.1 μ f, then maximum possible delay when $n = 6$ will be

$$\tau_{max} = 2n\,RC = 1.2 \text{ sec.}$$

Besides limited range of delay time application of passive circuits also has the deficiency that it does not allow us continuously to **change** delay time and requires during its setting variations of parameters of every link.

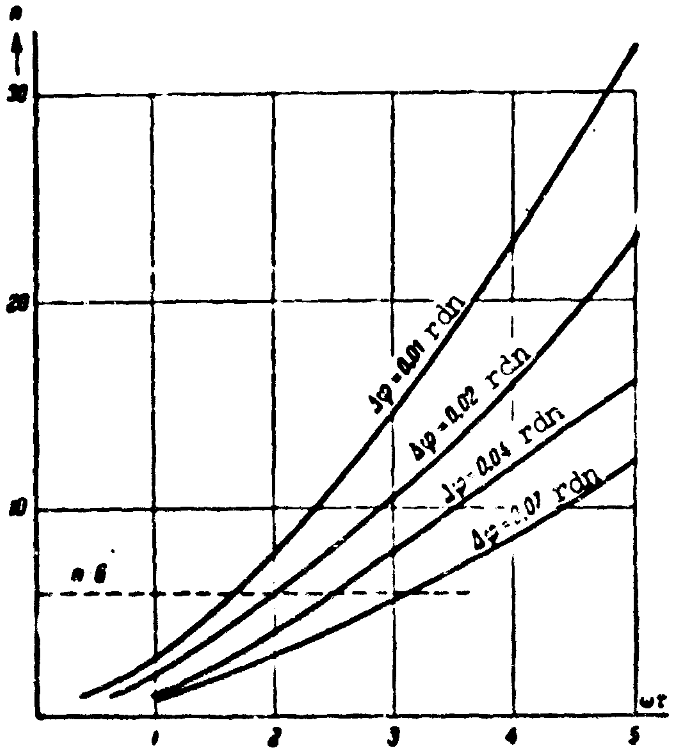

Fig. 203. Graph of dependence of number of sections on $\omega\tau$ with given phase error.

Application of active circuits has significant advantages in this respect. Let us consider one such circuit, reproducing the first two members of expansion of e^x into a Pade fraction series (see O. Perron [1]):

$$e^x = \lim_{(\mu+\nu)\to\infty} \frac{P_{\mu,\nu}(x)}{Q_{\mu,\nu}(x)}. \tag{11.27}$$

In Fig. 204 are shown comparative curves of [illegible] $= f(\omega\tau)$ with various n and various types of sections. Solid lines are curves for sections with transfer function of type (11.28), and dotted lines are for type (11.23). From the figure it follows that for values of $\omega\tau$ greater than 3, it is expedient to change to devices of the second type. Transfer function (11.28) can be realized both by passive (Fig. 202b) and active quadripoles. Application of active quadripoles allows us to simplify setting of required parameters and removes difficulties, connected with necessity in case of passive quadripoles to have inductance with high quality.

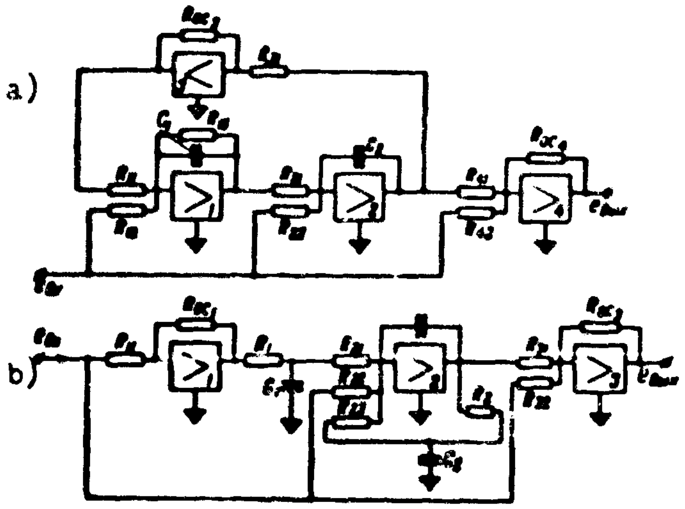

Fig. 205. Functional diagram of reproduction of delay.

Of special interest is application for these purposes of operational amplifiers (C. D. Morrill [1]). In Fig. 205 are shown two variants of such circuits. During composition of these circuits* it is more convenient to present the given transfer function (11.28) in the form

$$W(p) = 1 - \frac{12pT}{p^2T^2 + 6pT + 12}. \qquad (11.30)$$

The circuit shown in Fig. 205b is used when in the process of the experiment it is not required to change delay time. Amplifiers 1 and 3 here often can be taken from remaining set-up circuit, so that on creation of delay there is expended only one operational amplifier. Relationships for determination of parameters of these circuits are obtained for given value of τ by comparison of transfer functions of these circuits with expression (11.30). For the circuit of Fig. 205a we have:

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*According to methods of Ch. XII and Appendix I.

$$K_{22}=\frac{12}{\tau},\quad K_{13}=\frac{6}{\tau},\quad K_{21}K_{11}K_{31}=\frac{12}{\tau^2},\quad K_{21}K_{12}-K_{22}K_{11}.$$

Magnitude τ can be changed smoothly by setting voltage dividers and in steps—by change of magnitudes of capacities of capacitors C_1 and C_2. With physically possible values of transmission coefficients K_{ij} general range of time of delay constitutes from 0.05 to 100 sec. In Fig. 206 are shown oscillograms of work of circuit when $\tau=0.1$ and 0.90 sec. In initial period upon switching on the circuit there takes place the process of setting the voltage. Under conditions of modeling a CAP voltage at output of the delay block will scarcely change by jump and therefore process of setting up practically will not effect accuracy of simulating processes of control. If it is necessary to increase accuracy of reproduction of delay and to expand range of permissible values of $\omega\tau$, it is possible to use, for example, the first six terms of series (11.27), as C. D. Morrill recommends [1]. However for reproduction of such a transfer function there will be required 10 operational amplifiers.

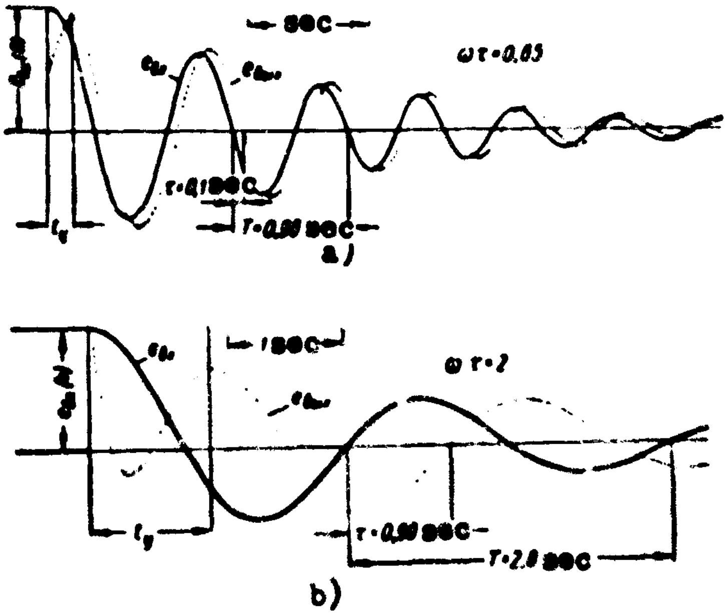

Fig. 206. Oscillograms of work of circuit of Fig. 205. t_y is setting up time.

A somewhat different approach to synthesis of transfer functions, with respect to location of their poles and zeroes, approximating the transfer function of the link with delay was developed by W. J. Cunningham [1].

Among devices, reproducing in principle accurately phase-frequency response and approximately gain-frequency are, delay units, based on use of capacitors and magnetic tape as memory units. Here the current values of the input magnitudes are continuously stored in the memory and continuously are selected from it after a given time delay.

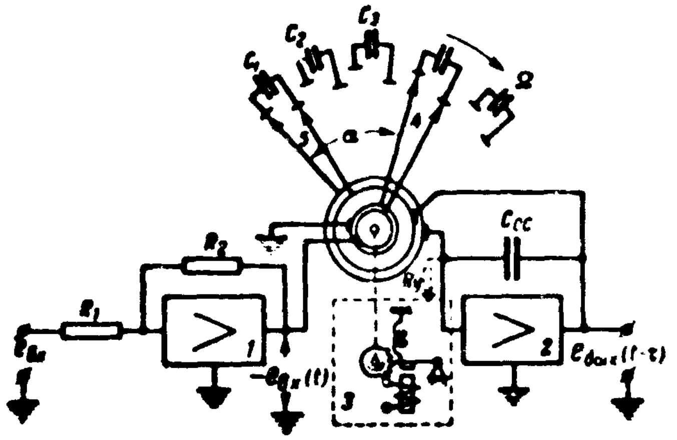

Fig. 207. Fundamental circuit of delay block with storage capacitors.

Principle of action of a device with remembering capacitors (V. V. Gurov [2]) consists of the following: for a definite, fairly small intervals of time output voltage of operational amplifier 1 (Fig. 207), representing input magnitude of the delay block is stored by charging capacitors C_1, C_2, . . . by charged brushes 4. Voltage of these capacitors after a time, equal to the time of delay, is transmitted by discharge brushes 5 to low-capacity capacitor C_{oc}, in the feedback circuit of output operational amplifier 2. Use of operational amplifiers here ensures practically instantaneous charge of capacitors C_1, C_2, . . . to value of voltage $e_{вх}$ due to the low value of the output impedance of amplifier 1. Discharge of capacitor C in the interval between two periods of charge, caused by external leaks R_y, will here be negligible, since the time constant of discharge is very great:

$$T = R_y C_{oc}(1 + K_y). \tag{11.31}$$

where K_y is the amplification factor of the operational amplifier with open negative feedback.

As the device, commutating capacitors C_1, C_2, . . . there are used stepping selectors or special collector commutators. In case of application of stepping selectors the brush contacts move, but in case of collector commutators—collectors move with constant speed.

Time delay τ is set by change of angular velocity Ω of rotation of motor 3 and angle α between brushes:

$$\tau = \frac{\alpha}{\Omega}. \qquad (11.32)$$

By force of finiteness of interval of time Δt between charge of two adjacent capacitors C_i and C_{i+1} in such device there is stored and reproduced with shift not a continuous curve $e_{\text{вх}}(t)$, but a stepped curve. This leads to error both in amplitude, and in time of delay of output magnitude. Error in delay time here does not exceed magnitude Δt, and error in amplitude can be estimated by the evident relationship

$$\Delta e_{\text{вх}} = \frac{de_{\text{вх}}}{dt} \Delta t, \qquad (11.33)$$

where $\Delta t = \frac{\alpha_1}{n\Omega}$, n is the number of storage capacitors, α_1 is the general angle, at which contacts of the commutator are located.

With the help of relationships (11.32) and (11.33) when $e_{\text{вх}} = e_{\text{вх}\,\max} \sin \omega t$ we find the maximum value of relative error in the form

$$(\delta e)_{\max} = \omega\tau \frac{\alpha_1}{\alpha_{\min} \cdot n}.$$

where $\alpha_{\min}$ is minimum gap between brushes.

When $\alpha_{\min} \cdot n = 10\alpha_1$

$$(\delta e)_{\max} = 0.1\omega\tau. \qquad (11.34)$$

From (11.34) it follows that for obtaining maximum error $(\delta e)_{\max} = 2\%$, magnitude $(\omega\tau)_{\max}$ must not be larger than 0.2. Besides error, caused by steepness of curve of output voltage, such a method of reproduction of delay introduces error due to finiteness of magnitude of capacity of capacitor C_{oc}. Absolute error can be estimated by expression

$$\Delta U_C = \frac{C_{oc}}{C + C_{oc}} \frac{de_{вх}}{dt} \Delta t.$$

Here C is the capacity of the storage capacitor C_i.

Using the earlier derived relationship for Δt and considering $e_{вх} = e_{вх\,max} \sin \omega t$, $(\Delta t)_{max} = \frac{0.1\tau}{n}$ we will receive the maximum value of relative error in the form

$$(\delta U_C)_{max} = 0.1 \frac{C_{oc}}{C + C_{oc}} \omega\tau. \qquad (11.35)$$

when $\omega\tau = 0.2$ and $(\delta U_C)_{max} = 0.25\%$, from (11.35) we obtain

$$C_{oc} \leq \frac{1}{7} C.$$

The fact that capacitor C_{oc} during switching on of storage copacitor is charged inaccurately up to voltage $(e_{вх})$, caused adjustment of the step-by-step variable signal not at once, but after several cycles of charge of capacitor C_{oc}. This as it were introduces parasitic inertial delay in the system. Usually capacitors C_i and C_{oc} for decrease of leak are selected as type KPG or PGS (with styroflex dielectric).

An original method of lowering error due to step nature of the output voltage curve was offered in the work of Ya. I. Grinya and P. N. Kopay-Gora [1]. This method consists of adding to each step of curve $e_{вых}(t)$ a linearly variable voltage. This voltage results from integration of difference of voltages on capacitors C_{oc}, obtained for the duration of two adjacent intervals of time t_i and t_{i+1}:

$$U_{пил} = \frac{1}{T} \int_{t_i}^{t_{i+1}} (U_{i+1} - U_i)\, dt = \frac{\Delta U_{i+1} \Delta t}{T}.$$

In order to receive linear smoothing of the stepped curve, it is necessary, that $U_{пил}(t_{i+1}) = \Delta U_{i+1}$. This is possible, if $T = \Delta t$. In case of application of integrating operational amplifier the given condition reduces to

$$RC = \Delta t.$$

Thus, with correct selection of transmission factor $\frac{1}{RC}$ of the integrating block summation of result of integration with current value of stepped output voltage gives output voltage of piecewise-linear form. We will estimate maximum value of relative error with such approximation. On basis of previously derived relationship (6.2) we have

$$(\Delta e_{\text{вых}})_{max} = \frac{1}{8}\left(\frac{d^2 e_{\text{вх}}}{dt^2}\right)_{max} \Delta t^2.$$

After substituting of value Δt when $e_{\text{вх}} = e_{\text{вх}\,max} \sin \omega t$ and Δt_{min} $\frac{\tau}{n} \cdot 10$ we obtain an expression for relative error:

$$(\delta e_{\text{вых}})_{max} = \frac{1}{800}(\omega\tau)^2.$$

When $(\delta e_{\text{вых}})_{max} = 2\%$ the permissible value of $\omega\tau \leq 4$. Thus, other conditions equal, introduction of piecewise-linear approximation as compared with stepped gives an increase of maximum permissible value of $\omega\tau$ by an order. However error from inaccurate transmission of voltage to capacitor $C_{\text{ж}}$ remains. If one were to select $C_{\text{ж}} \ll C$, then with preservation of low error δU_c it is possible to increase maximum permissible value of $\omega\tau$ to 4. Fundamental circuit of delay block, using considered principle of smoothing the stepped output voltage shown in Fig. 208. For preparation of circuit at the end of interval of time Δt for forming the following linear addition there is provided periodic discharge of capacitor C. By the given diagram delay blocks are industrially produced in these block commutation of two groups of 20 capacitors of type MGS of 0.1 microfarad is executed with the help of two stepping selectors (type ShI 25/8). Duration of interval can be fixed at 0.1 sec, 0.2 sec, 0.5 sec, and 1 sec, time lag τ—from 0.1 to 20 sec in steps. Accuracy of reproduction of output voltage constitutes ± 3% up to $(\omega\tau)_{max} \leq 5$.

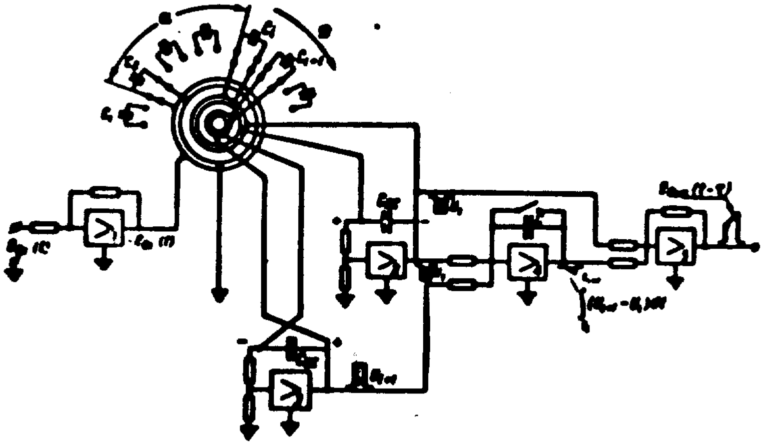

Fig. 208. Fundamental circuit of delay block with storage capacitors in which there is provided linear smoothing.

With the necessity of carrying out constant delay greater than 50 sec it is expedient to use principle of magnetic recording. In Fig. 209 is presented block-diagram of such an instrument (V. A. Ivanov [1]). Input signal, variable with low frequency, is modulated by Entrance signal variable with low frequency, is modulated in frequency by sweep oscillator of RC type. Magnitude of input signal can take value of ± 6 v; here the voltage frequency of oscillator changes linearly from 500 to 900 cps. Voltage of generator proceeds through power amplifier to recording head in which there is set the maximum value of amplitude of magnetizing field. The signal thus recorded on the magnetic tape, passing through a special device of "infinite cassette" type, induces in the head of reproduction with delay time a voltage, which moves to the input of the amplifier. Infinite cassette can contain from 0.5 to 250 m.

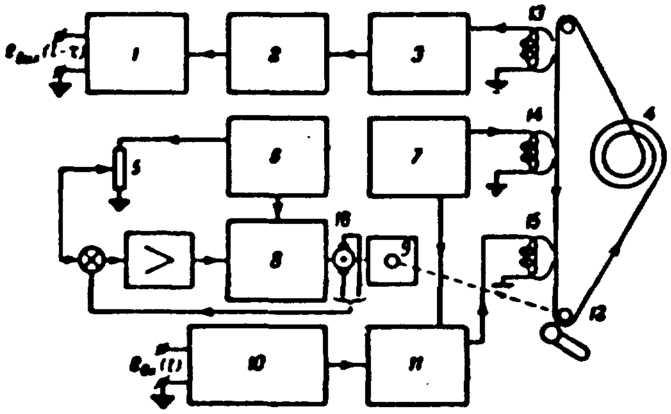

Fig. 209. Fundamental circuit of delay block with application of magnetic recording. 1—output device, 2—pulse counter, 3—amplifier with limitation, 4—infinite cassette, 5—speed setting, 6—stabilized rectifier, 7—high-frequency oscillator, 8—motor, 9—reductor, 10—generator of control pulses, 11—power amplifier, 12—leading shaft, 13—reproduction head, 14—erasing head, 15—recording head, 16—tachogenerator.

At the output of the amplifier-limiter we obtain a variable rectangular voltage tension variable in frequency, whose amplitude strictly is calibrated and does not depend on magnitude of emf induced in the reproduction head. After the amplifier-limiter the received square voltage pulses pass through a pulse counter, at whose output there is connected a capacitor, so that voltage on this capacitor is proportional to the number of pulses per unit time, i.e., frequency of recorded voltage or magnitude of input signal.

In Fig. 210a is brought the general form of delay block with application of magnetic recording as made by Academy of Sciences of USSR. In this block the belt-drive mechanism is moved by a d-c motor of type SL221. Speed of rotation changes by a change of excitation current. Fixed speed is kept constant by a system of automatic adjustment of speed. The delay block allows us to receive a delay time from 0.5 to 20 minute, and this range is covered smoothly. Magnitude of delay time is sustained with great stability (error near 0.1%). Error of reproduction of input signal does not exceed 1.5%. Maximum frequency of reproduced signals is 2-5 cps. In Fig. 210 (b, c, d) are brought several characteristic oscillograms, illustrating work of described delay block. On a delay block, made on magnetic drums, see also for K. W. Goff [2].

In Table IX are compared main technical characteristics of separate types of delay block.

Table IX

No. in Order	Type Delay Block	Limiting Values		Maximum Error	Note
		τ	$\omega\tau$		
1	**Block of 4 operational amplifiers by Fig. 201a diagram**	from 0.05 to 50 sec	3	± 2.5%	Error, phase, related to $(\omega\tau)_{max} = 3$
2	Block with application of storage capacitors and linear smoothing	from 0.1 to 20 sec	5	± 3%	Error, amplitude, related to $e_{max} = 100$ V when $\omega_{max} = 1$ cps.
3	Block with use of magnetic recording	from 0.5 sec to 20 min	from 12 to several thousand	± 1.5%	Error, amplitude, related to $\omega_{max} < 2-5$ cps.

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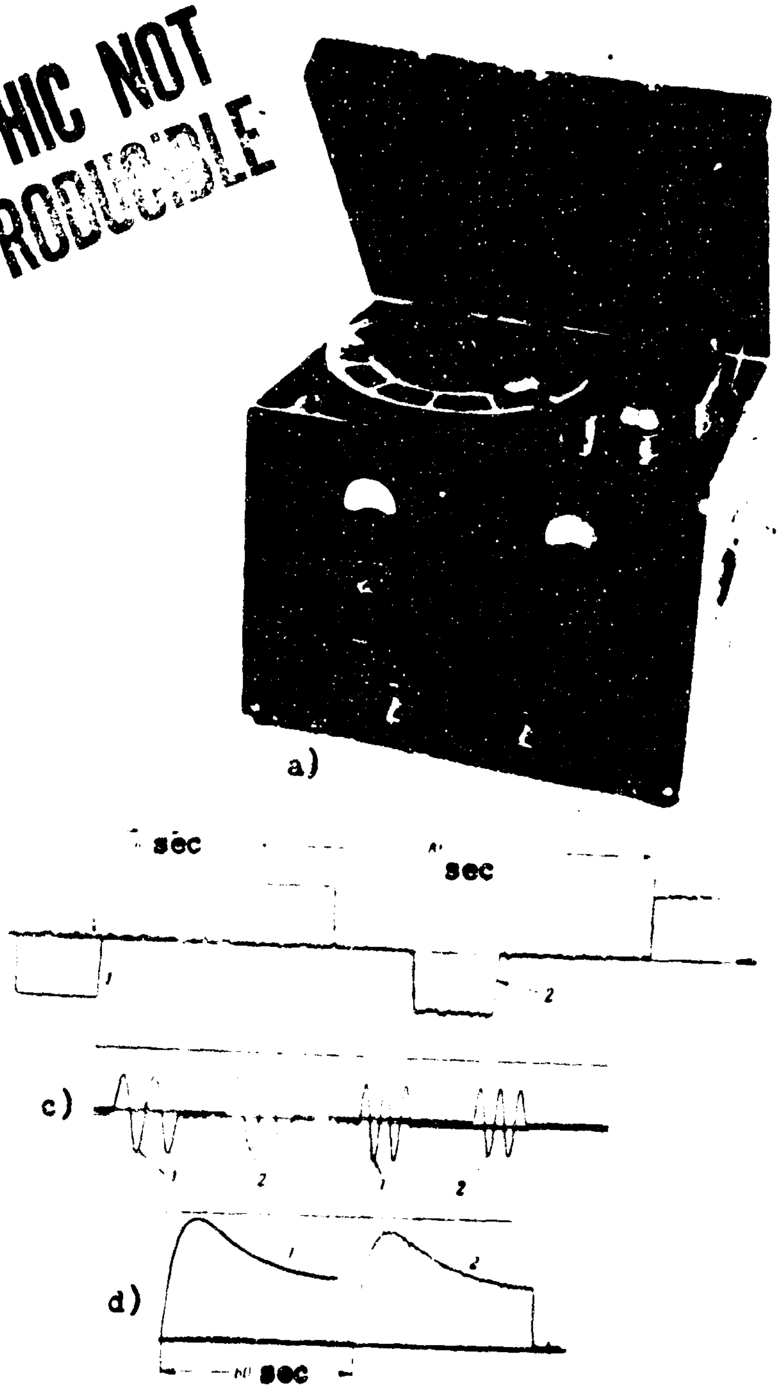

Fig. 210. a) general form of work of delay block with application of magnetic recording; b,c,d) oscillograms of shift with respect to time of rectangular, sinusoidal and aperiodic processes:
1—initial process, 2—delayed process.

If one were to consider that in systems of automatic control in practice $\omega\tau < \pi$, then from the data brought in the table it follows that with small time lags one can successfully use a circuit with operational amplifiers, and with large ones—delay blocks, based on magnetic recording. Delay blocks with storage capacitors as compared with circuits of amplifiers, other conditions being equal, are significantly more complicated, more expensive and less reliable.

5. Simulation of Systems of Automatic Control with Interferences

For a number of contemporary systems of automatic control (system of automatic racking of radar stations, etc.) of large significance is calculation of influence of interferences, penetrating the system simultaneously with useful signal. Problem of simulating of such CAP can be formulated in two ways. In one case the probmen is posed of more precise definition of parameters of system taking into account influence of interferences, introduced to model by reproduction of recordings of change of input signal, received by full-scale tests. In the other case there is studied influence on the process of adjustment of change of main statistical characteristics of interferences by addition to input signal of signal of interferences, produced by special generator with predetermined statistical characteristics. The last formulation of the problem usually occurs in the first stages of creation of the CAP, when it is possible to express only highly tentative assumptions about statistical characteristics of input signal.

With recordings of change of input signal, recieved in real conditions, reproduction is carried out variously depending upon nature of carrier of this recording. With use of magnetic recording one can successfully apply the delay block, described in the preceding section. With the help of this block it is possible to carry out both recording and reproduction of input signals. During reproduction of noises, registered on the tape, there is used a servo electronic-optical system, in many respects reminiscent of the cathode-ray functional generators considered in Chapter VII. Signal, controlling the position of the spot on the edge of the oscillogram during its movement before the screen of the cathode-ray tube, is used as the output signal of noises.

For generating noises in radio engineering they use ordinary electron tubes (most frequently diodes), fluctuations of whose current are amplified by multistage broad-band amplifiers. However for modeling CAP these generators are of little use, since during creation of noises in a narrow band of low frequencies (from 1 cps and lower) with power of the order of 0.1-1 w requires very great amplification. Replacement of vacuum tube by a gas-discharge tube (for example, thyratron) removes only partly the need to amplify the signal. For generating noises in the band of frequencies below 1 cps large advantages pertain to application of a ball generator (A. M. Petrovskiy [1], [2]) and a generator, made of voltage dividers (C. A. Wass [1]).

As can be seen from Fig. 211, he primary source of noises in a ball generator is a drum, filled with steel balls. Axis and lateral surface of drum are made of conducting, and the face covers are made from insulating material. To the axis and lateral surface of drum by slide contacts is applied a constant voltage. The drum is rotated by an electric motor through reductor with variable transmission ratio. Upon rotation of the drum the balls poured inside it create irregular contact between the wall of the drum and the axis, thanks to which current in the feed circuits continuously changes. The emf of interference is taken from a resistor, included in the feed circuit. As was shown in the work of A. M. Petrovskiy, in regions of low frequencies $(\omega < NF)$ the spectrum of frequencies of generated noises is practically uniform:

$$S(\omega) \simeq 2E_1^2 \bar{Y} \frac{1}{NF}. \qquad (11.36)$$

where $S(\omega)$ is the spectral density, E_1 is voltage of power source of drum with balls, $\bar{Y}$ is mean value of cylinder-axis conductance, N is the number of balls in the cylinder, F is angular velocity of rotation of drum in deg/sec.

Amplitude of output signal of such generator is subject to normal distributive law, since the instantaneous value of conductance Y_k is a function of a large number of independent random variables.

Number of balls in drum should be sufficiently great to ensure free fall of separate balls and sharp changes of conductance of drum. It has been practically fixed

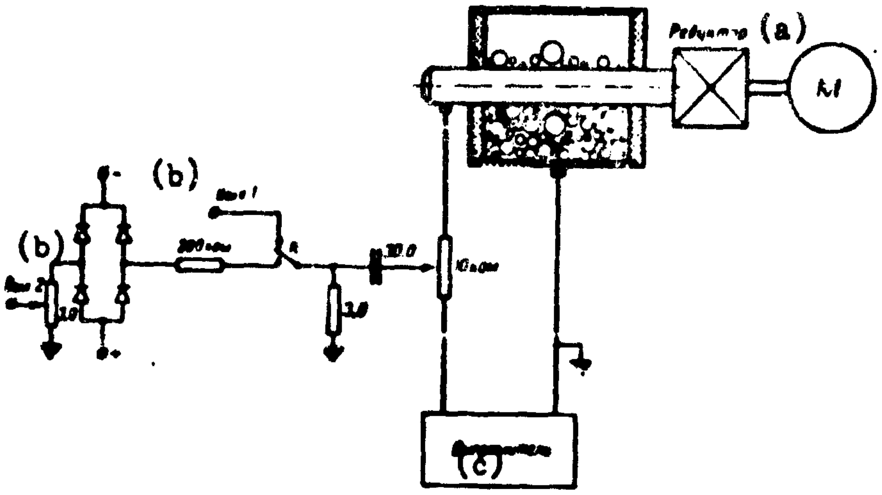

Fig. 211. Fundamental circuit of ball generator of noises.
KEY: (a) Reductor; (b) Output; (c) Rectifier.

that balls must occupy approximately half of the volume of the drum ($N \approx 50$ to 200). With decrease of angular velocity of rotation of drum power of noises in the frequency band, where $S(\omega)$ is constant, increases, however simultaneously there occurs narrowing of this band of frequencies. Narrowing of band of uniform spectrum of generated frequencies also results from application of limiters of amplitude at output of generator. With necessity of conversion of spectrum of frequencies of generated noises at output of generator there can be connected the corresponding filter. Usually the drum is fitted with balls of close diameters (from 0.5 to 4 mm); with filling by balls of strongly differing diameters fall of separate balls during rotation of the drum no longer is free, since one ball of large diameter draws after it a group of smaller ones. In this case fall of separate groups will be free which leads to appearance in output voltage of separate splashes. The latter can be successfully used in a number of cases of simulation.

Principle of construction of generator of fluctuating emf with use of switchable potentiometers is illustrated by the diagram of Fig. 212. Potentiometers will form a rectangular grid. Cursors of potentiometers, located in every vertical column, are joined to a common terminal $Ш_1$, $Ш_2$, . . . $Ш_n$, and these are joined to output

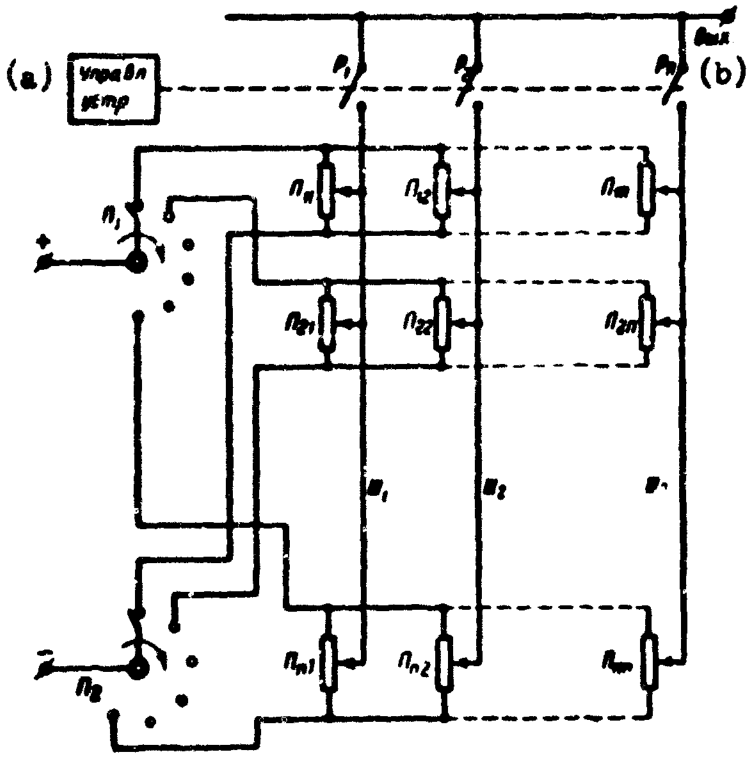

Fig. 212. Principle of construction of generator of fluctuating emf with use of switchable potentiometers.
KEY: (a) Control unit; (b) Output.

through contacts of relays $P_1, P_2, \ldots, P_n$. Relays are controlled from cyclical electronic meter through commutator. Distribution of amplitudes of output voltage is given by setting of cursors of potentiometers on the basis of experimental data or given characteristics of random process. Feed moves to potentiometers with the help of two switches $П_1$ and $П_2$. Strictly speaking, output voltage of such instrument cannot be called random. Possibility of repetition of signal from such generator is a known convenience, since it reduces the number of required solutions on models.

CHAPTER XII

INVESTIGATION OF DYNAMICS OF AUTOMATIC SYSTEMS WITH RATIONAL FRACTIONAL TRANSFER FUNCTIONS

In investigation of systems of automatic adjustment and control by electronic analog computers there often is the necessity of reproduction of rational fractional transfer functions of form

$$W(s)=\frac{b_m p^m+b_{m-1}p^{m-1}+\ldots+b_1 p+b_0}{a_n p^n+a_{n-1}p^{n-1}+\ldots+a_1 p+a_0}. \qquad (12.1)$$

where $m \leqslant n$; $b_m, b_{m-1}, \ldots, b_1, b_0$ and $a_n, a_{n-1}, a_{n-2}, \ldots, a_1, a_0$ are given constant coefficients, and p is a complex variable.

To such transfer functions leads, for example, approximate presentation of delay by Pade series, synthesis of correcting circuits, and also a number of problems of statistical dynamics of automatic control systems.

Application differentiators in setup circuits of these transfer functions is excluded due to sharp amplification of interferences, always attending output signal of operational amplifier; presentation of the right side as a given function of time is possible only in a very limited number of cases, when beforehand we know the law of change of input signal.

From literature (A. K. Ganulich [1], D. Michel [1], J. H. Laning and R. H. Battin [1], C. L. Johnson [1]) we know various methods of reproduction of these transfer functions with application of only integrators and summers. These methods can be reduced to four main ones:

1) direct integration;

2) decomposition of transfer function into partial fractions (method of transformation of structures);

3) expansion into first order equations;

4) combining derivatives.

These methods in literature were not compared and certain of them were mentioned only casually. Of interest also is establishment of the possibility of their propagation when coefficents of initial differential equations for (12.1) are given functions of time.

1. Method of Direct Integration

Let system of automatic control be described by equation of form

$$a_n y^{(n)} + a_{n-1} y^{(n-1)} + \dots + a_1 y^{(1)} + a_0 y = \\ = b_m x^{(m)} + b_{m-1} x^{(m-1)} + \dots + b_1 x^{(1)} + b_0 x \quad (n \geqslant m).$$

It is required to find y(t) for given constant coefficients a_i, b_i ($i = 1, 2, \dots, n$) and perturbation x, whose dependence on time is not given beforehand.

For finding rules of composition of the functional diagrams we solve the initial equation for highest order derivative $y^{(n)}$, if $n > m$, or when $n = m$ for the difference of highest order derivatives:

$$a_n y^{(n)} - b_n x^{(n)} = -(a_{n-1} y^{(n-1)} - b_{n-1} x^{(n-1)}) - \\ -(a_{n-2} y^{(n-2)} - b_{n-2} x^{(n-2)}) - \dots - (a_1 y^{(1)} - b_1 x^{(1)}) - (a_0 y - b_0 x).$$

Introducing the symbolic designation of operation of integration $\int dt = \frac{1}{p}$, $\int\int dt\, dt = \frac{1}{p^2}$, etc., we will receive

$$a_n y - b_n x = -\frac{1}{p}(a_{n-1} y - b_{n-1} x) - \frac{1}{p^2}(a_{n-2} y - b_{n-2} x) - \dots - \\ - \frac{1}{p^{n-1}}(a_1 y - b_1 x) - \frac{1}{p^n}(a_0 y - b_0 x). \qquad (12.2)$$

In order to preserve during set up initial values of coefficients a_i, b_i, we add to the left and right sides of expression (12.2) the term $-(a_n - 1)y$*). As a result we receive

$$y = -\frac{1}{p}(a_{n-1}y - b_{n-1}x) - \frac{1}{p^2}(a_{n-2}y - b_{n-2}x) - \dots - \frac{1}{p^{n-1}}(a_1 y - b_1 x) - \frac{1}{p^n}(a_0 y - b_0 x) + b_n x - (a_n - 1)y \qquad (12.3)$$

Expression (12.3) allows us directly to constitute functional setup diagram. Indeed, we designate in (12.3) the sum of terms, containing symbols of operation of integration by a new variable z_1, then we will receive equation

$$y = z_1 + b_n x - (a_n - 1)y. \qquad (12.4)$$

For further construction of the diagram we determine the value of the derivative of pz_1:

$$pz_1 = -(a_{n-1}y - b_{n-1}x) - \frac{1}{p}(a_{n-2}y - b_{n-2}x) - \dots - \frac{1}{p^{n-2}}(a_1 y - b_1 x) - \frac{1}{p^{n-1}}(a_0 y - b_0 x).$$

Introducing designation

$$z_2 = -\frac{1}{p}(a_{n-2}y - b_{n-2}x) - \dots - \frac{1}{p^{n-3}}(a_1 y - b_1 x) - \frac{1}{p^{n-2}}(a_0 y - b_0 x).$$

we receive

$$pz_1 = -(a_{n-1}y - b_{n-1}x) + z_2.$$

Continuing similar transformations, we will come to the generalized expression

$$pz_k = -(a_{n-k}y - b_{n-k}x) + z_{k+1} \quad (k = 1, 2, \dots, n), \; z_{n+1} = 0. \qquad (12.5)$$

Equations (12.4) and (12.5) lead directly to the setup diagram shown in Fig. 213. The total general number of required blocks when m = n here is n + 3. When m < n, obviously, it is possible to do without the output adder, and then number of blocks will be n + 2. The set up diagram for that case comes automatically from the one in Fig. 213 rejecting of all n — m — 1 couplings, which feed input signal x forward. Here naturally, there is the possibility to extract from the diagram derivatives of output coordinat y of order above n — m — 1. J. Matyas [1] first

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*It is necessary to turn attention to the fact that this method is useful to apply with sufficiently large $a_n > 0$. Otherwise one should preliminarily divide (12.2) by a_n.

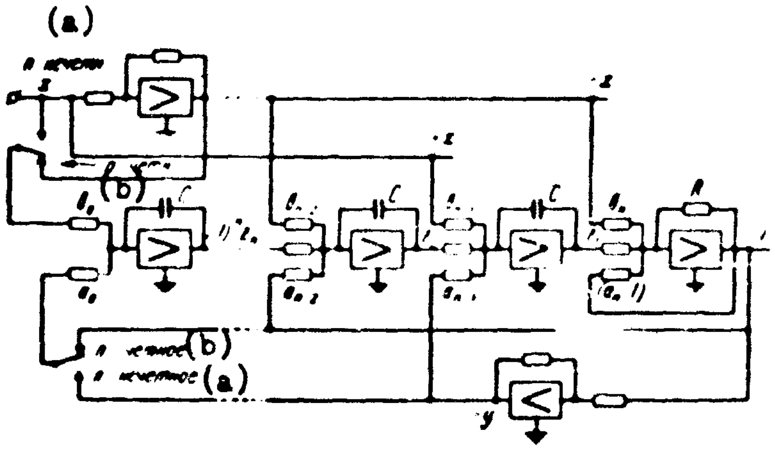

Fig. 21?. Setup diagram of differential equation of n-th order by method of direct integration.
KEY: (a) Odd; (b) Even.

indicated this possibility and formulated rules of composition of diagram for that particular case.

Initial conditions for integrators 1, 2 and n can be calculated by the given relationships for z_1 and z_k by generalized formula

$$z_k(0) = z_{n-j} = \sum_{1}^{n-j} [a_{i+j} y^{(i-1)}(0) - b_{i+j} x^{(i-1)}(0)]. \qquad (12.6)$$

where $j = n - k$, $i = 1, 2, 3, \ldots$

Main advantage of considered method is that setup is carried out with respect initial coefficients, and calculation of initial conditions is executed comparatively simply.

2. Decomposition of Transfer Function into Partial Fractions

x

This method is based on the fact that any transfer function W(p), where p is a complex variable, can be considered as a transfer function of a certain one-circuit system with negative feedback, which in the direct channel has transfer function $W_1(p)$, and in feedback circuits $W_2(p)$. On functions W_1 and W_2 are placed conditions, according to which the numerator of the first does not contain terms with p, and the numerator of the second can have a polynomial of p with degree, one smaller than the polynomial in the numerator of W(p).

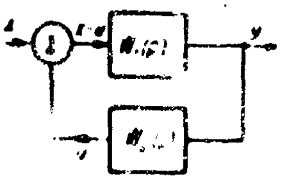

Fig. 214. Equivalent closed circuit of connection $W_1(p)$ and $W_2(p)$.

Indeed, let

$$W(p)=\frac{R(p)}{P(p)}=\frac{W(p)}{1+W_1(p)W_2(p)}. \qquad (12.7)$$

Functional diagram of connection of $W_1(p)$ and $W_2(p)$ is shown in Fig. 214. Let

$$W_1(p)=\frac{y(p)}{x(p)-u(p)}=\frac{1}{Q_1(p)}, \quad W_2(p)=\frac{u(p)}{y(p)}=\frac{Q_3(p)}{Q_2(p)}. \qquad (12.8)$$

We will find condition, which must be satisfied by $Q_1(p)$, $Q_2(p)$ and $Q_3(p)$, so that equality (12.7) is correct:

$$\frac{R(p)}{P(p)}=\frac{Q_2(p)}{Q_1(p)Q_2(p)+Q_3(p)}, \qquad (12.9)$$

whence

$$R(p)=Q_2(p), \quad P(p)=Q_1(p)Q_2(p)+Q_3(p). \qquad (12.10)$$

From condition (12.10) it follows that the degree of polynomial $Q_1(p)$ should be equal to the difference of degrees of polynomials P(p) and R(p), and from (12.8) and (12.9)—the degree of a polynomial $Q_3(p)$ should be one less than the degree of polynomial R(p). Using these peculiarities, it is possible to write the general form of polynomials $Q_1(p)$ and $Q_3(p)$:

$$Q_1(p)=\sum_{i=0}^{n-m} l_i p^i, \quad Q_3(p)=\sum_{i=0}^{m-1} m_i p^i, \qquad (12.11)$$

where l_i and m_i are constants to be determined.

For example, let $R(p) = b_0 + b_1p + b_2p^2 + b_3p^3$, and $P(p) = a_0 + a_1p + a_2p^2 + a_3p^3$. Then by (12.11) we obtain:

$$Q_1(p)=l_0, \quad Q_2(p)=b_0+b_1(p)+b_2p^2+b_3p^3,$$
$$Q_3(p)=m_0+m_1p+m_2p^2.$$

To determine l_0, m_0, m_1, and m_2 we substitute the found values of Q_1, Q_2 and Q_3 in (12.10). As a result we receive

$$a_0+a_1p+a_2p^2+p^3=l_0b_3p^3+(l_0b_2+m_2)p^2+(l_0b_1+m_1)p+l_0b_0+m_0,$$

whence the sought for coefficients will be:

$$l_0=\frac{1}{b_3}, \quad m_0=a_0-\frac{b_0}{b_3}, \quad m_1=a_1-\frac{b_1}{b_3}, \quad m_2=a_2-\frac{b_2}{b_3}.$$

Thus,

$$W_1(p)=\frac{1}{l_0}, \quad W_2(p)=\frac{m_0+m_1p+m_2p^2}{b_0+b_1p+b_2p^2+b_3p^3}.$$

$W_2(p)$ in turn can be presented

$$W_2(p)=\frac{W_{11}(p)}{1+W_{11}(p)W_{22}(p)}.$$

where

$$W_{11}(p) = \frac{1}{n_0 + n_1 p}, \quad W_{22}(p) = \frac{c_0 + c_1 p}{m_0 + m_1 p + m_2 p^3}.$$

For determination of unknown coefficients we will use on the basis of (12.10) the relationship

$$b_0 + b_1 p + b_2 p^2 + b_3 p^3 = (n_0 + n_1 p)(m_0 + m_1 p + m_2 p^2) + (c_0 + c_1 p).$$

Equating coefficients, we receive:

$$c_0 = b_0 - \left(b_2 - b_3 \frac{m_1}{m_2}\right) \frac{m_1}{m_2}, \quad c_1 = b_1 - \left[b_3 \frac{m_0}{m_2} + \frac{m_1}{m_2}\left(b_2 - b_3 \frac{m_1}{m_2}\right)\right].$$

$$n_0 = \frac{b_2 - b_3 \frac{m_1}{m_2}}{m_2}, \quad n_1 = \frac{b_3}{m_2}.$$

We will decompose $W_{22}(p)$ into partial components:

$$W'_{22}(p) = \frac{W'_{111}(p)}{1 + W'_{111}(p)\, W'_{222}(p)}, \quad W'_{111}(p) = \frac{1}{q_0 + q_1 p}.$$

$$W'_{222}(p) = \frac{r_0}{c_0 + c_1 p}.$$

Coefficients r and q we will determine from relationship

$$m_0 + m_1 p + m_2 p^2 =$$
$$= (q_0 + q_1 p)(c_0 + c_1 p) + r_0,$$

whence

$$q_1 = \frac{m_2}{c_1}, \quad q_0 = \left(m_1 - \frac{c_0}{c_1} m_2\right) \frac{1}{c_1}.$$

$$r_0 = m_0 - \left(m_1 - m_2 \frac{c_0}{c_1}\right) \frac{c_0}{c_1}.$$

Thus, the problem was reduced to setup of the following system of equations:

$$y = \frac{1}{l_0}(x - u).$$
$$(n_0 + n_1 p)\, u = y - u_1.$$
$$(q_0 + q_1 p)\, u_1 = u - u_2.$$
$$(c_0 + c_1 p)\, u_2 = r_0 u_1.$$

Functional and setup diagrams, corresponding to this system of equations, are shown in Figs. 215 and 216. Initial conditions with respect to new variables u, u_1, and u_2 can be found by the given initial conditions by the above mentioned system of equations.

As follows from Fig. 216, such a method of setup requires a large number of inverters as compared to method of direct integration. Furthermore, it is necessary to expend comparatively greater time on calculation of coefficients of transformed equations.

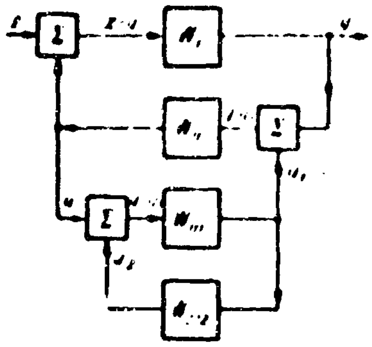

Fig. 215. Functional diagram, explaining method of decomposition of transfer function into partial fractions.

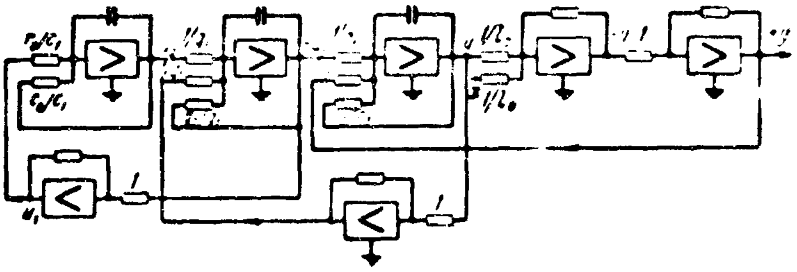

Fig. 216. Setup diagram, using method of decomposition of transfer function into partial fractions.

3. Expansion of Initial Inhomogeneous n-th Order Equation into a System of n Inhomogeneous First Order Equations

This expansion is not unique, however the best result is yielded by the method, described by **J. H. Laning and R. H. Battin** [1], according to which a linear differential equation with constant coefficients

$$\frac{d^n y}{dt^n} + a_{n-1}\frac{d^{n-1}y}{dt^{n-1}} + \dots + a_1\frac{dy}{dt} + a_0 y = \tag{12.12}$$

$$= b_0 x + b_1\frac{dx}{dt} + \dots + b_{n-1}\frac{d^{n-1}x}{dt^{n-1}} + b_n\frac{d^n x}{dt^n}$$

can be presented in the form of a system of linear first order differential equations:

$$\left.\begin{aligned} y &= y_1 + \alpha_n x, \\ \frac{dy_1}{dt} &= y_2 + \alpha_{n-1} x, \\ &\dots\dots\dots\dots \\ \frac{dy_{n-1}}{dt} &= y_n + \alpha_1 x, \\ \frac{dy_n}{dt} &= -a_{n-1} y_n - a_{n-2} y_{n-1} - \dots - a_1 y_2 - a_0 y_1 + \alpha_0 x. \end{aligned}\right\} \tag{12.13}$$

Indeed, excluding $y_1, y_2, \ldots, y_n$ from the last equation of system (12.13), we arrive at an equation of the form

$$\begin{aligned} \frac{d^n y}{dt^n} + a_{n-1}\frac{d^{n-1} y}{dt^{n-1}} + \dots + a_1 \frac{dy}{dt} + a_0 y = \alpha_n \frac{d^n x}{dt^n} + \\ + (\alpha_{n-1} + \alpha_n a_{n-1}) \frac{d^{n-1} x}{dt^{n-1}} + \dots + (\alpha_1 + a_{n-1}\alpha_2 + \dots + a_1 \alpha_n) \frac{dx}{dt} + \\ + (\alpha_0 + \alpha_1 a_{n-1} + \dots + \alpha_{n-1} a_1 + \alpha_n a_0) x. \end{aligned} \tag{12.14}$$

So that equations (12.13) and (12.14) were identical, it is necessary, that values of new coefficients α_i satisfy the following equalities:

$$\left.\begin{aligned} b_0 &= \alpha_0 + \alpha_1 a_{n-1} + \alpha_2 a_{n-2} + \dots + \alpha_{n-1} a_1 + \alpha_n a_0, \\ b_1 &= \alpha_1 \cdot 1 + \alpha_2 a_{n-1} + \dots + \alpha_{n-1} a_2 + \alpha_n a_1, \\ b_2 &= \alpha_2 \cdot 1 + \dots + \alpha_{n-1} a_3 + \alpha_n a_2, \\ &\dots\dots\dots\dots\dots\dots\dots\dots \\ b_{n-1} &= \alpha_{n-1} \cdot 1 + \alpha_n a_{n-1}, \\ b_n &= \alpha_n \cdot 1. \end{aligned}\right\} \tag{12.15}$$

The functional circuit is constructed for system of equations (12.13) in the form shown in Fig. 217. The total number of required blocks is n + 3. Coefficients $\alpha_0, \alpha_1, \ldots, \alpha_n$ easily are calculated on the basis of (12.15) by consecutive substitution of values α_i, starting with $\alpha_n = b_n$. Initial conditions with respect to new variables $y_1, y_2, \ldots, y_n$ are determined by system of equations (12.13) by given

$y(0), y^{(1)}(0), \ldots, y^{(n-1)}(0)$ and $x(0), x^{(1)}(0), \ldots, x^{(n-1)}(0)$.

As an example let us consider reproduction of transfer function

$$W(p) = \frac{p^2 - \frac{6}{\tau}p + \frac{12}{\tau^2}}{p^2 + \frac{6}{\tau}p + \frac{12}{\tau^2}},$$

approximating transfer function of a delay link $e^{-p\tau}$ by a Pade series where $\mu = \nu = 2$. Here $n = 2$, $a_0 = \frac{12}{\tau^2}$, $a_1 = \frac{6}{\tau}$, $a_2 = 1$, $b_0 = \frac{12}{\tau^2}$, $b_1 = -\frac{6}{\tau}$, $b_2 = 1$.

Equivalent system of equations on basis of (12.13) will be written in the form (zero initial conditions are assumed)

$$\frac{dy_2}{dt} = -a_1 y_2 - a_0 y_1 + \alpha_0 x, \quad \frac{dy_1}{dt} = y_2 + \alpha_1 x, \quad y = y_1 + \alpha_2 x,$$

where y corresponds to output signal $e_{\text{вых}}$ and x to input $e_{\text{вх}}$. Coefficients α_0, α_1 and α_2 on basis of preceding will be

$$\alpha_2 = b_2 = 1, \quad \alpha_1 = b_1 - a_1 = -\frac{12}{\tau}, \quad \alpha_0 = \frac{72}{\tau^2}.$$

Setup diagram is shown in Fig. a.

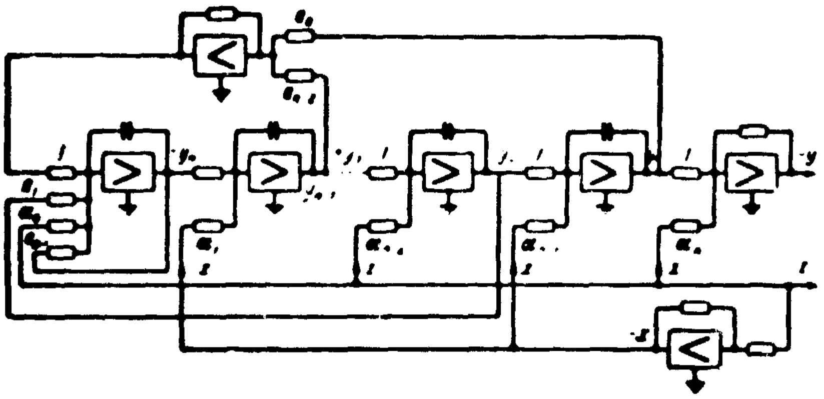

Fig. 217. Setup diagram of n-th order differential equation, composed by method of expansion of initial inhomogeneous equation into n inhomogeneous first order differential equations.

4. Method of Combining Derivatives

We divide initial equation (12.12) in two, introducing new variable

$$u = \frac{x}{p^n + a_{n-1}p^{n-1} + \dots + a_1 p + a_0}. \tag{12.16}$$

As a result we obtain

$$y = b_n \frac{d^n u}{dt^n} + b_{n-1} \frac{d^{n-1} u}{dt^{n-1}} + \dots + b_1 \frac{du}{dt} + b_0 u. \tag{12.17}$$

Expression (12.16) can be rewritten in differential form:

$$\frac{d^n u}{dt^n} + a_{n-1} \frac{d^{n-1} u}{dt^{n-1}} + \dots + a_1 \frac{du}{dt} + a_0 u = x. \tag{12.18}$$

For composition of functional setup diagram it is necessary at first to "setup" equation (12.18) by method of lowering the order of the derivative, and then form the sought for variable y in the form of the sum of derivatives of u with corresponding coefficients. Values of derivatives $\frac{d^i u}{dt^i}$ are obtained directly from corresponding outputs of integrators during solution of equation (12.18). Certain simplification of the setup circuit can result if in equation (12.17) we exclude

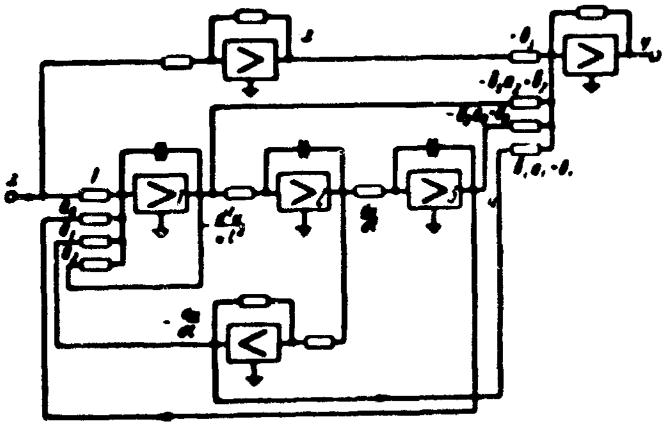

Fig. 218. Diagram of setup of differential equation, composed by method of combining derivatives.

$\frac{d^n u}{dt^n}$ by substitution of its value from equation (12.18). As a result we pass to equations

$$\frac{d^n u}{dt^n} + a_{n-1}\frac{d^{n-1}u}{dt^{n-1}} + \dots + a_1\frac{du}{dt} + a_0 u = x.$$

$$y = (-b_n a_{n-1} + b_{n-1})\frac{d^{n-1}u}{dt^{n-1}} + \dots + (-b_n a_1 + b_1)\frac{du}{dt} + (-b_n a_0 + b_0)u + b_n x.$$

The functional setup diagram by these equations for m = n = 3 is shown in Fig. 218. In general for setup it is necessary to have n + 3 computing blocks. For determination of coefficients during setup there is not required fulfillment of labor-consuming calculations.

5. Methods of Simulation of Differential Equations with Variable Coefficients

Methods of simulation of differential equations with variable coefficients are considered sufficiently enough by J. H. Laning and R. H. Battin [1] and J. Matyas [3]. Therefore we will limit ourselves here only to brief remarks.

Composition of functional setup diagram in case of solution of equation (12.12) with coefficients variable in time can be done by direct integration or method of transition to an equivalent system of first order differential equations.* Main distinction of these methods consists in method of determination of new variable coefficients for equations to be set up. Indeed, for setup by method of direct integration we replace equation (12.12) by equivalent equation

$$\sum_{i=0}^{n}(-1)^{i}(\alpha_i y)^{(i)}=\sum_{i=0}^{m}(-1)^{i}(\beta_i x)^{(i)}. \qquad (12.19)$$

where y,x are the same variable as in (12.12) and $\alpha_i(t)$, $\beta_i(t)$ are new functions of time.

Using properties of adjoint linear operators, Matyas [3] showed that new variable coefficients should be coupling with the following old relationships:

$$\beta_{m-k}=\sum_{i=0}^{k}(-1)^{(m-i)}\frac{(m-i)!}{(m-k)!\,(k-i)!}b_{m-i}^{(k-i)} \qquad (k=0,1,2,\ldots,m),$$

$$\alpha_{n-k}=\sum_{i=0}^{k}(-1)^{(n-i)}\frac{(n-i)!}{(n-k)!\,(k-i)!}a_{n-i}^{(k-i)} \qquad (k=0,1,2,\ldots,n).$$

Functional setup diagram of equation (12.19) is obrained by method, mentioned above for differential equation (12.12) with constant coefficients with only this difference, that in corresponding places there are connected dividers of variable coefficients for setting $\beta_{m-k}(t)$ and $\alpha_{n-k}(t)$.

Transition to an equivalent system of first order differential equations is executed just as in the case of constant coefficients, but now new coefficients $\alpha_n(t), \alpha_{n-1}(t), \ldots, \alpha_0(t)$ must be considered certain functions of time. Determination of these functions of time by initial variable coefficients can be done on the basis of the work of J. H. Laning and R. H. Battin [1] by recurrence formula:

$$a_0(t)=b_0(t),$$

$$a_i(t)=b_i(t)\sum_{k=0}^{i-1}\sum_{r=0}^{i-k}C_{n-k-r}^{i-k-r}\,a_{i-k-r}(t)\frac{d^r v_k(t)}{dt^r}.$$

From comparison of the two considered methods it follows that transition to equivalent system of first order differential equations requires fulfillment of less calculating work, since in setup there participate all initial variable coefficients $a_j(t)$ in unconverted form. During solution of the problem by the method of direct integration it is necessary anew to calculate all variable coefficients in the equations to be set up.

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*Methods of decomposition of transfer function into partial fractions and combining of derivatives turn out to be invalid, since they lead to change of places of differential operators which is impermissible with variable coefficients.

Comparison of considered methods of simulation of a rational fractional transfer functions allows us to draw the following conclusions:

1. Reproduction of rational fractional transfer functions and initial differential equations for them with the help of electronic models without differentiating elements is possible to carry out by several methods. With non-zero initial conditions there must also be known values of perturbation and its n — 1 derivatives at the initial moment of time.

2. Minimum number of operational amplifiers in functional setup circuit in general case constitutes n + 3, where n is the order of setup differential equation and does not depend on method of setup. An exception is the method of decomposition of transfer function into partial fractions, leading to functional circuits with a large number of operational amplifiers.

3. Simplest from the viewpoint of volume of required preparatory work is method of combining derivatives. This method is applicable only in problems with constant coefficients.

4. During resolution of problems with variable coefficients one should give preference to method of transition to equivalent system of first order differential equations, requiring a minimum of auxiliary calculations.

CHAPTER XIII

SIMULATION OF NONLINEAR SYSTEMS OF AUTOMATIC CONTROL.

Necessity of solution of nonlinear problem appears every time it is necessary to consider behavior of system of automatic control with output beyond the limits of small delfections of regulated magnitude, to consider limited power of object and actuating mechanism of regulator, limitation of certain coordinates of system, and also a number of pecularities, accompanying physical realization of system (dry friction, zone of insensitivity, gap in transmissions, etc.). Often nonlinear connections are introduced in system to achieve optimum processes of adjustment.

In all enumerated cases it is necessary not only to develop stability, but also to select structure and parameters of systems, ensuring given character of flow of process of adjustment for all possible perturbations in system.

To avoid errors* during simulation of nonlinear ACS one should turn special attention to correctness of recording of differential equations. In connection with this it is expedient to distinguish reproduction of nonlinear dependences in electronic and inertial elements of control systems. As was already mentioned in Chapter V nonlinearities, met in systems of automatic control, can be divided into typical ones, those leading to elementary functions (sin x, cos x, xy, x/y, etc.), and artitary ones, obtained from experiment.

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*Besides author these same errors were indicated by G. I. Monastyrshin [1] who offered to replace verbal extension of definition and graphs by introduction of functions sign.

If for simulation of arbitrary nonlinearities and nonlinearities which lead to elementary functions, we need special nonlinear blocks (See Chapter V, VI, VII and VIII), then during simulation of typical nonlinearities we can use operational amplifiers in combination with diode limiters or electromechanical relays.

Typical nonlinearities are what we usually call nonlinearities, connected with intermittent changes of transmission factor of separate sections, appearing at one or another value of input or output magnitude. Such intermittent change of transmission factor usually results from presence of zone of insensitivity, dry friction, gaps in transmissions, relay characteristics, limitation of coordinates, speeds and accelerations in nodulus, loop hysteresis in elements of regulator and controlled process*.

1. Simulation of System of Automatic Control with Typical Nonlinear Characteristics in Electronic Elements

Limitation of coordinates in modulus. In real systems of automatic control for constructive considerations and due to power limitations usually the range of change of coordinates is limited.

During simulation of such systems of electronic integrator it is necessary that output voltage of one or another operational amplifier, representing the coordinate interesting us, after reaching a certain predetermined value does not change further. This can be realised by connecting a diode limiter in the feedback circuit or at the output of operational amplifier. In Fig. 219a is depicted one possible scheme of connecting a diode limiter in feedback circuit of operational amplifier. At low values of input voltage diodes $Д_1$ and $Д_2$ are locked by voltage + E and —E from outside source and operational amplifier when $R_1 = R_2$ has transmission factor $K = 1$.

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*Questions of simulating typical nonlinearities already were touched upon in in literature by various authors (See T. N. Sokolov [1], A. A. Fel'dbaum [3], C. D. Morrill and R. F. Baum [2], B. Ya. Kogan [4], C. A. Meneley [1] and others).

When under the influence of increasing input voltage, output voltage attains absolute magnitude value E, one diode will unlock, and now transmission factor of amplifier will be

$$K = \frac{\left(1 + \frac{r_0}{r + r_1 + r_{ik}}\right) r_1 + r_1}{R_2 + r_1\left(1 + \frac{r_1}{r + r_0 + r_{ik}}\right) + r_1} \cdot \frac{R_2}{R_1} \qquad (13.1)$$

When $R_1 = R_2 = 1$ megohm, $r_1 = 500$ om, $r_0 = 5000$ om, $r = 33$ kilohm, $r_{ik} = 10$ kilohm, we will take $K = \frac{5657}{1001000} = 5.65 \cdot 10^{-3}$.

It is obvious that with such a small transmission factor of block it is possible with accuracy sufficient for practice to consider magnitude on its output constant during change of input magnitude.

Changing magnitude of resistance R_2, it is possible to affect steepness of change of output voltage of block in the interval between the boundaries of limitation. Especially important is the case where $R_2 \to \infty$. Here, by in force of the very large gain factor of amplifier it is possible to consider that output voltage reaches its limit with the slightest change of input voltage (Fig. 219c). Such a block reproduces z-form nonlinear characteristics.

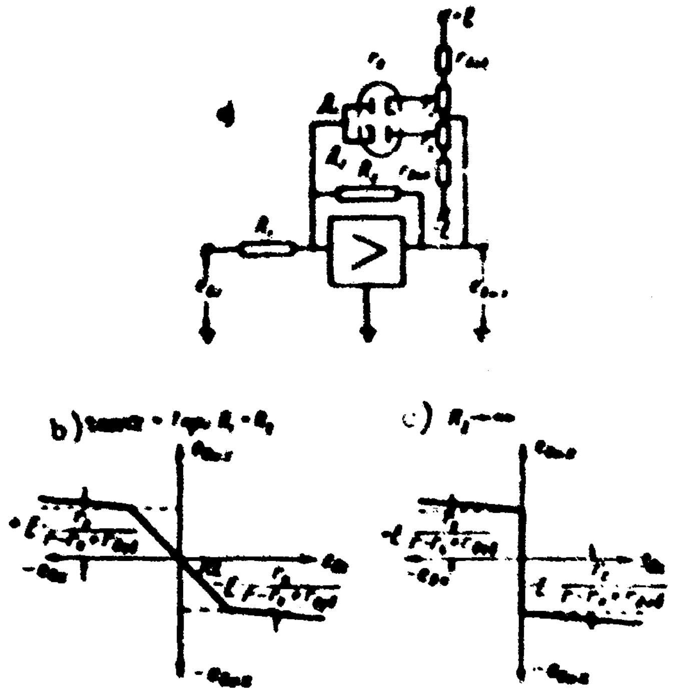

Fig. 219. Operational amplifier with diode limiter in feedback circuit.

Diagram of connection of limiter to output of operational amplifier (See R. I. Medkeff and R. I. Parent [1]) is shown in Fig. 220a. At low output voltage of block $e^{*}_{вых}$ diodes $Д_1, Д_2, Д_3$ and $Д_4$ pass current, and difference of potentials between points a and b is equal to zero. Therefor $e_{вых} = e^{*}_{вых}$. Depending upon magnitude of E and r for definite value of $e_{вых} = e_{вых.гран} = E\frac{R_{вых}}{R_{вых}+r}$ diodes $Д_1$ and then $Д_4$ lock, and when $e^{*}_{вых.гран} < 0$ diodes $Д_2$ and $Д_3$ lock, as a result of which voltage on output resistor with further increase of $e^{*}_{вых}$ remains unchanged and equal to $E\frac{R_{вых}}{R_{вых}+r}$. Such limiter as compared with considered one has only the advantage that it gives more accurate cutoff of output voltage. However, it has significantly higher output impedance, increasing with growth of voltage of limitation, and it requires twice the number of diodes.

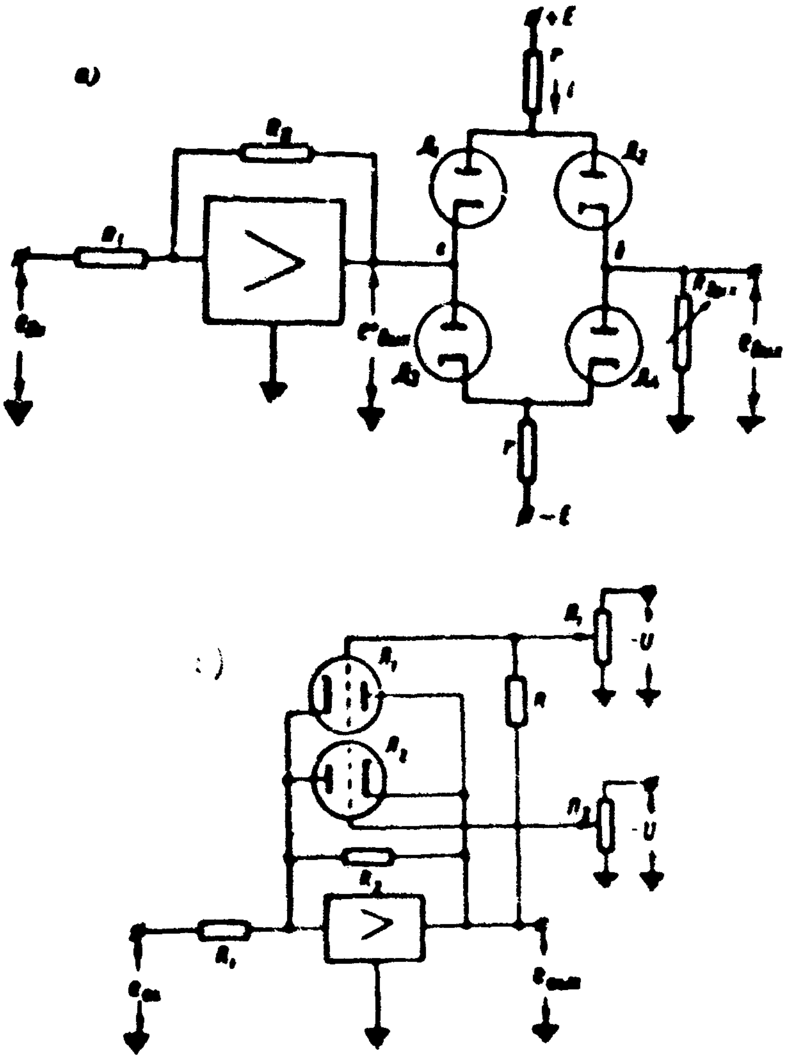

Fig. 220. Operational amplifier with diode limiter at output.

Accurate setting of voltage of limitation is also given by the circuit of a triode limiter (Fig. 220b), connected in the feedback circuit of the operational amplifier. Here triodes in zone of linear change of output voltage remain locked due to negative voltage between grid and cathode.

Slope of characteristic of circuit when diodes are conducting turns out to be smaller than for diodes due to removal of resistance of divisor r_0 and decrease of internal resistance of triode with zero grid potential.

Reproduction of zone of insensitivity. If diode limiter, depicted in Fig. 219a, is connected in series with input impedance of operational amplifier, then transmission factor of such block will change just as transmission factor of the section possessing zone of insensitivity changes (Fig. 211a). Combination of two diode limiter, one, in series with input impedance and the other, parallel to feedback impedance of operational amplifier, allows us to reproduce static characteristics of a section of a system of automatic control, possessing zones of insensitivity with simultaneous limitation of output magnitude in modulus.

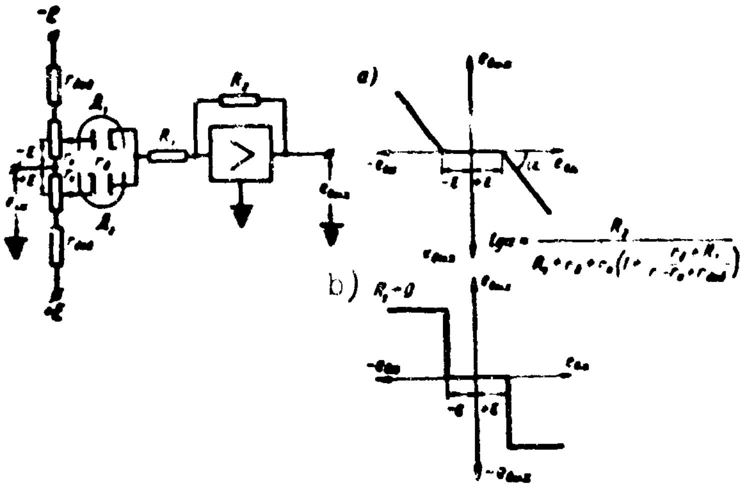

Fig. 221. Scheme for simulating a zone of insensitivity.

If in scheme of Fig. 221 we change magnitude of resistance R_1, then it is possible to change steepness of voltage build-up at output and when $R_1 \to 0$ we can get practically instantaneous build-up. With limitation of output voltage of subsequent block in Fig. 209a or the very same block in Fig. 220 it is possible to reproduce characteristic of a relay with restoration coefficient, equal to one (Fig. 221b).

Simulation of transmission gaps. For reproduction of static characteristics of a section containing a gap in kinematic circuit, there should be routed components, reproducing the area of insensitivity, and also devices keeping constant the value of output coordinate with change of direction of motion of driving component until the whole gap of the kinematic circuit is taken out. Upon such consideration they usually assume that the driven component does not possess a moment of inertia, but is under influence of small moment of griction, as a consequence of which it keeps its position, antedating change of direction of motion of master component.

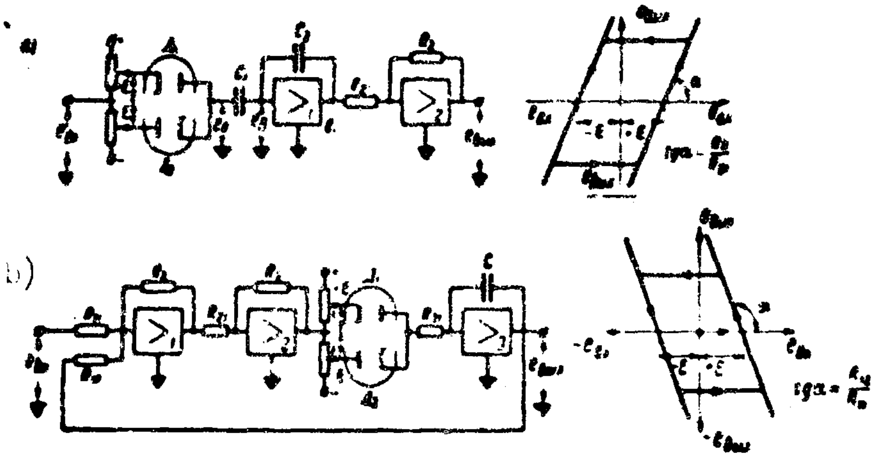

Fig. 222. Simulating gaps in transmissions.

In the scheme of Fig. 222a as the memory component there is used an operational amplifier with capacitor in feedback circuits and at input. Transmission of such decisive amplifier will be:

$$K = -\frac{C_1}{C_2}. \tag{13.2}$$

When we disconnect the input circuit output voltage keeps its previous value thanks to very slow discharge of capacitor C_2. Capacitor C_1 with accuracy up to e_δ always is charged to voltage $e_{вх} - \varepsilon$ and therefore automatically, ensures secondary switching on of circuit with decrease of input voltage by a magnitude, equal to 2ε.

Scheme of Fig. 222b is built on principle of servo system. Input magnitude here is compared with output on first operational amplifier. In the strengthening errors amplification channel are connected in series a diode limiter, switching on the circuit only when input magnitude exceeds output by magnitude $\pm\varepsilon$, and an integrating operational amplifier with a large amplification factor, at whose output voltage is preserved during breaking of the channel of error amplification by diode limiter. In number of operational amplifiers the diagram of Fig. 22a is more economical. To decrease error of first diagram due to resistance, introduced by diode limiter to input of first operational amplifier, one should decrease capacitance of capacitors utilised in this circuit.

With help of these diagrams it is possible also approximately to reproduce characteristics of steady-state mode of a section, possessing magnetic hysteresis.*

Simulation of static relay characteristics. A scheme for reproduction of static relay characteristics taking inot account restoration coefficient shown in Fig. 223a (B. Ya. Kogan [4]). It consists of two operational amplifiers 1 and 2 with limiters in feedback circuit, changing like relays output voltage with change of sign of input voltage, and one summing amplifier 3. In absence of input signal due to constant voltages $+U_0$ and $-U_0$ fed to inputs of amplifiers 1 and 2, on output of summing amplifier voltage turns out to be equal to zero. When with growth of input voltage polarity of total voltage changes, for example at input of amplifier 1, then on output of summing amplifier 3 there will appear with a jump voltage $2E$. Part of this voltage, taken from the divider, moves in the form of positive feedback to input of amplifiers 1 and 2. Therefore, with decrease of input signal

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*For scheme of modeling of family of static hysteresis loops see article of V. G. Vasil'yev, V. A. Zverev [1].

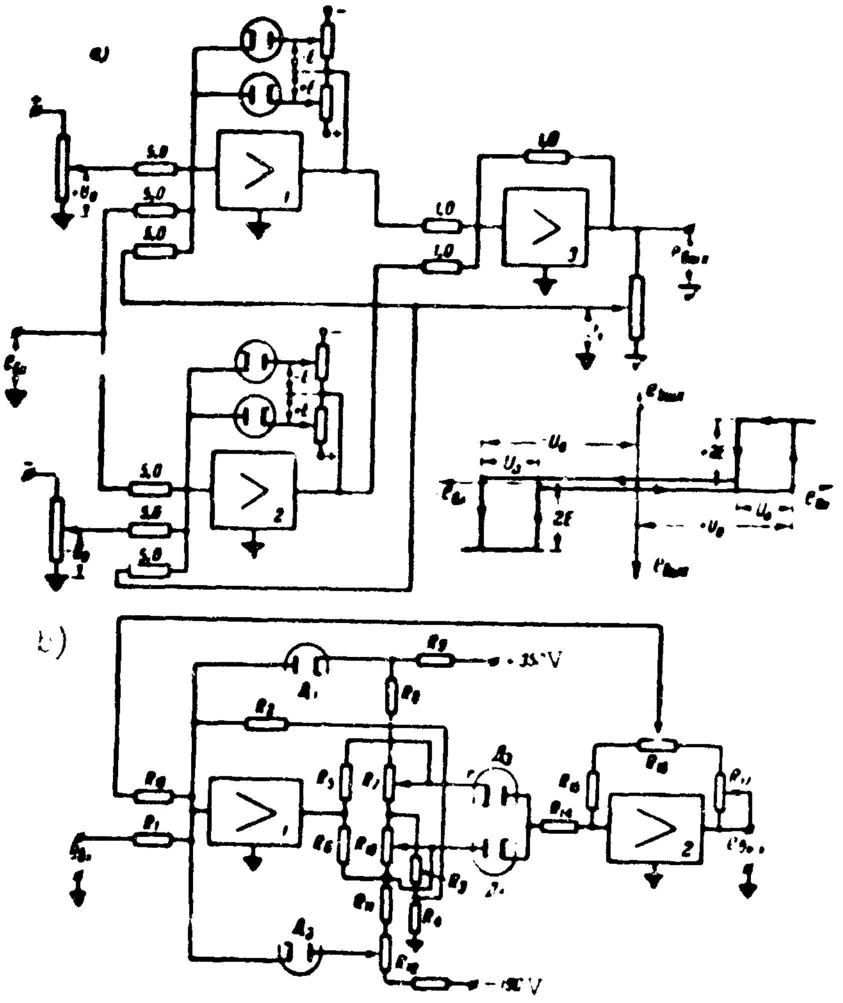

Fig. 223. Schemes for simulating static relay characteristics.

change of sign of output voltage of amplifier 1 occurs now with a value of $e_{вх} = U_0$, but with a value of $e_{вх} = U_0 - U_\Delta$. The lower part of the scheme works analogously with opposite on input signal.

On oscillograms of Fig. 224a are shown dependences of $e_{вых}$ on $e_{вх}$, received with the help of considered scheme for various values of U_0, E and U_Δ. In Fig. 223b is presented a diagram of modeling of static relay characteristics on two operational amplifiers, offered by A. I. Manukhin [1]. Reduction of number of operational amplifiers here is attained by limitation of possibilities of the circuit. U_Δ here can change from 0 to 4 v, U_0 from 0 to 6 v and 2E from 24 to 100 v.

Combination of a relay element and memory unit in the form of integrating amplifier in a circuit with negative feedback (C. A. Meneley [1]) allows us to reproduce approximately the static characteristic of a linear potentiometric pickup taking into account step nature of potentiometer.

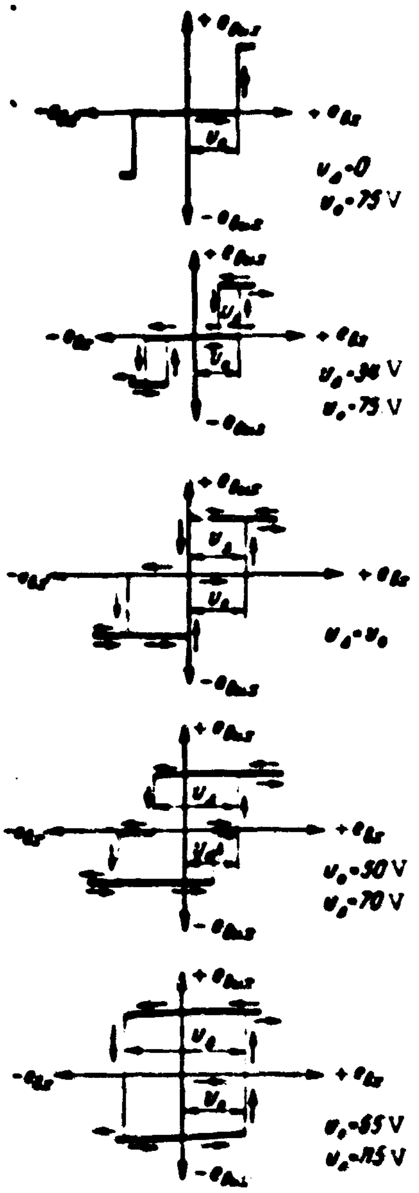

Fig. 224. Oscillograms of work of scheme from Fig. 223.

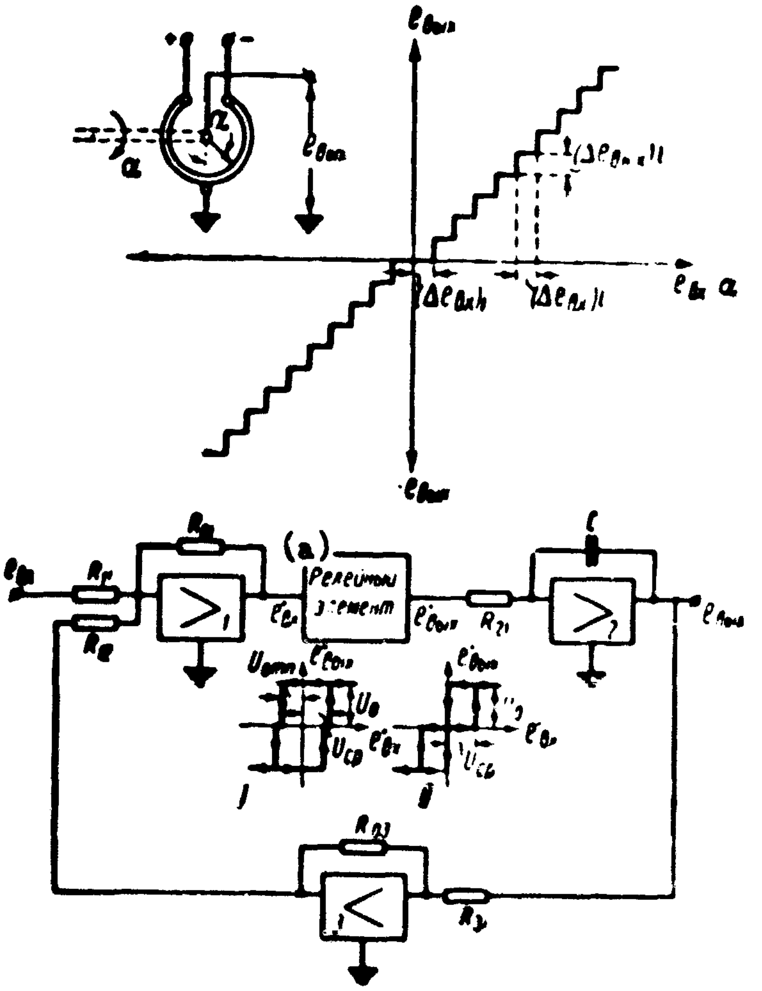

Fig. 225. Characteristic of potentiometric pick-up taking into account step nature of potentiometer and fundamental circuit of its electronic model.
KEY: (a) Relay element.

In Fig. 225 is shown fundamental circuit of such a device. Depending upon form of characteristic of relay element (I or II in Fig. 225) it is possible approximately to model the step nature of potentiometer accordingly without calculation or taking into accout the gap in kinematics of the cursor drive.

Relay element can be made in various ways: by thyratrons, neon tubes or more accurately by the diagram of Fig. 223 (See M. A. Shnaydman [3]).

The connection between parameters of the diagram, the characteristic of the step are given by relationships:*

$$(\Delta e_{in})_1 = (\Delta e_{in})_2 - (\Delta e_{in})_3 = \dots = (\Delta e_{in})_n = (U_{cp})\frac{R_{11}}{R_{01}},$$
$$(\Delta e_{out})_1 = (\Delta e_{out})_2 = \dots = (\Delta e_{out})_n = \frac{R_{12}}{R_{01}}\frac{R_{21}}{R_{02}}(U_{cp}).$$

Speed of voltage build-up of a step will be

$$\left(\frac{de_{out}}{de_{in}}\right)_{\text{Step}} = \frac{U_0}{CR_0}.$$

Comparison of diagrams on Figs. 219, 220, 221, 222, 223 and 224, shows that for reproduction of typical nonlinearities of electronic elements of systems of automatic control it is sufficient to have a diode limiter and a certain number of usual operational amplifiers.

Let us consider as an example simulation of several typical nonlinear problems of automatic control on dc electronic analog computers.

Simulating relay system of automatic control. Equations of investigated systems are given in the form

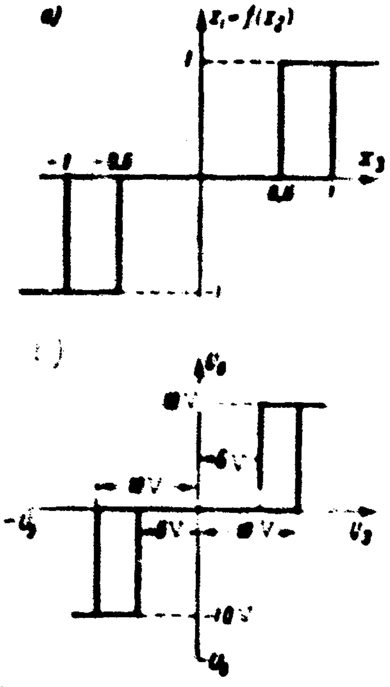

Fig. 226. Relay characteristics.

$$\left.\begin{aligned}(p^2 + 2.5p + 1)x_2 &= -kx_1,\\ x_1 &= f(x_2).\end{aligned}\right\} \qquad (13.3)$$

Function $f(x_2)$ is given by graph (Fig. 226a). It is required to determine character of transients with initial conditions $x_2(0) = 1$, $\dot{x}_2(0) = 0$ and various values of k, namely: k = 304, k = 400, k = k = 200 and k = 100.

The setup diagram on model for given equations (13.3) is shown in Fig. 227. The dotted line circles that part of the diagram which reproduces the relay characteristic.

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*In these relationships the minus sign is omitted and it is taken that $U_{release} = 0$.

Equations for voltages for every block of the setup diagram will be:

$$U_1 = -(K_{11}U_6 + K_{12}z_2U_3 + K_{13}z_1U_4).$$

$$U_2 = -\frac{1}{p}K_{21}U_1.$$

$$U_3 = -\frac{1}{p}K_{31}U_2.$$

$$U_4 = -K_{41}U_2.$$

$$U_6 = f(U_3).$$

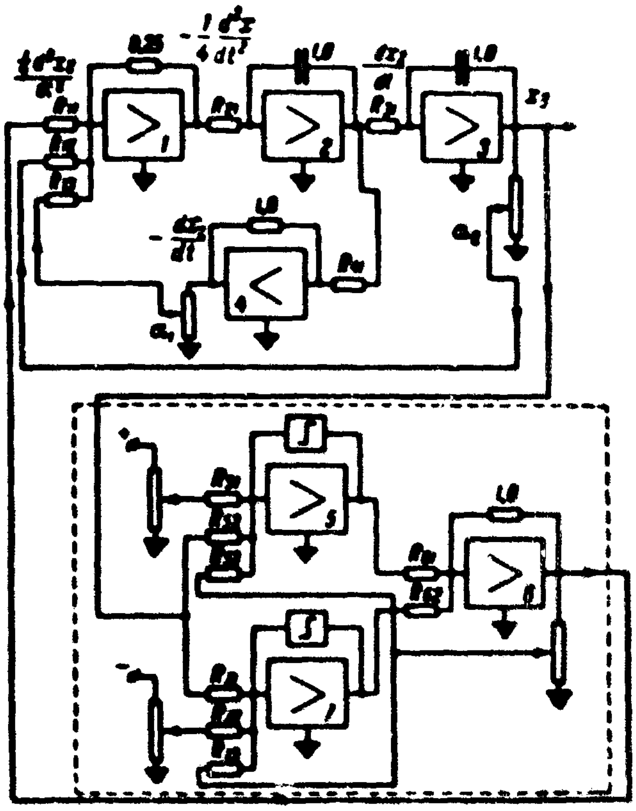

Fig. 227. Functional diagram of simulation of relay system of automatic control.

Solving the resulting system of equations for U_3, which represents coordinate x_2, we receive

$$U_3(p^2 + K_{21}K_{13}K_{41}z_1p + K_{21}K_{31}K_{12}z_2) = K_{21}K_{31}K_{11}f(U_3).$$

After transformation of variable $x_2 = M_{x_2}U_3$; $x_1 = M_{x_1}U_6$ we receive

$$x_2(p^2 + K_{21}K_{13}K_{41}z_1p + K_{21}K_{31}K_{12}z_2) = -M_{x_1}K_{21}K_{31}K_{11}f\left(\frac{x_2}{M_{x_2}}\right).$$

Comparing with initial equations and taking that $M_{x_1} = \frac{x_1(0)}{U_1} = \frac{1}{10} = 0.1$ and $M_{x_2} = M_{x_1}$, we have

$$K_{21}K_{13}K_{41}\alpha_1 = 2.5, \quad K_{21}K_{31}K_{12}\alpha_2 = 1, \quad \frac{M_{x_2}}{M_{x_1}} K_{21}K_{31}K_{11} = k,$$
$$K_{21}K_{31}K_{11} = k,$$
$$U_4 = f(U_3), \qquad U_3 = \frac{x_2}{M_{x_2}}.$$

Dependence $U_4 = f(U_3)$ with the taken numerical values of scale is shown in Fig. 226b.

Since coefficient k significantly differs from remaining coefficients and thereby causes difficulty during setup (possibility of output of blocks beyond the limits of linearity), it is expedient to introduce a time scale.

If one sets $t = \frac{1}{3}\tau$, then initial equations will take the form

$$\frac{d^2x_2}{d\tau^2} = -0.835\frac{dx_2}{d\tau} - 0.111x_2 - \frac{k}{9}x_1.$$

Now transmission factors of separate blocks can be determined from relationships:

$$K_{21}K_{13}K_{41}\alpha_1 = 0.835,$$
$$K_{21}K_{31}K_{12}\alpha_2 = 0.111,$$
$$K_{21}K_{31}K_{11} = \frac{k}{9}.$$

Values of every transmission factor for various k are brought in the following table.

Table X

k	K_{11}	K_{21}	K_{31}	K_{12}	K_{13}	K_{41}	α_1	α_2
300	8.33	0.25	0.25	4	1	1	0.835	0.111
400	11.10	0.25	0.25	4	1	1	0.835	0.111
200	5.56	0.25	0.25	4	1	1	0.835	0.111
100	2.78	0.25	0.25	4	1	1	0.835	0.111

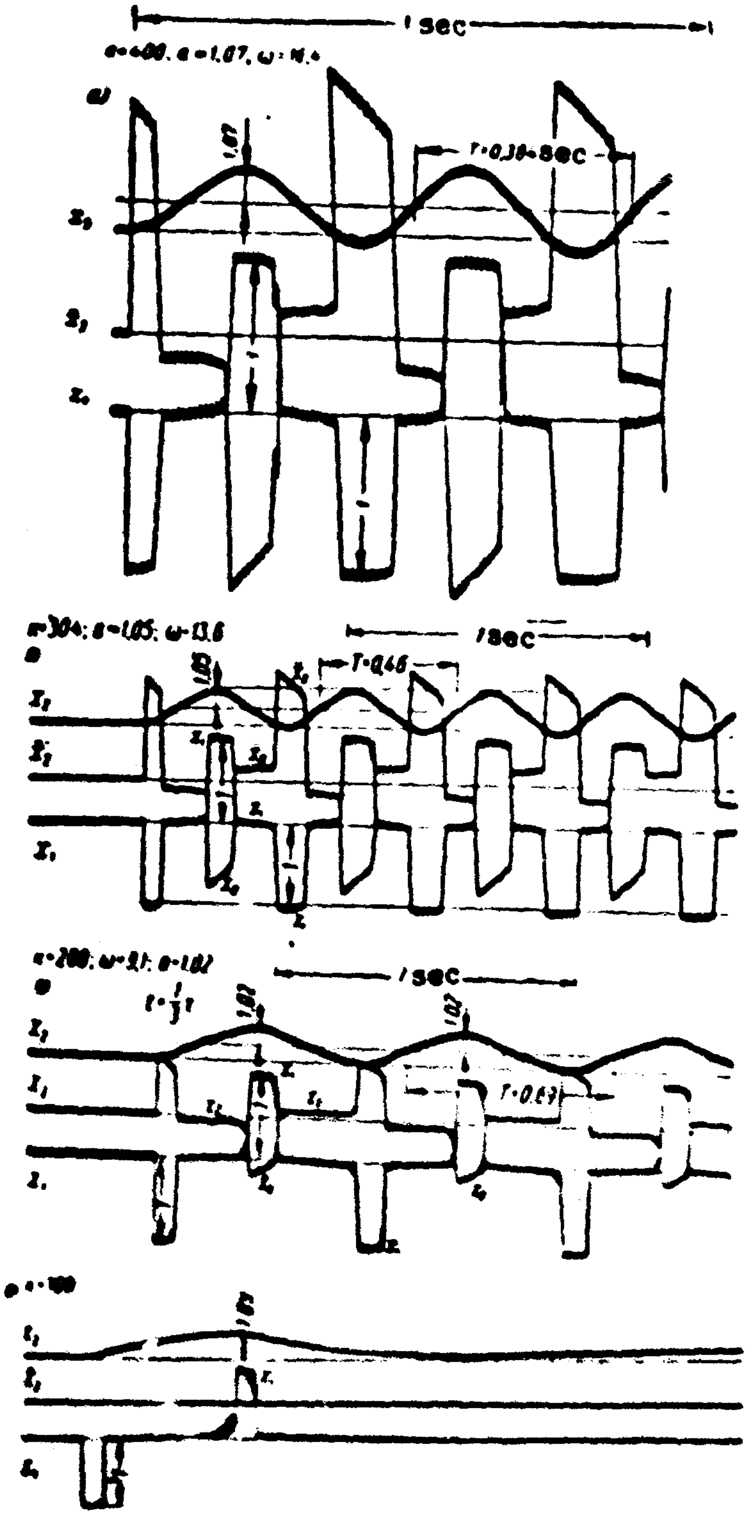

Fig. 228. Oscillograms of control processes in relay system.

In Fig. 228 a, b, c, d are given oscillograms of transients for various k. Comparison of results from oscillograms with data ; calculation, by pointwise transformation,* are shown in this table:

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*Calculation by I. M. Smirnova (a—amplitude of oscillations).

Table XI

(a) Параметр k	(b)	(c)	(b)	(c)
304	13,6	13,6	1,07	1,05
400	16,4	16,4	1,13	1,07
200	9,75	9,1	1,03	1,02
100	—	—	—	—

KEY: (a) Parameter; (b) Calculated; (c) Experimental.

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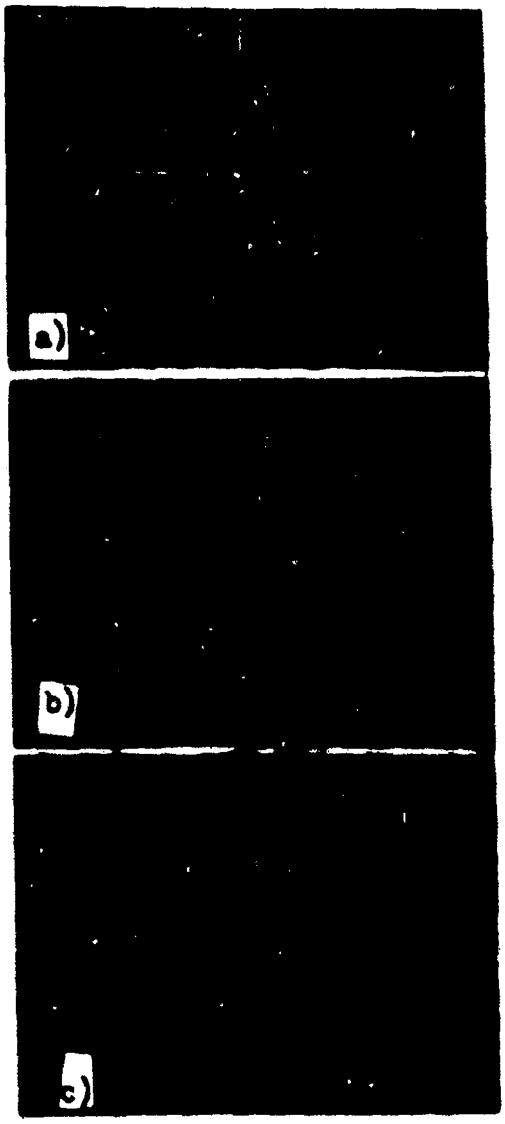

Fig. 229. Phase portraits of relay system of automatic control.

From comparison of data of the table can see sufficiently well coincidence of calculating and experimental values. A picture of phase plane, photographed from screen of cathode-ray oscilloscope for three values k = 304, 400 and 100 and various initial conditions, is shown in Fig. 229a, b, c.

Results of experiment confirm presence of a stable limit cycle when k = 400, 304 and 200.

Simulating electro-hydraulic servo system with calculation for limitation of current of frame of electromechanical converter. In Fig. 230 is brought the fundamental circuit of electro-hydraulic servo system, used in device, intended for conversion of output voltage of electronic model into angle of rotation of platform (V. A. Khokhlov [1]). Such converters are required in a number of cases of simulation with elements of the control loop.

Equation of motion of separate elements of diagram on the assumption that load of hydraulic servomotor is small and nonlinearity is developed only in limitation of current of electromechanical converter, will be:

equation of hydraulic servomotor

$$T_{\text{гм}}\frac{d\varphi}{dt} = k_{\text{гм}}\rho_1. \qquad (13.4)$$

where φ is angle of rotation of servomotor, ρ_1 is displacement of valve, $k_{\text{гм}}$ is proportionality factor;

equation of motion of valve with hydraulic amplifier

$$T_1\frac{d\rho_1}{dt} + \rho_1 = \rho. \qquad (13.5)$$

where T_1 is time constant, and ρ is dislocation of needle of hydraulic amplifier;

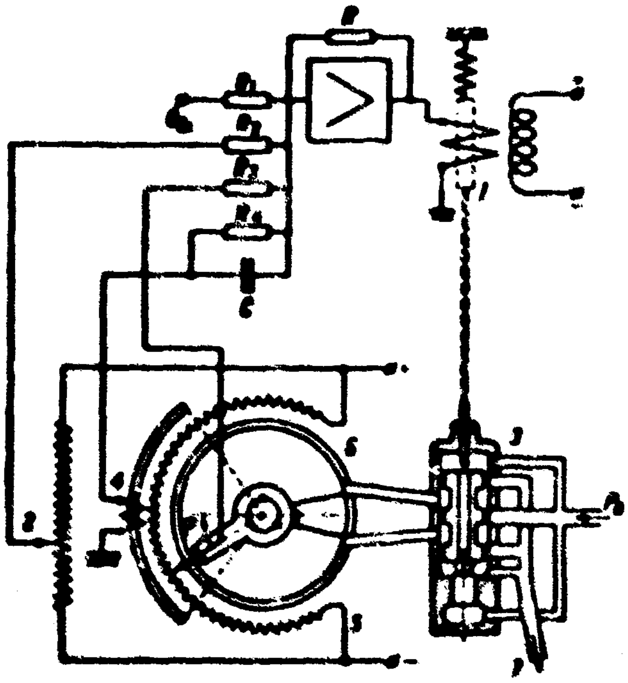

Fig. 230. Fundamental circuit of electro-hydraulic servo system. 1—electromechanical converter, 2—potentiometer of setting of table, 3—hydraulic amplifier, 4—pickup of speed, 5—potentiometer of feedback, 6—hydraulic actuator, 7—discharge.

equation of electromechanical converter (relay)

$$T_2^2\frac{d^2\rho}{dt^2} + T_3\frac{d\rho}{dt} + \rho = k_p I_y. \qquad (13.6)$$

where I_y--current of control in relay coil, variable within limits $A < I_y < A$, K_p is proportionality factor, T_2, T_3 are time constants;

equation of electronic amplifier

$$I_y = S_y\left[e_{\text{вх}} - k_{\text{ос}}\left(\varphi + T_4\frac{d\varphi}{dt} + T_5^2\frac{d^2\varphi}{dt^2}\right)\right]. \qquad \text{(1[illegible])}$$

where S_y is steepness of amplifier, k_{oc}--feedback factor, T_4, T_5 are time constants of differentiating circuits with respect to first and second derivatives.

Problem consists of finding influence of limitation of control current on the transient in the system with the following numerical parameters:

$$T_{\text{гм}} = 1.16\ \text{sec}. \quad k_{\text{гм}} = 105\ 1/\text{см}. \quad T_1 = 0.38 \cdot 10^{-2}\ \text{sec}.$$
$$T_2 = 0.15 \cdot 10^{-4}\ \text{sec}^2. \quad T_3 = 0.18 \cdot 10^{-2}\ \text{sec}. \quad k_p = 0.13 \cdot 10^{-2}\ \text{см/ма}.$$
$$S_y = 12.8\ \text{ма/в}. \quad k_{\text{ос}} = 57.3\ \text{v/deg}. \quad -50\ \text{ма} < I_y < 50\ \text{ма}.$$
$$T_4 = 20 \cdot 10^{-2}\ \text{sec}. \quad T_5^2 = 0.8 \cdot 10^{-5}\ \text{sec}^2.$$

Functional diagram of setup of these equations is shown in Fig. 231. Taking scales of representation of initial variable

$$M_{e_{\text{вх}}} = M_{e_{\text{вых}}} = 0.1. \quad M_t = 0.1. \quad M_p = M_{p_{\text{н}}} = 0.1 \text{ and } M_{I_y} = 11.$$

we receive transmission factors of separate computer blocks:

$$K_{11} = 9.5. \quad K_{12} = 10. \quad K_{13} = 1.2. \quad K_{21} = 10. \quad K_{31} = 6.67.$$
$$K_{41} = 4. \quad K_{42} = 26.3. \quad K_{51} = 9.05. \quad K_{61} = 6.73. \quad K_{62} = 6.73.$$
$$K_{63} = 1. \quad K_{71} = 1.$$

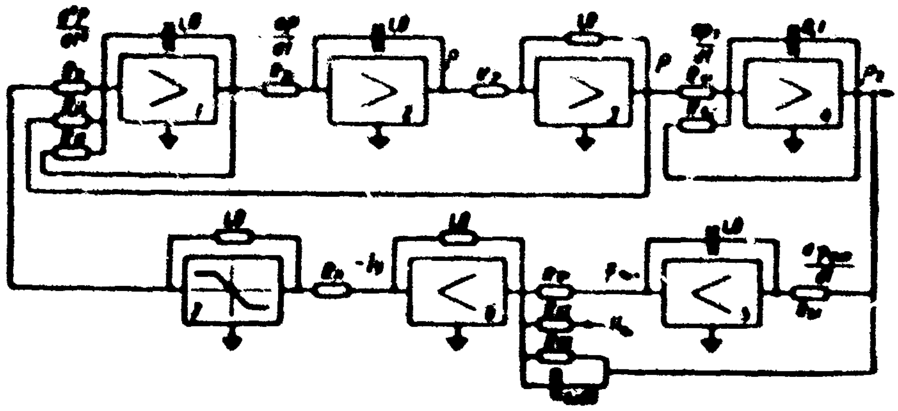

Fig. 231. Functional diagram of simulation of electro-hydraulic servo system.

Results of oscilloscoping transients during step change of input signal are shown in Fig. 232a, b accordingly for a linear problem and taking into account limitation of current I_y. Oscillograms show noticeable lowering of speed of reaching steady-state value with limitation of current.

It is necessary to indicate that during construction of converting devices large difficulties arise, caused by the fact that natural parameters of the device must not distort processes of adjustment, obtained in the simulation. Therefore, the transmission band of the converter should be at least one order wider than the transmission band of the investigated system.

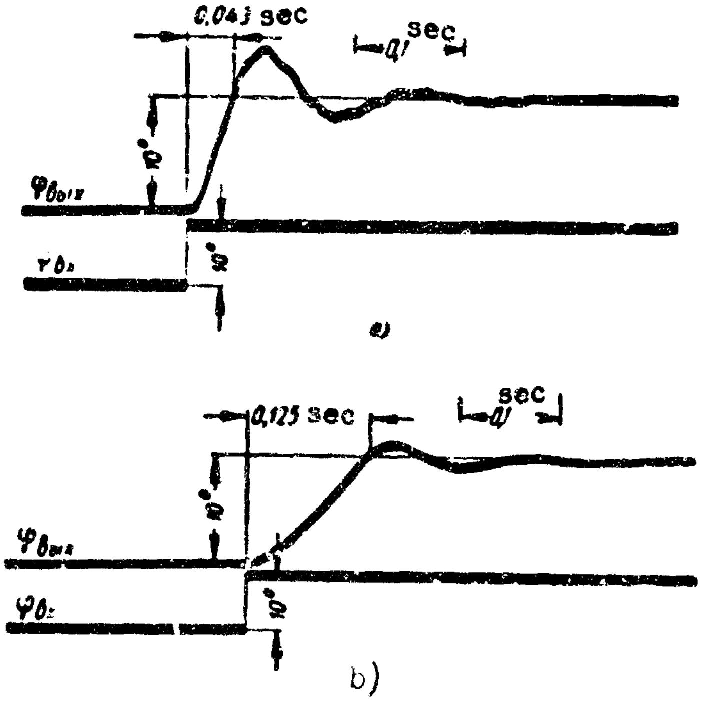

Fig. 232. Oscillograms of processes of adjustment in electro-hydraulic servo system, obtained during simulation.

At present converters are constructed on the basis of a servo system, converting voltage into angle of rotation, voltage into angular velocity of rotation of platform and, finally, in the form of a model of physical analogy, converting voltage of model into moment acting on shaft of a mechanical system with one degree of freedom. One example of a converter, based on combined use of enumerated principles, is described in work of L. N. Fitsner [3].

2. Simulation of Systems of Automatic Control with Typical Nonlinearities in Inertial Elements

Simulation taking into account limitation of coordinates. During simulation of limitations of coordinates of electronic elements (for example, limitation of output voltage of electronic amplifier) it makes no difference where the amplifier is coupled in, at input, output or in feedback circuit of operational amplifier.

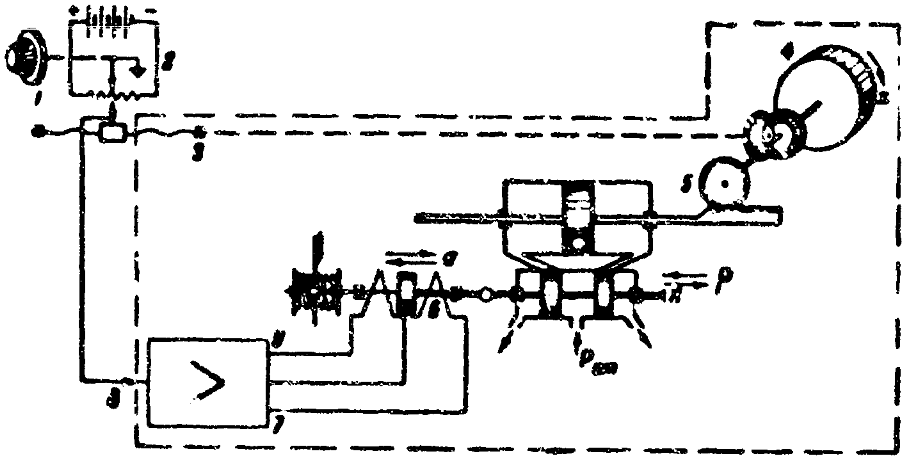

Fig. 233. Diagram of electro-hydraulic servo system, from book of G. Korn and T. Korn [1]. 1--pickup of input signal, 2--potentiometric pickup of mismatch, 3--feedback, 4--load, 5--servomotor, 6--converter, 7--amplifier, 8--valve.

Completely otherwise stands the matter during reproduction of limitations of coordinates in inertial elements. For example let us consider simulation of the hydraulic actuating mechanism of the servo system, described in the book of G. Korn and T. Korn [1]. Equation of motion of this actuating mechanism (Fig. 233) in linearized form will be:

a) for electronic amplifier

$$U = k_1\delta; \tag{13.8}$$

b) for electromechanical converter (relay, solemoid, etc)

$$(T_2p + 1)\sigma = k_2U; \tag{13.9}$$

c) for valve

$$(T_4T_3p^2 + T_3p + 1)\rho = k_3\sigma, \tag{13.10}$$

d) for servomotor

$$T_5px = k_5\rho. \tag{13.11}$$

In reality corrdinates U, ρ and x cannot change infinitely. Thanks to finite value of voltage of power supplies and load output voltage of amplifier can change linearly only within limits $-A_1 \leqslant U \leqslant A_1$. Thanks to limited productivity of oil pump and finite area of apertures, covered by valve, $-A_2 \leqslant \rho \leqslant A_2$ and, consequently, speed px, developed by piston, will also be limited. Also limited will be movement of piston $x\ (-A_3 \leqslant x \leqslant A_3)$.

After coordinates U, ρ and x reach limits, these equations of motion become invalid. Assuming that vibration in mounts does not occur and limitations of coordinates do not set on simultaneously, we arrive at the following possible cases:

$$1.\ \left.\begin{array}{l} (T_2p + 1)\sigma = k_2A_1, \\ (T_4T_3p^2 + T_3p + 1)\rho = k_3\sigma, \\ T_5px = k_5\rho \end{array}\right\} \text{when } |k_1\delta| \geqslant A_1 = \text{const.}$$

$$2.\ \left.\begin{array}{l} U = k_1\delta, \\ (T_2p + 1)\sigma = k_2U, \\ p\rho = 0, \quad \rho = A_2, \\ T_5px = k_5A_2 \end{array}\right\} \begin{array}{l} \text{when } |k_1\delta| < A_1, \\ \text{but } |k_3\sigma| \geqslant A_2. \end{array}$$

$$3.\ \left.\begin{array}{l} U = k_1\delta, \\ (T_2p + 1)\sigma = k_2U, \\ (T_4T_3p^2 + T_3p + 1)\rho = k_3\sigma, \\ px = 0, \\ x = A_3 \end{array}\right| \begin{array}{l} \text{when } |k_1\delta| < A_1, \\ \qquad |k_3\sigma| < A_2, \\ \dfrac{k_5}{T_5}\left|\displaystyle\int_0^t \rho\, dt\right| \geqslant A_3. \end{array}$$

It is obvious that during simulation it is necessary to have the possibility of reproducing all enumerated cases. Frequently, they disregard such detailed recording of equations, as a result of which they allow incorrectness into solution of the problem.

In Fig. 234a is shown setup diagram of computer blocks of the model, brought in the mentioned book of G. Korn and T. Korn, for solution of the placed problem.

From analysis of this diagram it follows that after working of the output limiter integrating amplifiers 4 and 5 will continue to integrate input signals, and up to moment of reverse movement of servomotor on these integrators there can be established voltage of any magnitude (within limits of linearity). These voltages will represent initial conditions with respect to coordinates $\frac{d\varphi}{dt}$ and φ for reverse movement and will lead to motions of system, in principle differing from those, which should take place in reality.

In Fig. 234b is shown a diagram of simulation, without the indicated deficiencies. Limitation of the coordinate of the inertial element here is attained by forced conversion to zero of its speed by means of short-circuiting the feedback circuit of the corresponding operational amplifier. For every limited coordinate here there are introduced two additional units: a comparator and a unit of the sign of acceleration. For the purpose of greatest graphicness for commutation, in the circuit there are used electromagnetic and polarized relays (ПР). Analogous commutation when indispensable can be fulfilled on diode keys.

Simulating an inertial actuating mechanism taking into account dry friction on output shaft. Dry friction in an electronic element usually is reproduced by the same circuits, which are used for obtaining the static characteristic of clearance.

For an inertial element such an approach leads to incorrect results. As it is known, equation of inertail actuating mechanism taking into account moment of freiction on output shaft can be reduced to the form

$$\left.\begin{array}{l}(Tp+1)\omega = k\sigma - \frac{1}{c} M_{\text{тр}} \operatorname{sign} \omega \\ \left(\text{when } \omega \neq 0 \text{ or if } \quad \omega = 0, \text{ but } |k\sigma| > \frac{1}{c} M_{\text{тр}}\right). \\ \text{and} \qquad\qquad x = \text{const} \\ \left(\text{when } \omega = 0 \text{ и } |k\sigma| < \frac{1}{c} M_{\text{тр}}\right).\end{array}\right\} \quad (13.12)$$

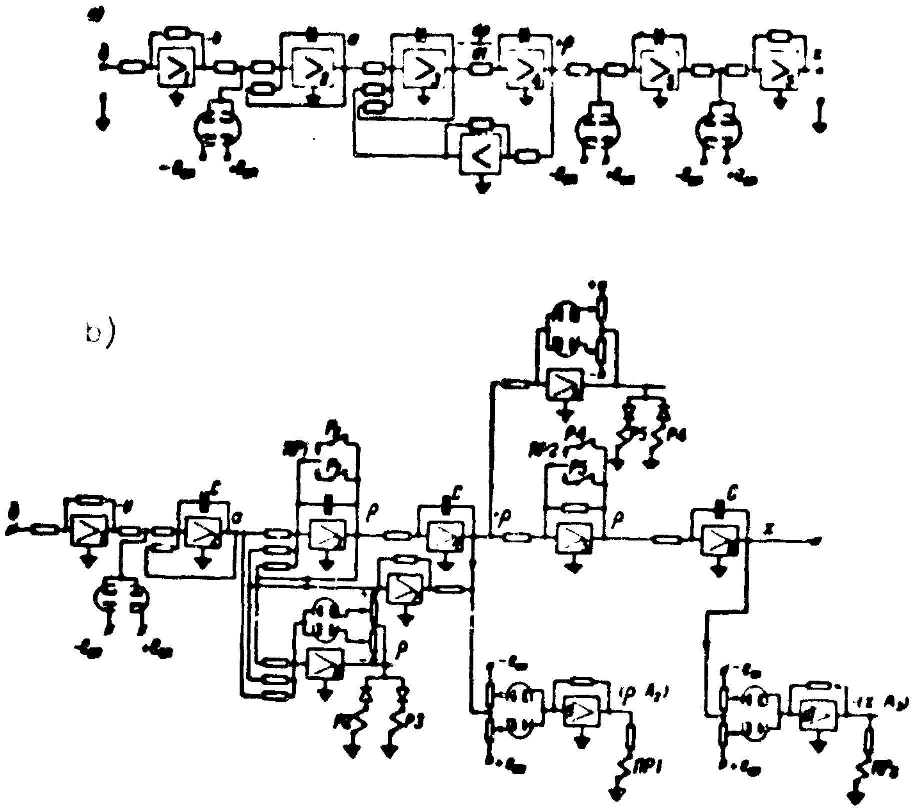

Fig. 234. Functional diagram of simulation of CAP of Fig. 233.

Here T is time constant of actuating mechanism, x is output coordinate of actuating mechanism, ω is speed of output shaft, σ is control signal, k—amplification factor, M_{Tp} is moment of dry friction, ϵ is coefficient of self-levelling.

Moment of friction when $\omega = 0$ is equal in modulus and opposite in direction to total effective moment $k\sigma$ as lond as the latter does not excel limit value $\frac{1}{\epsilon} M_{Tp}$.

In Fig. 235a is brought diagram of solution of this equation on a model. Until the input signal is less than voltage, which represents friction U_{Tp} , the circuit works as a unique relay servo system. Operational amplifier 2 here reproduces the relay characteristic. Indeed, with output voltage $|U_2| < |E|$ gain factor of amplifier due to breaking of feedback is very great and the least change of input signal leads to appearance at output of signal +E or -E depending upon sign of input signal. If the input signal determines speed, then output signal of such block will

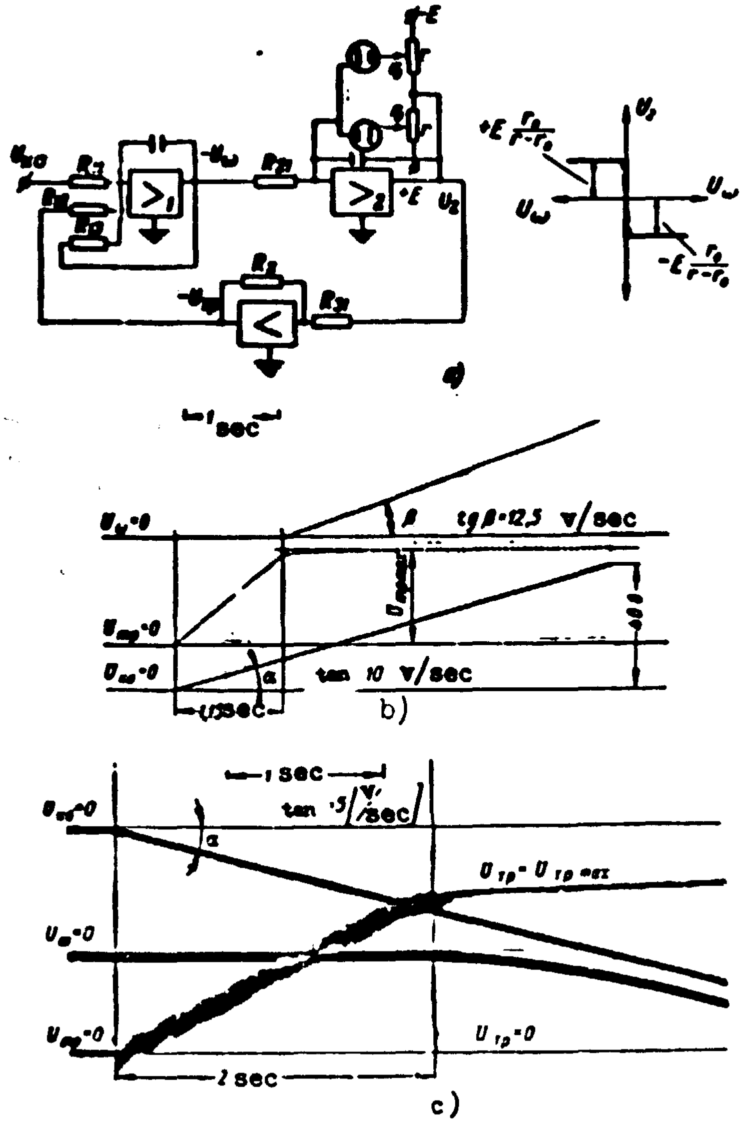

Fig. 235. Diagram of simulation of dry friction and oscillograms, illustrating its work.

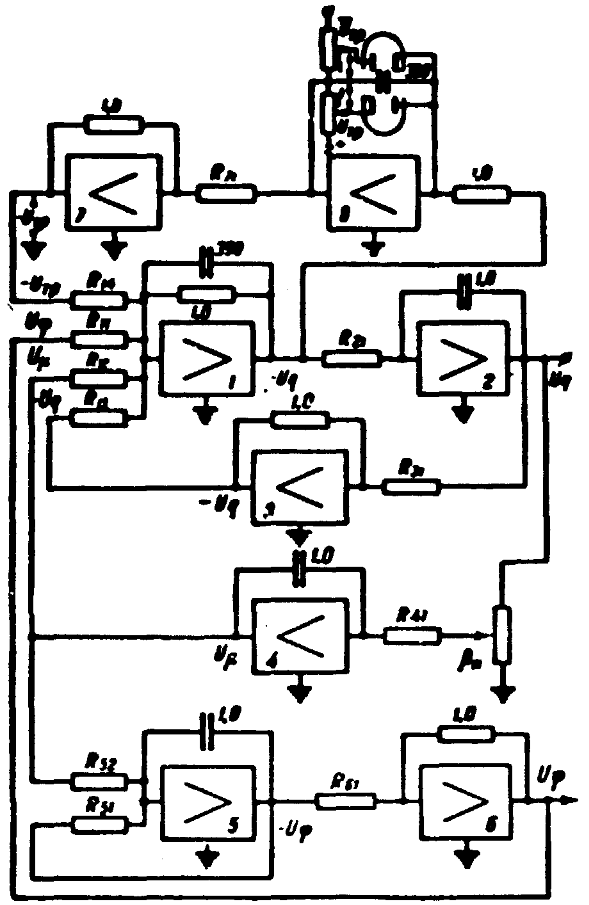

Fig. 236. Diagram of simulation of control system of speed of steam turbine with dry friction in valve of servomotor.

represent moment of friction.

Output voltage of first block, which is the mismatch signal of the considered servo system, will be minute (since gain factor of subsequent block is very great), and, therefore, as long as $U_{\sigma} < U_{\tau p}$, $U_{\omega} = 0$, and consequently, $U_{\tau p}$ in magnitude will follow U_{σ}.

After signal U_{σ} exceeds value of $U_{\tau p\,\max}$, the latter does not change further and processes in system occur in accordance with given equation for constant value of $U_{\tau p\,\max}$. For removal of natural oscillations, appearing in circuit of model during work in mode of relay servo system, one must connect in feedback circuit of operational amplifier 2 small capacitance of order $C \approx 300$ pf.

In Fig. 235b, c are brought oscillograms of change of separate voltages in the circuit of Fig. 235a when time constant is $T = 1$ sec and 0.4 milliseconds. The last case takes place during calculation of dry friction, for example, in low-inertia sensor of a regulator.

As an example let us consider simulation of the system of automatic control of speed of steam turbine taking into account friction in valve of servomotor.*

During investigation of the system of automatic control of speed of steam turbine on stand in the VTI (All Union "Order of Red Banner of Labor" Scientific Research Institute of Heat Engineering in name of F. E. Dzerzhinskiy) there were revealed natural oscillations of the control system in the presence of friction, artificially introduced in sensor of control circuit with diagonal coupling. There was formulated the problem of reproducing these phenomena on an electronic model and of investigating influence of magnitude of introduced friction on character of transients, amplitude and frequency of natural oscillations.

Equations, which describe motion of investigated system of automatic control, according to the data of VTI have form:

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*Work on simulation of this CAP is being conducted by the author with G. A. Kirokosyants.

equation of sensor

$$T_k \frac{d\eta}{dt} + \eta = \mu + \varphi - |r| \operatorname{sign}\left(\frac{d\eta}{dt}\right); \qquad (13.13)$$

equation of valve

$$\sigma = -\eta; \qquad (13.14)$$

equation of servomotor

$$T_c \frac{d\mu}{dt} = \sigma; \qquad (13.15)$$

equation of controlled process

$$T_a \frac{d\varphi}{dt} + \alpha\varphi = (1 - \alpha)\mu. \qquad (13.16)$$

where η is coordinate of sensor, μ is coordinate of servomotor σ is coordinate of valve, φ is regulated magnitude (speed), α is coefficient of self-levelling, r is force of dry friction, T_K, T_C, T_a are time constants of system.

Equations are given in relative magnitudes.

Functional setup diagram of these equations on electronic model is shown in Fig. 236. For reproduction of friction we use the earlier considered circuit. The connection between transmission factors of separate blocks and coefficients of differential equations is determined from equations:

$$\frac{1}{T_k} = K_{13}K_{31}K_{21}, \qquad \frac{1}{T_k}|r| = K_{31}K_{11}K_{21}U_{\text{тр}},$$

$$\frac{1}{T_k} = K_{11}K_{21}M_{\varphi}, \qquad \frac{\alpha}{T_a} = K_{51}M_{\varphi},$$

$$\frac{1}{T_c} = K_{41}З_{\sigma}M_{\mu}, \qquad \frac{1-\alpha}{T_a} = K_{52}\;K_{61}M_{\varphi},$$

$$\frac{1}{T_k} = K_{12}K_{21}M_{\mu}.$$

On oscillograms of Fig. 237 for illustration of received results there are brought transients in system at initial value of coordinates φ (0) = -1, η (0) = 0, μ (0) = 1 and values of parameters T_k = 0.22 sec; T_c =0.3 sec, a =0.316, T_a = 0.41 sec, M_μ = 1.16, M_φ = 2.3 β_n =0.92 without and with of frictional force r = 0.1.

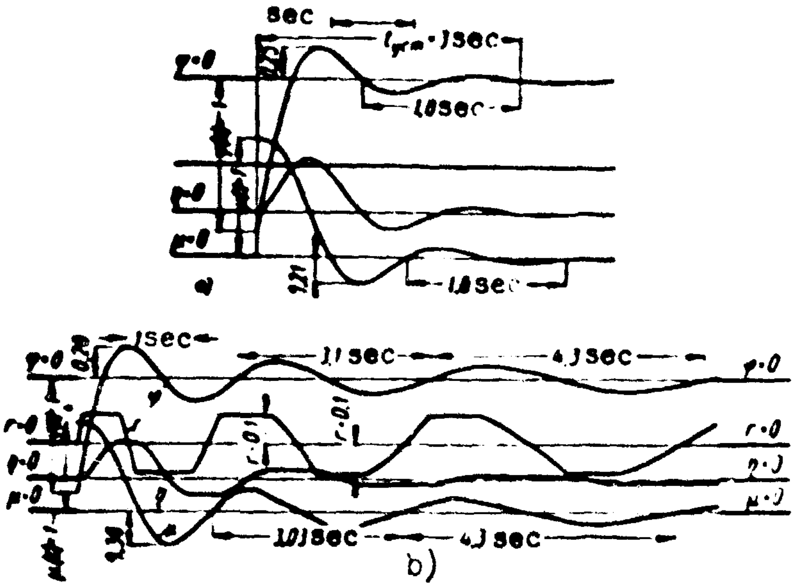

Fig. 237. Oscillograms of transients in system of automatic control of speed of steam turbine. a) for linearised system, b) with calculation for dry friction in valve of servomotor.

As follows from the oscillograms, with these values of parameters of system and its structure friction in sensor leads to appearance of natural oscillations and considerably worsens quality of process.

Simulation of inertial actuating mechanism taking into account gaps in kinematic circuit, inertial load and friction on output shaft. Let us consider equation of motion of actuating mechanism, in which motor is coupled with regulating unit by means of gear transmission, possessing clearance (Fig. 238). We will also consider inertia of the regulated unit and moment of load, created by it on output shaft of reductor.* Equation of motion of such dynamic system will vary depending upon whether there is selected a gap in transmission or not.

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*As far as this author knows, simulation of such a problem was first considered by A. A. Fel'dbaum [4]; see also A. A. Fel'dbaum and S. P. Onufryuk [1].

Take the following designations: J_1 is given moment of inertia of driving shaft, J_2 is moment of inertia of driven shaft, δ_1 is coordinate of driving shaft, δ_2 is coordinate of driven shaft axis, ε is gap of transmission, $M_{\text{дв}}$ is moment, developed by motor of actuating mechanism on driving shaft, $M_{\text{н}}$ is counteracting moment, developed by load (in particular case this can be moment of dry friction).

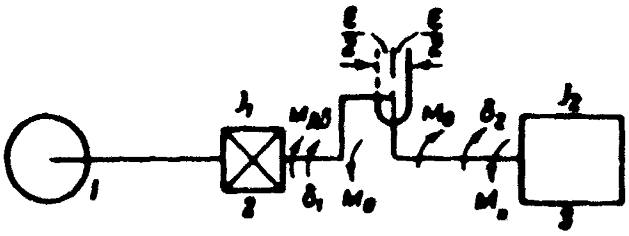

Fig. 238. Toward deriving equations (12.21). 1—motor of actuating mechanism, 2—reductor, 3—regulated unit.

Breaking of system due to presence of transmission clearance will take place whenever $|\delta_1 - \delta_2| < \frac{\varepsilon}{2}$. Here equations of motion of driving and driven elements will be independent:

$$J_1 \frac{d^2\delta_1}{dt^2} = M_{\text{дв}}. \tag{13.17}$$

$$J_2 \frac{d^2\delta_2}{dt^2} = -M_{\text{н}}. \tag{13.18}$$

When gap is selected $\delta_1 = \delta_2 = \delta$, system will move as a whole. Here we will receive

$$(J_1 + J_2)\frac{d^2\delta}{dt^2} = M_{\text{дв}} - M_{\text{н}}. \tag{13.19}$$

During solution of this problem on a simulator it is possible to use two different methods. Introducing for consideration moment of reaction of driven element to driving and of driving to driven* (suggested by A. A. Fel'dbaum) it is possible to preserve recording of equation of motion of considered system in the form of system of two equations both for motion in zone of gap, and for motions with selected gap. In another method of solution of problem transition from system of two independent equations (13.17) and (13.18) to equation (13.19) is carried out automatically with departure from clearance limits.

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*Physically this moment of reaction can be treated as a moment of elastic strains, appearing in reductor during transmission of moving moment to load.

Determining in first case moment M_0 as

$$M_0=\begin{cases} k\left(\delta_1-\delta_2-\frac{\varepsilon}{2}\right) & \text{when } |\delta_1-\delta_2|>\frac{\varepsilon}{2},\quad \delta_1>0, \\ 0 & \text{when } |\delta_1-\delta_2|\leqslant\frac{\varepsilon}{2}, \\ k\left(\delta_1-\delta_2+\frac{\varepsilon}{2}\right) & \text{when } |\delta_1-\delta_2|>\frac{\varepsilon}{2},\quad \delta_1<0, \end{cases} \tag{13.20}$$

we write equations of motion of considered actuating mechanism in the form

$$\left.\begin{aligned} J_1\frac{d^2\varphi_1}{dt^2}&=M_{\text{дв}}-M_0,\\ J_2\frac{d^2\varphi_2}{dt^2}&=M_c-M_0.\end{aligned}\right\} \tag{13.21}$$

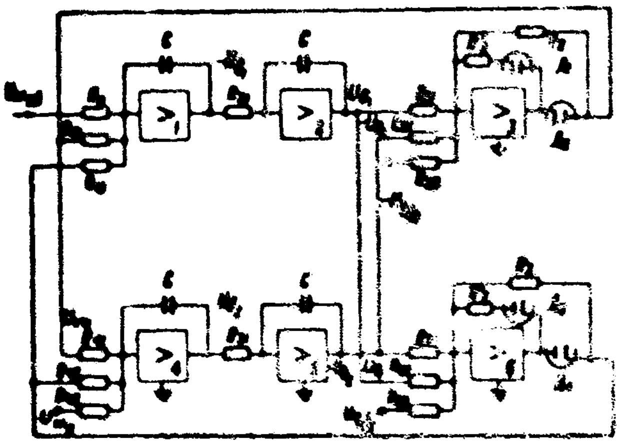


Fig. 239. Functional diagram of simulation of processes in inertial elements containing gaps.

Thus, problem of simulation of inertial actuating mechanism in the presence of gap in transmission boils down to solution of system of nonlinear equations, into which there enters nonlinear function M_0 (determined by relationship (13.20)). When $M_0 = 0$ (motion in zone of gap), system of investigated equations becomes free, and when $M_0 \neq 0$ it is connected. Nonlinear function M_0 as it were commutates system of equations (13.21). Forming of this function is most conveniently carried out by two operational amplifiers, united in series with diode elements.

Fig. 239 is brought complete diagram of simulation of considered problem.

series with diode elements.

In Fig. 239 is brought complete diagram of simulation of considered problem. It differs from that offered by A. A. Feld'baum only in smaller number of operational amplifiers and coverage of diodes $Д_1$ and $Д_3$ by a feedback circuit. The latter pursue the goal of decreasing error, caused by nonlinearity of volt-ampere characteristic of diodes with small plate voltages. Diodes $Д_3$ and $Д_4$ coupled in circuit of auxiliary feedback of amplifiers 3 and 6, serve to switch in this feedback during break of main diodes $Д_2$ and $Д_4$ This ensures preservation of voltage at summing points of operational amplifiers 3 and 6 in all regimes at a very low level and allows us to receive output voltage in blocks 3 and 6 practically equal to zero when diodes $Д_2$ and $Д_4$ are locked.

When motion is in the zone of clearance ($\delta_1 - \delta_2 - \frac{\varepsilon}{2} < 0$ or $\delta_1 - \delta_2 + \frac{\varepsilon}{2} > 0$), diodes $Д_2$ and $Д_4$ are locked and $Д_1$ and $Д_3$ are unlocked and variables δ_1 and δ_2 change independently (accordingly under the influence only of $M_{дв}$ and M_c).

Outside the clearance zone$\left(|\delta_1 - \delta_2| \geqslant \frac{\varepsilon}{2}\right)$depending upon direction of rotation either diode $Д_2$, or diode $Д_4$ opens and the circuit as a whole should reproduce equation (13.19).

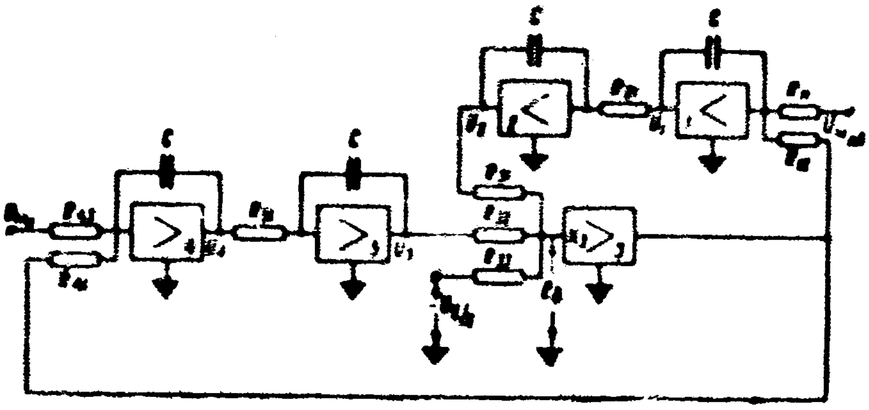


Fig. 240. Equivalent scheme for limit case of Fig. 239.

In Fig. 240 is depicted diagram of model for that limit case under the condition that $\delta_1 > 0$. With change of direction of motion ($\delta_1 < 0$) diagram of Fig. 240 in structure remains as before, but in it operational amplifier 3 should be replaced by operational amplifier 6 together with connected input impedances.

We will find those limitations, which it is necessary to put on parameters of diagram (Fig. 240), so that transients in it are described by equation (13.19). Output voltage of amplifier 3 taking into account switched in feedback circuit of integrators can be written in the form

$$U_3 = -\frac{U_5 \frac{Y_{32}}{Y_{31}} T_{12} T_{21} p^2 + U_{M_{30}} - U_{\dot{\gamma}} \frac{Y_{33}}{Y_{31}} T_{12} T_{21} p^2}{\frac{Y_{31} + Y_{32} - Y_{33}}{Y_{31}} \cdot \frac{T_{12} T_{21}}{K_3} p^2 + 1}. \qquad (13.22)$$

where $T_{12} = R_{12}C.$ $T_{21} = R_{21}C.$ $Y_{33} = \frac{1}{R_{33}}.$ $Y_{32} = \frac{1}{R_{32}}.$ $Y_{31} = \frac{1}{R_{31}}.$

With sufficiently large gain factor K_3, when in working range of frequencies the following is valid

$$\left| \frac{Y_{31} + Y_{32} + Y_{33}}{Y_{31}} \cdot \frac{T_{12} T_{21}}{K_3} \omega^2 \right| \ll 1,$$

expression (13.22) can, with accuracy sufficient for practice be presented in the form

$$U_3 = -U_5 \frac{Y_{32}}{Y_{31}} T_{12} T_{21} p^2 - U_{M_{30}} + U_{\dot{\gamma}} \frac{Y_{33}}{Y_{31}} T_{12} T_{21} p^2. \qquad (13.23)$$

Equation, connecting output voltage of fifth block with input of fourth, will be

$$U_5 = \frac{1}{T_{51} p^2} \left(\frac{1}{T_{41}} U_3 + \frac{1}{T_{43}} U_{M_4} \right). \qquad (13.24)$$

where

$$T_{51} = R_{51}C. \quad T_{41} = R_{41}C. \quad T_{43} = R_{43}C.$$

In the model, presented in Fig. 240, voltage U_5 represents output coordinate $\dot{\gamma}$.* Therefore, equations (13.23) and (13.24), describing processes in considered circuit should be solved for voltage U_5.

After excluding U_3 and transition to originals taking into account that $U_{\dot{\gamma}}$ = const, we receive

$$\left(1 + \frac{T_{12} T_{21}}{T_{41} T_{51}} \frac{Y_{32}}{Y_{31}}\right) \frac{d^2 U_5}{dt^2} = -\left(\frac{1}{T_{41} T_{51}} U_{M_{30}} - \frac{1}{T_{43} T_{51}} U_{M_4} \right). \qquad (13.25)$$

---

*With opposite sign.

Assuming $T_{41} = T_{43}$ and considering that scales k_M of input magnitudes are selected identical, i.e., $U_{M_{10}} = k_M \cdot M_{a0}$. and $U_{M_n} = k_M M_n$, we will receive

$$\left(T_{41} \cdot T_{51} + T_{12}T_{21}\frac{Y_{32}}{Y_{31}}\right)\frac{d^2\Omega_5}{dt^2} = k_M(M_{an} - M_n). \tag{13.26}$$

So that equation (13.26) is identical to equation (13.19), it is necessary, that when $U_5 = -k_5\delta_7$ we have

$$\frac{Y_{32}}{Y_{81}} = \frac{Y_{61}}{Y_{62}} = 1; \quad k_5T_{41}T_{51} = k_MJ_2; \quad k_5T_{12}T_{21} = k_MJ_1.$$

where k_5 is scale factor of output magnitude.

Main deficiency of presented method of solution is inclination of circuit of model to generate undamped oscillations. Indeed, considering circuit of Fig. 240 in the form of one amplifier with complicated feedback, we can verify that in its circuit for a finite value of gain factor there must appear undamped oscillations. If one were to consider that gain factor K_3 is not a constant magnitude, and by force of inertness of the amplifier represents a certain function of frequency, then the possibility of loss of stability will become evident. Presence of undamped oscillations leads to distortion of transmission of signals of main process of adjustment. Use of usual methods of removal of these oscillations leads to lowering upper limit of frequencies of signals, developed without distortion in such simulation of the investigated servo system.

In connection with this it is useful to consider another method of solution of the formulated problem, in which simultaneously on models there is not reproduced the main process of adjustment and oscillations, caused by elastic deformations in transmission, but there is carried out automatically transition from one differential equations (13.17) and (13.18) to another (13.19) and back depending upon state gap in transmissions.

If Fig. 241a is presented a complete fundamental circuit of the model, necessary during solution of problem by the considered method. In circuit are shown those switching operations, which provide transition from equation (13.17) and (13.18) to equation (13.19) and vice versa. These operations are executed by normally-closed diode keys K_1, K_2, K_3 and normally open ones K_4 and K_5. Change of state of key is carried out by commutating voltage $\pm U_k$, appearing every time after selection of gap in transmission. With appearance of commutating voltage occurs cross-connection of input voltages to blocks 1 and 4 and disconnection of block 5 from block 4. Simultaneously, with this transmission factor of blocks 1 and 4 increases which reproduces increase of general moment of inertia of system due to moment of inertia of output axis.

So that after selection of clearance we have equality $\delta_1 = \delta_2$, in the circuit is an additional sixth block, forming circuit of negative feedback for blocks 5 and 3. Selecting great gain factor of this circuit (by decrease of R_{52}), it is possible to achieve satisfactory accuracy of tracking. Voltage $U_{M_м}$ is fed to block 4 so that at output of this block after selection of clearance we receive voltage, proportional to δ_2. This voltage during reverse input in zone of clearance serves as voltage of initial conditions and ensures, thereby, correct repeated union of blocks 4 and 5.

Fundamental circuit of normally-closed diode key, shunting input impedance of operational amplifiers is shown in Fig. 241b. From this diagram it is possible to obtain the diagram of normally closed key K_3. With change of polarity of voltages U_k and U_{∞}, we obtain diagram of keys K_4 and K_5.

Impedances of normally-closed key are selected from relationships:

in absence of commutating voltage

$$\pm U_{оп} Y_{13} \mp e_{вх\,max} Y_{11} \lesseqgtr 0. \qquad (13.27)$$

in the presence of commutating voltage

$$\mp U_{к} Y_{14} \pm U_{оп} Y_{13} \pm e_{вх\,max} Y_{11} \gtrless 0. \qquad (13.28)$$

Hence

$$Y_{13} \geqslant \frac{e_{\text{вх max}}}{U_{\text{оп}}} Y_{11}. \quad Y_{14} \geqslant Y_{13} \frac{U_{\text{оп}}}{U_{\text{к}}} + \frac{e_{\text{вх max}}}{U_{\text{к}}} Y_{11} \geqslant \frac{e_{\text{вх max}}}{U_{\text{к}}} 2Y_{11}.$$

If $U_{\text{оп}} = \pm 30\text{ v}, \; e_{\text{вх max}} = U_{\text{к}} = \pm 100\text{ v}$, then

$$Y_{13} = \frac{10}{3} Y_{11}. \quad Y_{14} = 2Y_{11}.$$

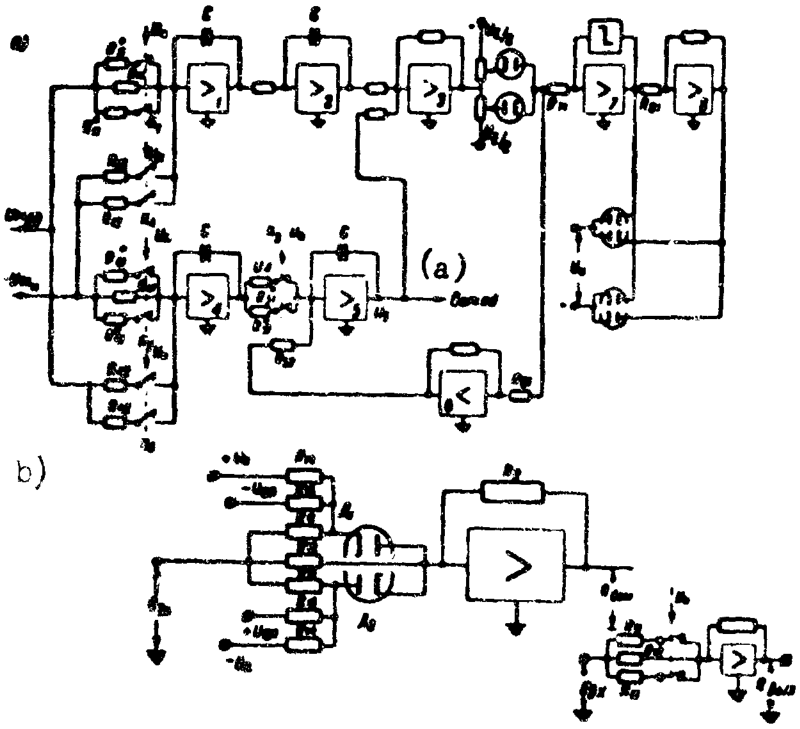


Fig. 241. Diagram of simulation of processes in inertial elements with gap, based on application of electronic keys.
KEY: (a) Output.

Conductance Y_{11} usually is determined from required values of transmission factor of block in every position of key. Let there be given two required values of transmission factors of block K_1 and K_2 accordingly for closed and opened state of key. Then

$$K_1 = \frac{Y_{\text{вых}}}{Y_2} = 2\frac{Y_{11}}{Y_2} + \frac{Y_{12}}{Y_2},$$
$$K_2 = \frac{Y_{12}}{Y_2}.$$

Hence

$$Y_{11} = Y_2 \frac{K_1 - K_2}{2}. \tag{13.29}$$

Considered examples of simulation of certain nonlinear problems also give an idea of methods of simulating problems of adjustment, requiring realization of automatic transition from certain equations to others, differing not only in coefficients, but also in structure. Apparently, presented methods may also be used for realization of automatic transition of a CAP from one law of adjustment to another depending upon phase and nature of the transient.

3. Examples of Solution of Nonlinear Problems of Automatic Control

During investigation of systems of automatic control by electronic models it is possible to obtain not only particular solutions, correct for the given numerical values of parameters, but also, which is very valuable, to reveal the total picture of possible motions for givne structure of the system and, thus, to make recommendations on selection of parameters. Let us consider the method of formulating such investigations with two examples of very simple systems of industrial control.*

As the first example let us consider system of automatic control of inertial object of first order, regulated by relay astatic regulator. Here we will assume presence of delay τ in transmission of controller action. In these conditions considered system of automatic control is described by following system of differential equations:

$$T_0 \frac{d\varphi}{dt} + \varphi = k_0 \mu (t - \tau), \quad \frac{d\mu}{dt} = f(\varphi). \qquad (13.30)$$

where T_0 is time constant of object, k_0 is amplification factor of object, τ is time lag, μ is controlling action, φ is controlled variable, and function $f(\varphi)$ is determined thus:

$$f(\varphi) = \begin{cases} 0 \text{ when } -\varphi_{\text{н}} < \varphi \leqslant \varphi_{\text{н}}. \\ \pm A \text{ when } \varphi > \varphi_{\text{н}} \text{ and } \varphi < -\varphi_{\text{н}}. \end{cases}$$

---

*These examples were considered by author on the advice of A. Ya. Lerner.

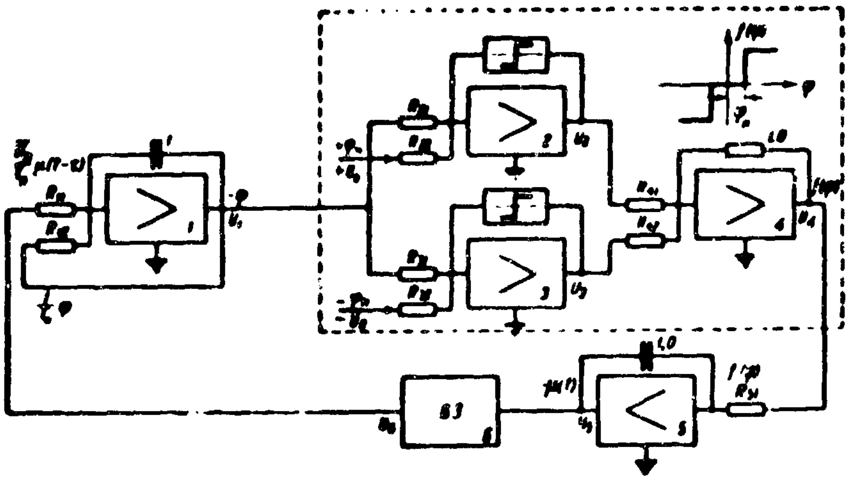


Fig. 242. Functional diagram of simulation of astatic relay system of automatic control with constant delay. БЗ is delay block. Dotted line circles circuit, reproducing relay characteristics.

The problem is set in the following form: to find decomposition of plane of parameters of system k_0 and $\frac{\tau}{T_0}$ into regiens with attenuating process and natural oscillations for various values of the zone of insensitivity of relay $\frac{\varphi_н}{\varphi_{max}}$ (φ_{max} is maximum value of controlled variable) and initial conditions $\varphi(0) \neq 0$, $\mu(0) = 0$.

Setup diagram is shown in Fig. 242. During simulation there was established a fixed value of magnitude A. Change of magnitude A is equivalent to change of k_0. Magnitude T_0 for convenience was taken equal to 1.0 sec. Zone of insensitivity of relay was set equal to in $\frac{\varphi_н}{\varphi(0)} = 0.1$ and 0.05. Zone of insensitivity is related to $\kappa\ \varphi(0)$, since experiment $\varphi(0) = \varphi_{max}$. Results of simulation for value $\varphi(0) = 1$ is shown in Fig. 243a, and b. In Fig. 243a is brought decomposition of plane of characteristic parameters of system, and in Fig. 243b is the dependence of amplitude φ_{ak} and frequency f_{ak} of natural oscillation on $\frac{\tau}{T_0}$. These materials allow us to make selection of main parameters of object and regulator on first stages of projection, and also formulate a number of valuable conclusions about properties of considered control system.

As a second example of simulation of control process let us consider the same object as in the preceding case, but with a proportional-pulse controller. Equations

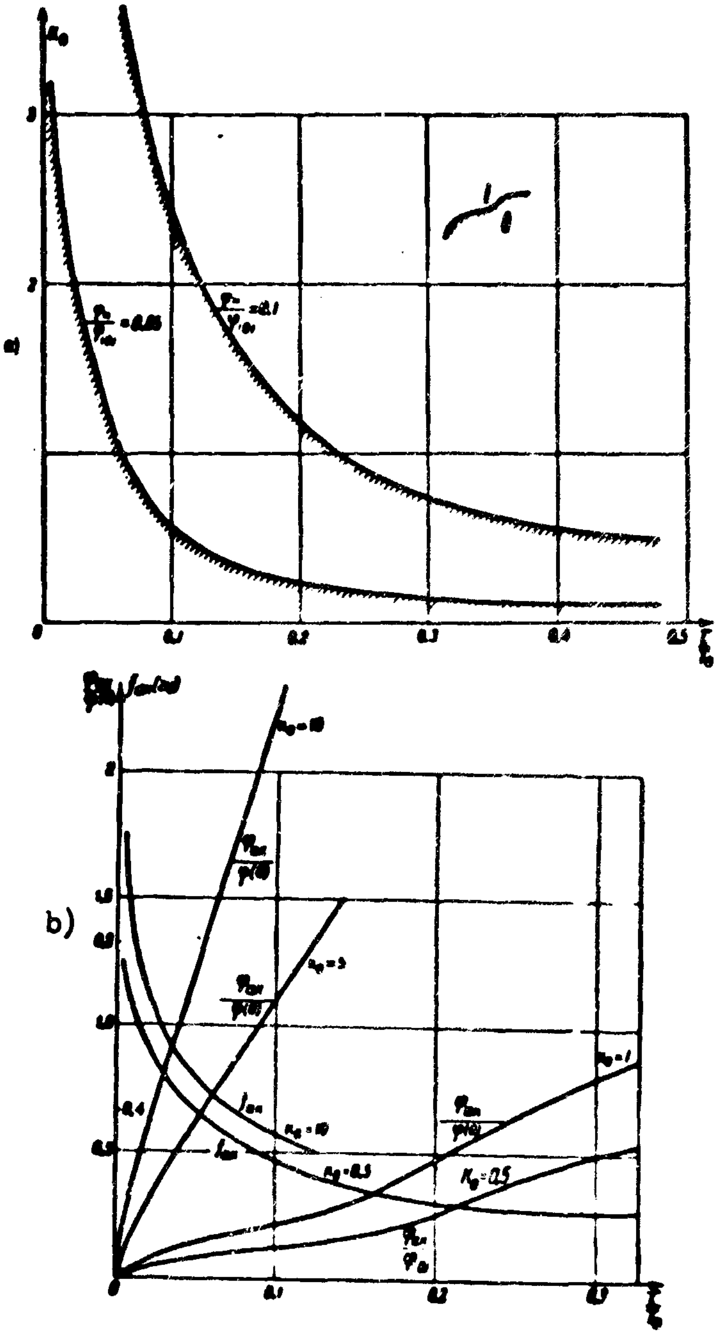


Fig. 243. a) Boundaries of regions with attenuating processes and stable natural oscillations in plane of parameters k_0 and $\frac{\tau}{T_1}$ for astatic relay CAP with delay. I--region of stable oscillations, II--region of attenuating processes. b) Dependence of frequency and amplitude of natural oscillations on $\frac{\tau}{T_1}$ with various values of k_0 and $\frac{\varphi_n}{\varphi(\infty)}$.

describing process of adjustment in this case are:

$$\left.\begin{aligned} T_0 \frac{d\varphi}{dt} + \varphi &= k_0 \mu (t - \tau), \\ T_c \frac{d\mu}{dt} \qquad &= f(\varphi - \varepsilon), \\ T_н \frac{d\varepsilon}{dt} + \varepsilon &= \beta T_н \frac{d\mu}{dt}. \end{aligned}\right\} \qquad (13.31)$$

where T_c is time constant of servomotor, ε is coordinate of the driftless stabilizer $T_н$ is time constant of the driftless stabilizer, β is proportionality factor of the driftless stabilizer. Operating characteristic of servomotor $f(\varphi - \varepsilon)$ is given by graph of Fig. 244.

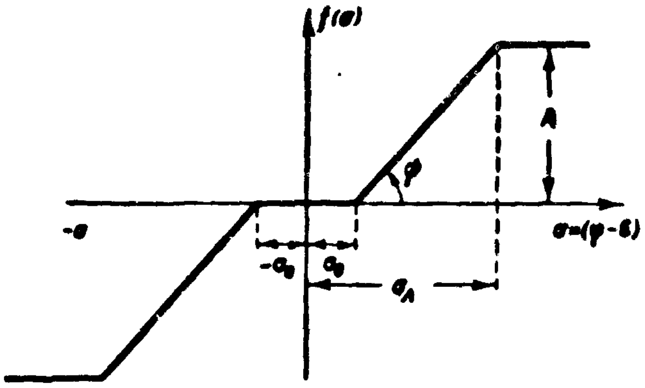


Fig. 244. Characteristic of actuating mechanism.

We find, as before, decomposition of plane of parameters of system k_0 and $\frac{\tau}{T_0}$ into values regions with stable self-excited and attenuating processes for two values of zone of insensitivity of servomotor and various values of coefficient of the driftless stabilizer on the condition that $T_н = T_0$ (condition "tuned driftless stabilizer". A. A. Voronov [1]). Functional setup diagram is shown in Fig. 245. We take also $\frac{T_0}{T_c} = 10$, $A = 1$, $\tan \varphi = 1$ and $T_0 = 1$ sec. Change of magnitude A and tan φ ends to increase or decrease of general gain factor. Change of k_0 works in the same direction. In Fig. 246 are given results of conducted investigations. Oscillograms of Fig. 247a and b illustrate effect of introduction of a driftness stabilizer in considered system. As follows from these oscillograms, without a driftless stabilizer in system impermissible natural oscillations is set.

Introduction of the driftless stabilizer removes natural oscillation and ensures reduction of duration of transients approximately to one half-period of natural oscillations.

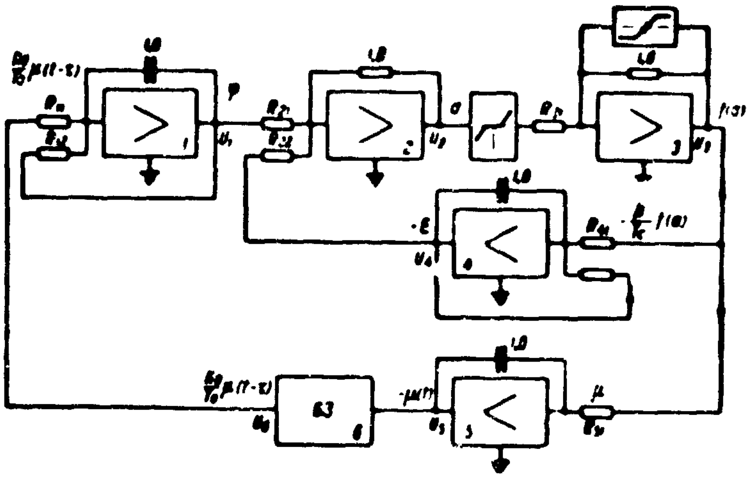


Fig. 245. Functional diagram of simulating a CAP by proportional-plus-floating controller. БЗ-delay block.

Investigation of transients in systems of automatic control with nonlinear damping. Lately nonlinear feedbacks have found wide application for improvement of quality of transients in systems of automatic control. However, calculation of transients in such systems, usually executed by approximate numerical methods, turns out to be very labor-consuming. We will show as an example determination of transients in such systems by a simulator.

In Fig. 248 is shown a servo system (See Dzh. L'yuis [? Lewis or Louis] [1]), in which for improvement of quality of transients there is introduced additional damping, opposite in sign to signal of main damping and proportional to product of performance speed by the magnitude of error signal. In beginning of process, when error signal is great, additional damping considerably decreases total damping in system and, thereby, ensures maximum possible build-up speed of process. With decrease of error signal additional damping decreases and system is effectively braked.

If we disregard inductances of armature and excitation coil of tachogenerator, and also the delay, introduced by amplifiers K_1 and K_2, then equation of motion of considered servo system can be reduced to the form

$$J_1 \frac{d^2 x_{\text{вых}}}{dt^2} + A \frac{d x_{\text{вых}}}{dt} + B (x_{\text{вых}} - x_{\text{вх}}) \frac{d x_{\text{вых}}}{dt} + D x_{\text{вых}} = D x_{\text{вх}}. \qquad (13.32)$$

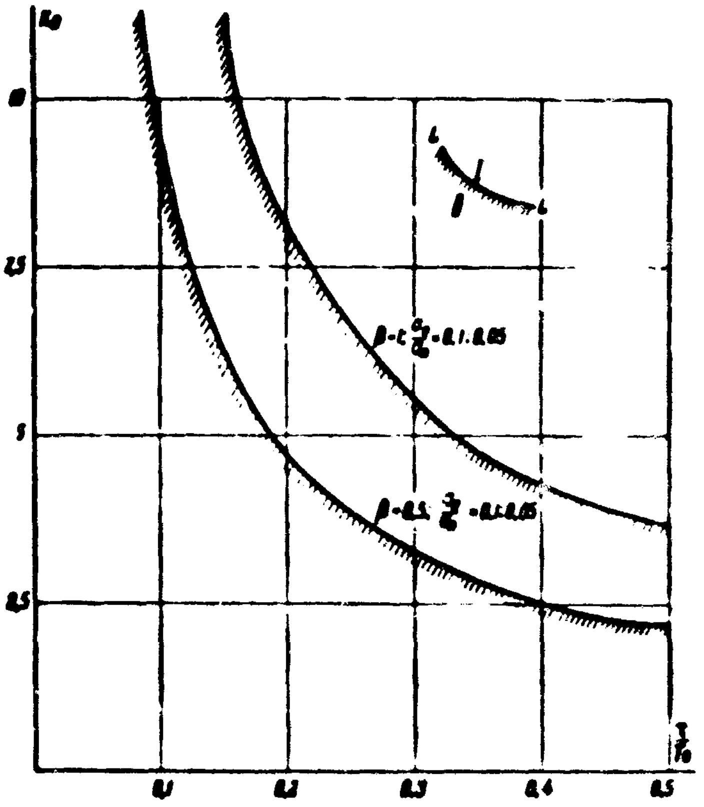
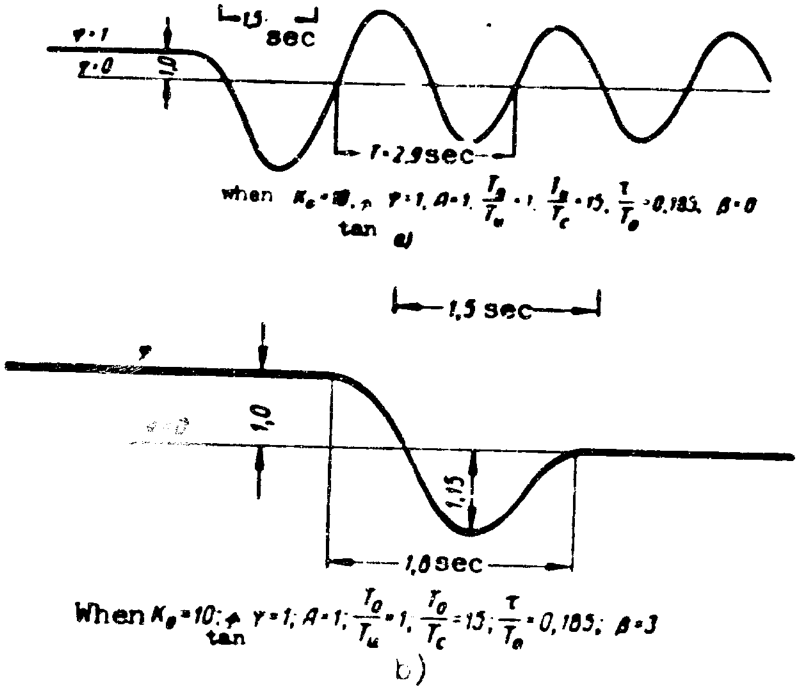


Fig. 246. Boundaries of regions with attenuating and divergent processes in plane of parameters k_0 and $\frac{1}{T_1}$ with various values of $\frac{\delta}{\delta_0}$. I—region of divergent oscillations, II—region of attenuating processes, L—L—boundary of unstable limit cycles.

Coefficients J_1, A and B are determined by parameters of motor, tachogenerator and amplifiers.

Problem consists of determining character of transients in system for preselected parameters $\left(\frac{A}{J_1} = 8, \frac{B}{J} = 7.2, \frac{D}{J_1} = 15.2\right)$, step change of input signal $x_{\text{вх}} = 1$ and zero initial conditions.

Functional setup diagram is shown in Fig. 249. Here to obtain identical effect during supplying and removal of perturbation into the multiplier there is introduced the modulus of error signal. Obtaining of modulus of variable is attained



when $K_e = 10,\ \underset{\tan}{\varphi}\ \gamma = 1,\ A = 1,\ \frac{T_0}{T_u} = 1,\ \frac{T_0}{T_c} = 15,\ \frac{\tau}{T_a} = 0{,}185,\ \beta = 0$

a)

When $K_\theta = 10;\ \underset{\tan}{\varphi}\ \gamma = 1;\ A = 1;\ \frac{T_0}{T_u} = 1,\ \frac{T_0}{T_c} = 15;\ \frac{\tau}{T_a} = 0{,}105;\ \beta = 3$

b)

Fig. 247. a) Transients in system without isodroma; b) transient conditions with tuned driftless stabilizer.

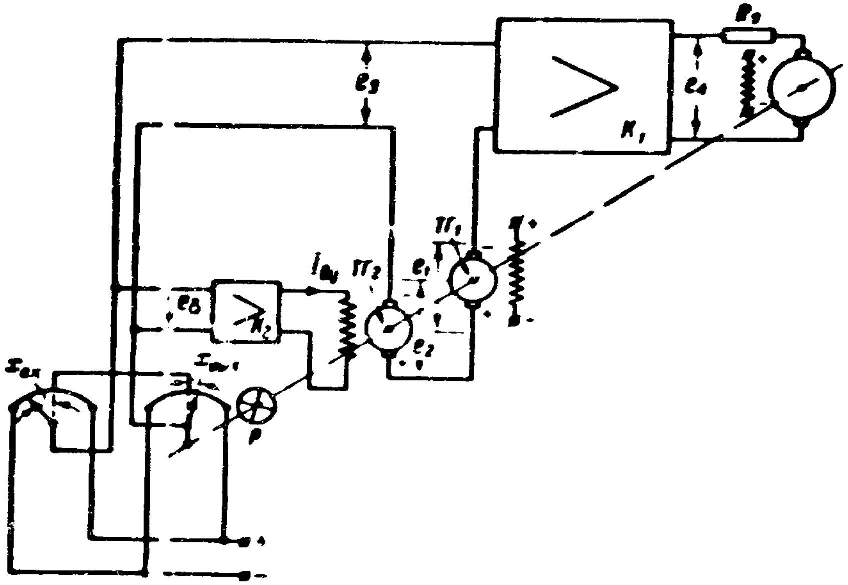


Fig. 248. Fundamental circuit of servo system with nonlinear damping (by Dzh. L'yuis [Lewis] [1]). [illegible] and [illegible] tachogenerators P—reductor, K_1 and K_2—amplifiers.

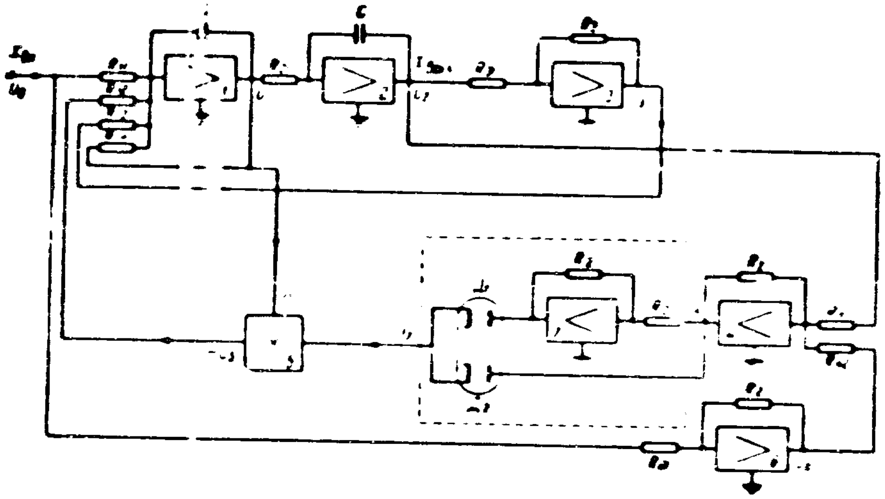


Fig. 249. Functional setup diagram of equation of motion of servo system with nonlinear damping.

by use of additional sign-inverting amplifier 6 and two diodes $Д_1$, $Д_2$ coupled by a scheme of full-wave rectification.

Equations of voltages for separate computing blocks of the functional diagram have the form:

$$
\begin{aligned}
U_1 &= -\frac{1}{p}(K_{11}U_0 + K_{12}U_5 + K_{13}U_3 + K_{14}U_1).\\
U_2 &= -\frac{1}{p}K_{21}U_1.\\
U_3 &= -K_{31}U_2.\\
U_4 &= -(K_{41}U_2 + U_6K_{42}).\\
U_5 &= -\beta_5 U_1 U_7.\\
U_6 &= -K_{61}U_0.\\
U_7 &= |U_4|.
\end{aligned}
$$

Solving this system of equations for voltage U_2, representing in the circuit initial variable $x_{\text{вых}}$, we will receive

$$
\begin{aligned}
\frac{d^2U_2}{dt^2} + K_{14}\frac{dU_2}{dt} - K_{12}\beta_5|(K_{61}\cdot K_{42}U_0 - K_{41}U_2)|\frac{dU_2}{dt} &+\\
+ K_{13}K_{31}K_{21}U_2 &= K_{11}K_{21}U_0.
\end{aligned}
$$

Introducing transformation of variables $x_{\text{вх}} = M_{\text{вх}}U_0$, $x_{\text{вых}} = M_{\text{вых}}U_2$ and assuming $M_t = 1$, we will recieve after comparison with initial equation (12.32):

$$
\frac{A}{J_1} = 8 = K_{14}, \quad \frac{B}{J_1} = 7.2 = \frac{K_{12}\beta_5K_{42}}{M_{\text{вых}}},
$$

$$
\frac{K_{12}\beta_5K_{41}K_{42}M_{\text{вых}}}{M_{\text{вх}}} = \frac{B}{J_1}, \quad \frac{D}{J_1} = 15.2 = K_{21}\cdot K_{13}\cdot K_{31} = K_{11}\cdot K_{21}\frac{M_{\text{вых}}}{M_{\text{вх}}}.
$$

Assuming $K_{41} = K_{42}$ and $K_{61} = 1$ and considering that $\beta_5 = \frac{1}{10}$, we select scales of presentation of variables. Having in mind best use of full scale of model, we let $x_{\text{вх}} = 1$ be represented in installation by voltage $U_0 = 100$. Then $M_{\text{вх}} = M_{\text{вых}} = 0.01$. Here transmission of factors of blocks will be:

$$
K_{14} = 8, \quad K_{61} = 1, \quad K_{21} = 3, \quad K_{11} = 5.07, \quad K_{31} = 5, \quad K_{13} = 1.01,
$$

$$
K_{41} = K_{42} = 1, \quad K_{12} = 0.72.
$$

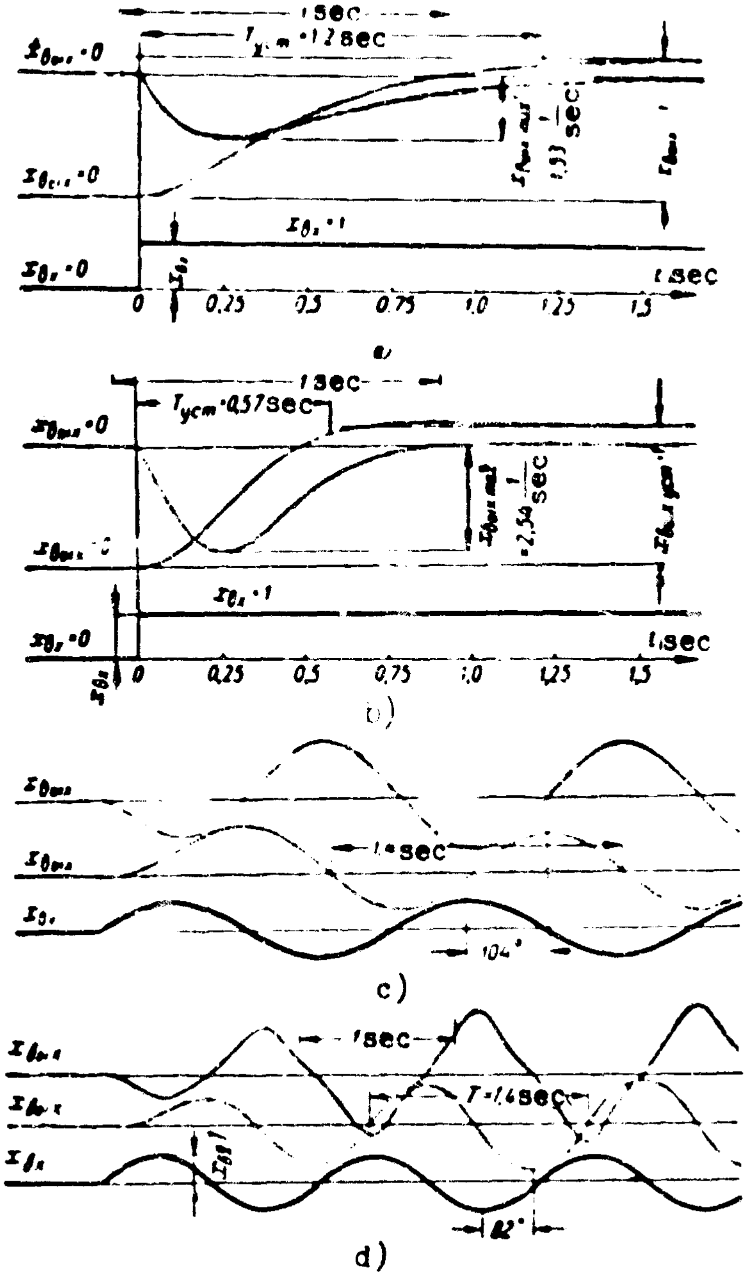


Fig. 250. Transients in servo system: a) with linear damping and step input; b) with nonlinear damping and step input; c) with linear damping and sinusoidal input; d) with nonlinear damping and sinusoidal input.

In Fig. 250 are shown oscillograms of transients in the circuit for two cases: with supplying of unit perturbation a and b and with sinusoidal input c and d. As can be seen from the oscillograms, duration of transients in system with unit perturbation is reduced approximately to half of that with linear variant system, possessing fairly great damping.

With sinusoidal input there is attained certain decrease of phase shift, but there appear significant nonlinear distortions. The latter indicates once again the well-known fact that by reaction of nonlinear system to unit perturbations one must not judge its behavior during perturbations of other types.

CHAPTER XIV

METHODS OF SIMULATING AUTOMATIC SYSTEMS WITH DYNAMIC RANGE OF CHANGE OF VARIABLE, EXCEEDING DYNAMIC RANGE OF ELECTRONIC MODEL

As is known, dynamic range of the best models of electonic analogs does not exceed 10^3. However, in solving a number of problems of automation it is necessary to deal with cases, when range of change of variable considerably exceeds this figure. Among these cases one can mention solution of problem of moving a flying object into the zone of self-guidance, processes of nuclear reactor startup, solution of certain problems of automation of chemical processes, etc. In solving these problems use of electronic models requires application of special methods. Of these methods one should mention: 1) decomposition of total interval of change of variables into subranges with limits, permissible for electronic model, of change and realization of automatic transition from one step to the next with corresponding change of scales of representation of variables and transmissions factors of separate blocks; 2) transformation of initial equations into logarithmic presentation of variables.

Below both enumerated methods are expounded in the example of investigation of processes of starting a nuclear reactor (B. Ya. Kogan [6]). Application of electronic models also to reproduction of processes of starting considerably expands region of their application, especially as trainers.

1. Fundamental Equations of Reactor Startup

To show the main laws of starting a reactor we will write the equation of kinetics in simplified form. With this aim we will take "one group" theory of calculation of delayed neutrons, and also the assumption that all allocated power goes for heating the reactor, and the effective multiplication factor linearly depends on position of the moderating rod and temperature. Upon these assumptions we have:

$$\left.\begin{aligned} \frac{dn}{dt} &= -\frac{\beta n}{l} - \frac{1-\beta}{l}\delta k_{эф} n + \lambda C + S. \\ \frac{dC}{dt} &= \frac{\delta k_{эф}\beta}{l} n + \frac{\beta}{l} n - \lambda C. \\ \frac{d\overline{T}}{dt} &= \mu n. \end{aligned}\right\} \qquad (14.1)$$

where $\delta k_{эф} = at - \alpha\overline{T}$.

Here are accepted the following designations : n—total number of neutrons at moment of time t; $\delta k_{эф}$ —effective multiplication factor; β —percent of delayed neutrons in total number of fission neutrons; l—average lifetime of prompt neutron; C—number of radioactive fragments of equivalent group of delayed neutrons; λ -disintegration constant of radioactive fragments of equivalent group; S—number of neutrons, radiated by outside source; $\overline{T}$ —average temperature of reactor; μ —proportionality factor, determined by physical properties of material of active zone and geometry of reactor; a—speed of change of reactance with advancement of rod; α —temperature coefficient of reactance.

By solving system (14.1) one must receive dependences n(t), C(t) and $\overline{T}$(t) for linear movement of rods and given initial conditions.

Neutron power of reactor and concentration of nuclei-emitters of delayed neutrons here can change in very wide limits (for example, from 0 to 10^{10}).

2. Solution of Equations of Startup by Sections

Solution of equations of startup by sections can be carried out both manually and automatically. In the first case solution is conducted up to the moment, when voltage, representing in the model initial variable n, reaches 100 v. Then the installation is changed to the "stop" mode, we record values of all voltages on integrators toward the end of the period of solution. Before beginning the following stage of solution the new initial conditions are set in a scale, corresponding this section of the solution. As shown below, values of transmission factors of certain computing blocks of the model must change with each transition to a new scale of representation of variables.

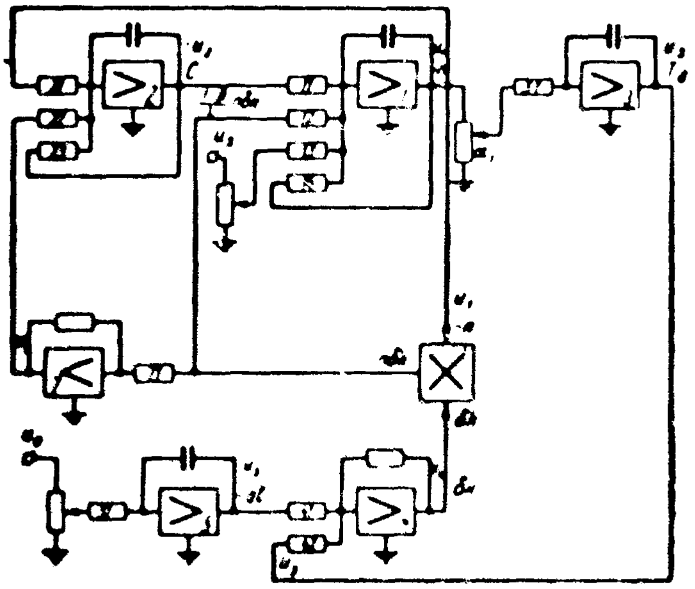


Fig. 251. Setup diagram of equations of reactor.

Indeed, relationships for voltages in the setup diagram of equations (14.1) (Fig. 251) will be:

$$\left.\begin{aligned}
\frac{dU_1}{dt} &= -K_{14}U_1 + 0.01K_{13}U_1U_4 - K_{12}a_2U_3 - K_{11}U_2,\\
\frac{dU_2}{dt} &= -K_{21}U_1 - 0.01K_{22}K_{21}U_1U_4 - K_{22}U_2,\\
\frac{dU_3}{dt} &= -a_1K_{31}U_1,\\
U_4 &= \int_0^t K_{41}K_{31}U_0^*\,dt - K_{42}U_3.
\end{aligned}\right| \qquad (14.2)$$

Introducing scales of representation of initial variables $n = M_nU_1$, $C = M_CU_2$, $\bar{T} = M\,U_3$, $\delta k_{\rho} = M_kU_4$, $S = M_SU_S$ and considering time scale to be one, we will receive the following relationships between coefficients of initial equation, scales and transmission factors of separate blocks:

$$\begin{gathered}
K_{14} = \frac{\beta}{l}, \quad K_{21}\frac{M_C}{M_n} = \frac{\beta}{l}, \quad M_kK_{41}K_{31}U_0^* = a,\\
\frac{0.01K_{13}}{M_k} = \frac{1-\beta}{l}, \quad \frac{K_{22}K_{21}\cdot 0.01M_C}{M_kM_n} = \frac{1}{l}, \quad \frac{M_k}{M_T}K_{42} = \alpha,\\
K_{11}\frac{M_n}{M_C} = \lambda, \quad K_{22} = \lambda, \quad \frac{M_n}{M_S}K_{12}a_2 = 1, \quad a_1K_{31}\frac{M_T}{M_n} = \mu.
\end{gathered}$$

From analysis of these relationships it follows that during change of scales M_n and M_C during transitions from one section of the excursion to the next it is simultaneously necessary to change values of a_2 and a_1. Thus, for example, if in process of startup M_n increases with transition from one section to the next by an order, then coefficient should be increased by the same magnitude a_1 and coefficient a_2 should decrease.

During process automation of simulation of startup of reactor with variable scales of representation of variables it is necessary to anticipate accumulation in new scale of voltage of initial conditions for the following stage of solution. This can be realized in principle by various memory units. In the work of B. Ya. Kogan [6] it was with this aim that it was proposed to double part of operational elements of set in order, that when one part is occupied in solution, the other

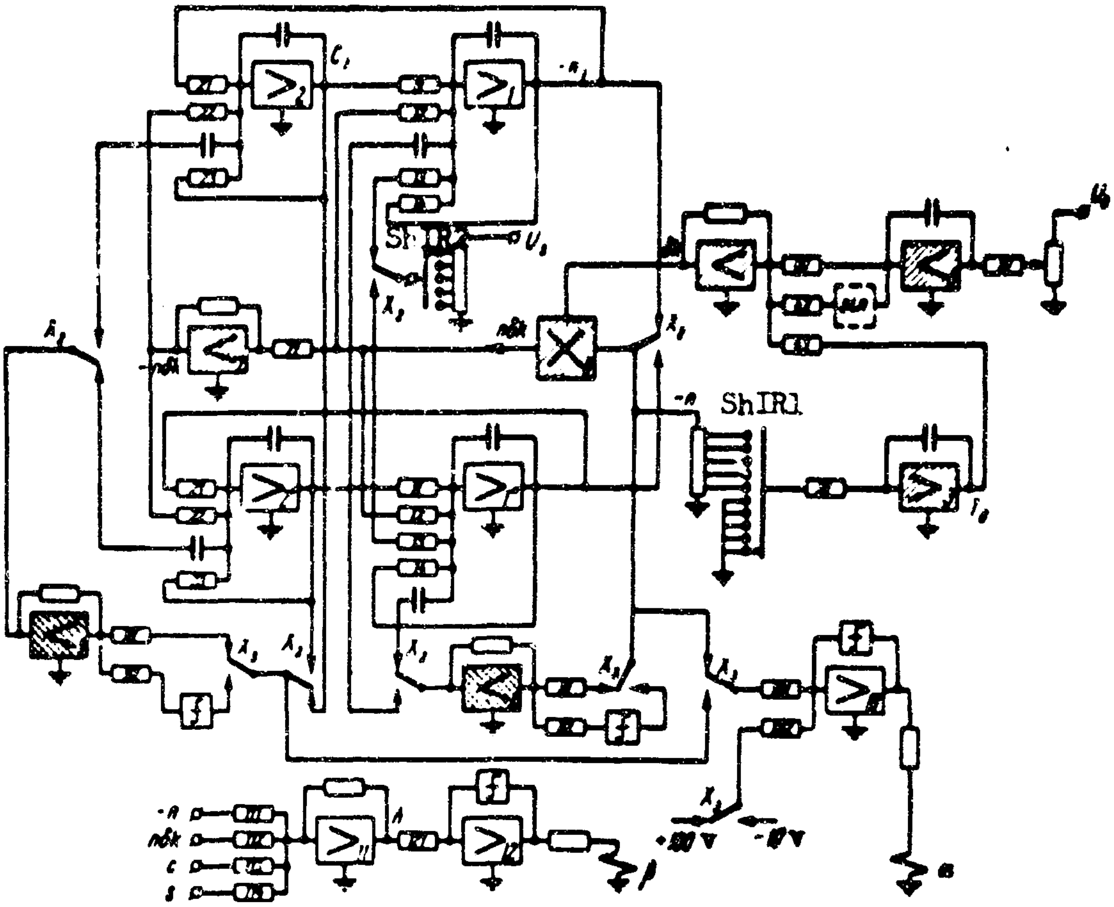


Fig. 252. Setup diagram of equations of reactor taking into account automatic change of scales during transition from one section of startup to the next.

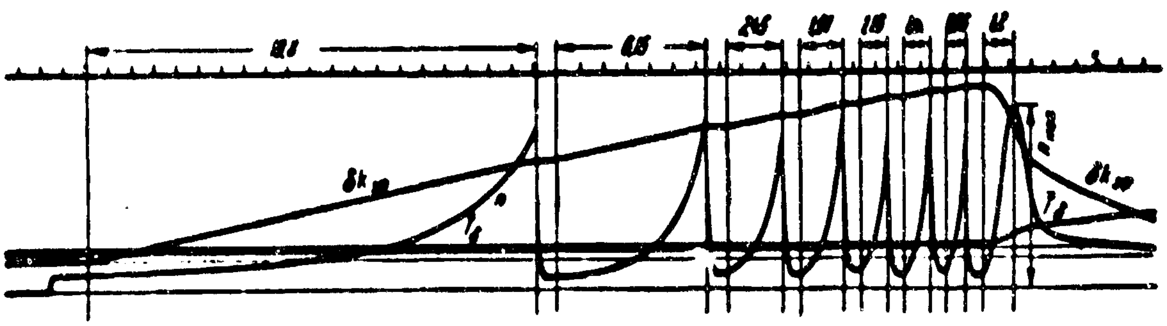


Fig. 253. Oscillogram of starting process of reactor by steps.

is prepared for solution, storing in new scale the initial conditions.

In Fig. 252 is shown setup diagram taking into account automatic transition from section to section.

Control circuit of simulator should with such method of solution ensure automatically a definite sequence of succession of regimes of each group of computing elements, participating in the solution. With this aim they use two signals: one, obtained from comparison circuit, switching on relay a when voltage, representing variable n, attains a magnitude of 100 v, and the other form the delay circuit. With growth of neutron power $\bar{T}$ grows and, consequently, thermal negative feedback increases by force of which neutron power starts to fall and as a result of an oscillatory process arrives at steady-state regime. Fall of neutron power under the influence of input of thermal negative feedback can in a number of cases be so large that for its reproduction it will be necessary, just as in build-up, to go through the whole process in sections. Here, naturally, with decrease of power it is necessary to decrease scale of representation of variables n and C.

Since process of change of concentration of nuclei-emitters of delayed neutrons will lag behind process of change of neutron power, then with decrease of power one must decompose the solution into intervals, being oriented on the process of change of C. For this purpose in the control circuit from operational amplifiers 11 and 12 there results a signal during change of sign of derivative $\frac{dn}{dt}$. Derivative $\frac{dn}{dt}$ is obtained by repeated summation of all components of first equation of system (14.1). Switching on of relay β excites auxiliary relay X_3, executing all necessary commutation for transition to tracking for U. Transmission factors of amplifiers 8 and 9 $K_{91} = K_{81} = 0.1$, and $K_{92} = K_{82} = 10$. So that output voltage of amplifiers 8 and 9 not exceed 100 v in process of tracking C, at inputs of 92 and 82 there are connected auxiliary limiters, preventing input into these amplifiers of a voltage exceeding 10 v. Simultaneously with change of scales M_n and M_c, it is necessary to change on every interval values of a_0 and a_1, but in the opposite direction. For this there are used reversible stepping selectors ШИР1 and ШИР2, changing direction of motion after every switching on of relay. In Fig. 253, as an example, there is brought the oscillogram of the process of starting of the reactor, obtained by the above method.

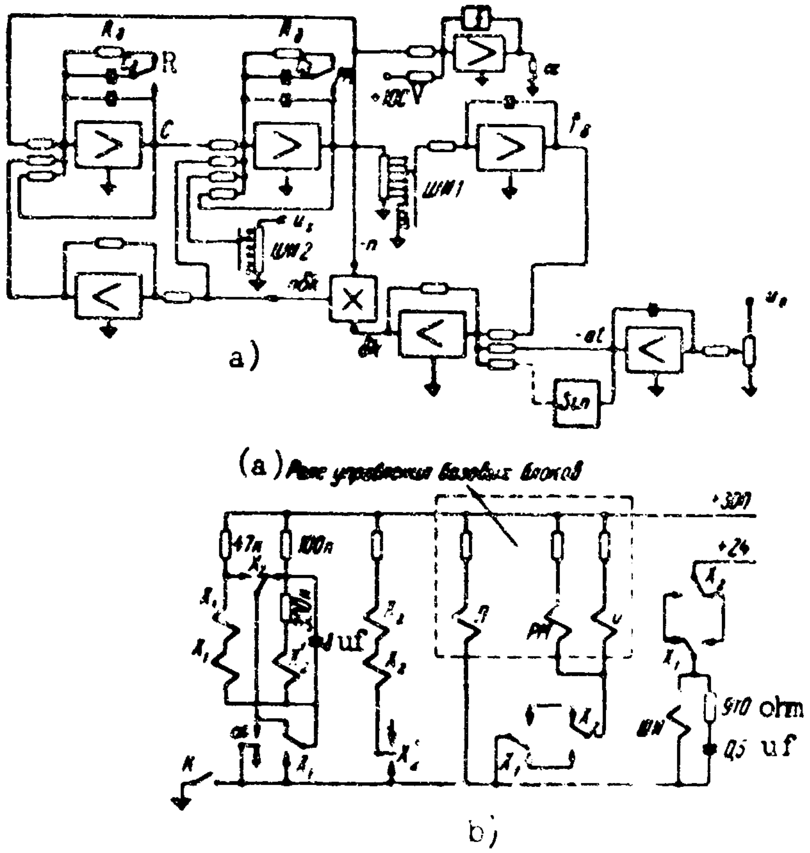


Fig. 254. A) Setup diagram with application of additional capacitors; B) Control of model with additional capacitors. KEY: (a) Control relay of base blocks.

When in process of starting of magnitudes n and C due to action of thermal feedback are not lowered more than an order or when one is interested only in the process of build-up power, it is possible to considerably simplify the scheme of automatic solution of problem by sections. With this aim at the beginning of every subsequent stage of solution for setting voltages of integrators C and n, decreased by 10 times, there is used connection for the preparatory period of additional capacitors parallel to capacitors of these integrating blocks* (Fig. 254a). It is natural that in periods of work these additional capacitors must be disconnected and reliably discharged. Since additional capacitors do not participate in process of solution, they need not have a polystyrene or styroflex dielectric.

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*Simultaneously with author this method was offered by N. Filipchak, V. Filipchak, T. Zelinskiy [1].

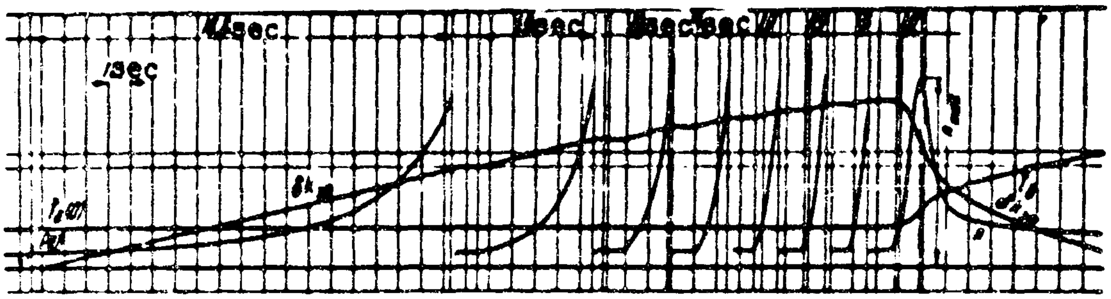

Fig. 255. Oscillogram of starting process with application of circuit with additional capacitors.

Magnitude of capacity of additional capacitor $C_{\text{д}}$ is by evident relationship

$$C_{\text{д}} = C\left(\frac{U_{\text{к}}}{U_{\text{н}}} - 1\right) \quad \text{when} \quad \frac{U_{\text{к}}}{U_{\text{н}}} = 10, \quad C_{\text{д}} = 9C. \tag{14.3}$$

where C is capacity of integrator, U_k is voltage at end of preceding working interval, U_H is voltage, which should be at the beginning of the subsequent working section.

Control and setup diagram with such a method of change of scales is considerably simplified (See Fig. 254a and b). Oscillogram of starting process, taken with the help of application of discharging capacitors, is shown in Fig. 255.

3. On Solution Error

Error in given interval of solution is caused by the following main factors: inaccuracy of setting of transmission factors of separate computer blocks, static error of multiplier, error of linear computing elements, inaccuracy of setting of initial conditions.

In accuracy of work of comparison circuits and scattering in operation of control relays need not in principle affect accuracy of solution. Their influence shows only on magnitude and number of intervals of solution.

Comparison of results of investigation of errors shows that one of the main components is caused by error of multiplier. In Table XII is brought comparison of data, received from solution of the problem of starting on digital computer and on an electronic analog with application of multipliers of various type.

Table XII

(b) Мощность (g) Время от начала пуска, *сек*	(a) Значения мощности реактора при пуске, полученные с помощью					(f) Примечание
	(c) цифровой электронной вычислительной машины	(d) электронной модели с электронным множительным устройством		(e) электронной модели с электромеханическим множительным устройством		
	$\lg (n)_{ц}$	$\lg n$	δn	$\lg n$	δn	
32	4,301	4,36	17%	4,318	4%	(h) Относительная погрешность $\delta n = \frac{(n)_{ц} - n}{(n)_{ц}}$
100	3,16					
152		3,29		3,29		
200	3,25	3,255		3,255		
300	3,26	3,258		3,258		
900	3,26	3,25		3,25		

KEY: (a) Value of power of reactor during starting, received by; (b) Power; (c) Digital computer; (d) Electronic analog with electronic multiplier; (e) Electronic analog with electromechanical multiplier; (f) Note; (g) Time from beginning of starting, sec; (h) Relative error.

Comparison of data of table indicates expediency of application of multipliers of increased accuracy. In a number of cases good results can be achieved with the help of an electromechanical multiplier (See, for example, G. Korn and T. Korn [3]). Considering that $\delta k_{ф}$ changes comparatively slowly and can be fed to inertial input of the multiplier, application of such a multiplier will not put limitation on transmission band of signal.

4. Investigation of Starting Processes with the Help of Equations, Preliminarily Converted to Logarithmic Representation of Power.

Expounded method is based on the fact that the system of initial differential equations (14.1) is solved not for power n, but for magnitude y, the inverse of the reactor period:

$$y = \frac{1}{n}\frac{dn}{dt}.$$

Here $\int y\,dt = \int \frac{1}{n}\,dn = \ln n$ and consequently, $y = \frac{d(\ln n)}{dt}$.

Designating for brevity C/n = x, S/n = W we find:

$$\frac{dx}{dt} = \frac{d\frac{C}{n}}{dt} = \frac{1}{n}\frac{dC}{dt} - \frac{C}{n}\frac{1}{n}\frac{dn}{dt}, \quad \frac{dW}{dt} = \frac{d\frac{S}{n}}{dt} = \frac{1}{n}\frac{dS}{dt} - \frac{S}{n}\frac{1}{n}\frac{dn}{dt}. \qquad (14.4)$$

Dividing the first two equations of system (14.1) by magnitude n and considering $\frac{dS}{dt} = 0$, we will write taking into account (14.1) a system of transformed equations (14.1) in the form:

$$\left.\begin{aligned} &\frac{dx}{dt} = -xy - \frac{\delta k_{эф}\beta}{l} - \lambda x + \frac{\beta}{l}, \quad \frac{d\bar{T}}{dt} = \mu n,\\ &\frac{dW}{dt} = -Wy, \quad y = -\frac{\beta}{l} + \frac{1-\beta}{l}\delta k_{эф} + \lambda x + W,\\ &\delta k_{эф} = at, \; -\alpha\bar{T}, \quad n = e^{\int y\,dt}. \end{aligned}\right\} \qquad (14.5)$$

Variables x, y in this system change in significantly narrower limits than in the initial one which allows us to investigate the process of starting as a whole. For solution of this system on electronic model there are required nonlinear computing elements: two multipliers* and one functional generator.

If we designate by y not the derivative of $\ln n$, but simply $\ln n$ itself, then we come to a system, equivalent to the one brought earlier, with the same setup diagram (Fig. 256). If one were to apply electromechanical or pulse-time multipliers, then by one multiplier there can be obtained both sought for products, having one general co-factor.

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*It is necessary to indicate that the requirement of accuracy of multiplier, delivering the product $W \cdot y$, may be not so high, since with growth of power n magnitude W begins to vanish and starting from a certain value n it can be completely ignored. Influence of thermal negative feedback at beginning of starting process practically is inconspicuous due to small values of temperature $\bar{T}$, and therefore, this coupling actually can be connected, only starting from a certain value of $\bar{T}$, i.e., after definite time after the starting process.

Equations of voltages for circuit of Fig. 256 will be:

$$\left.\begin{aligned}
U_1 &= -(K_{11}U_{01} + K_{12}U_{10} + K_{13}U_8 + K_{14}U_6).\\
\frac{dU_2}{dt} &= -K_{21}U_1.\\
U_3 &= -\nu e^{\xi U_2}.\\
\frac{dU_4}{dt} &= K_{41}U_3.\\
\frac{dU_5}{dt} &= -K_{51}U_{02}.\\
U_6 &= -(K_{61}U_4 + K_{62}U_5).\\
U_7 &= -0.01U_1 U_8\\
\frac{dU_8}{dt} &= (K_{81}U_7 + K_{82}U_8 + K_{83}U_5 + K_{84}U_4).\\
U_9 &= -0.01U_1U_{10}.\\
\frac{dU_{10}}{dt} &= -K_{101}U_9.
\end{aligned}\right\} \qquad (14.6)$$

Section of scales ν and ξ for functional generator is executed proceeding from requirement that when $U_2 = 100$ v, $U_3 = 100$ v.

Relationships for determination of scales of representation of variables will be: $y = -M_yU_1$, $n = -M_nU_3\bar{T} = M_TU_4$, $\delta k_{эф} = M_{\delta k}U_8$, $W = M_wU_{10}$, $\int y\,dt = M_jU_2$.

After substitution in initial system of equations of voltage and comparison with given system (14.5) we will receive:

$$M_yK_{11}U_{01} = \frac{1}{l}.\quad M_j = \frac{M_y}{K_{21}}.\quad \frac{M_n}{M_T}K_{41} = \alpha.\quad \frac{M_y}{M_W}K_{12} = 1.\quad \nu M_n = 1.\quad K_{62}K_{51}U_{02}M_n = a.$$

$$\frac{M_y}{M_x}K_{13} = \lambda.\quad \frac{\xi K_{21}}{M_y} = 1.\quad K_{62}K_{51}U_{02}M_x = a\frac{\beta}{l}.\quad \frac{M_y}{M_W}K_{14} = \frac{1-\beta}{l}.\quad \frac{K_{81}M_T}{M_x} = \mu.\quad \frac{K_{84}M_x}{M_T} = a\frac{\beta}{l}.$$

From these relationships it follows that selection of scales ν and ξ influenc value of transmission factors K_{21}, K_{41} and magnitude of scale M_n, and the latter in turn effects the maximum value of power, which can be represented by the electronic model. Limiting steepness, reproduceable by functional generator, to magnitude $S_\phi = 10$, we will take

$$U_3 = -\frac{100}{e^{10}}e^{\frac{U_2}{10}}. \qquad (14.7)$$

It is obvious that when $U_2 = 100$v, $U_3 = -100$ v, and when $U_2 = 0$, $U_3 =$ $= (100/e^{10}) < 0.005$ v.

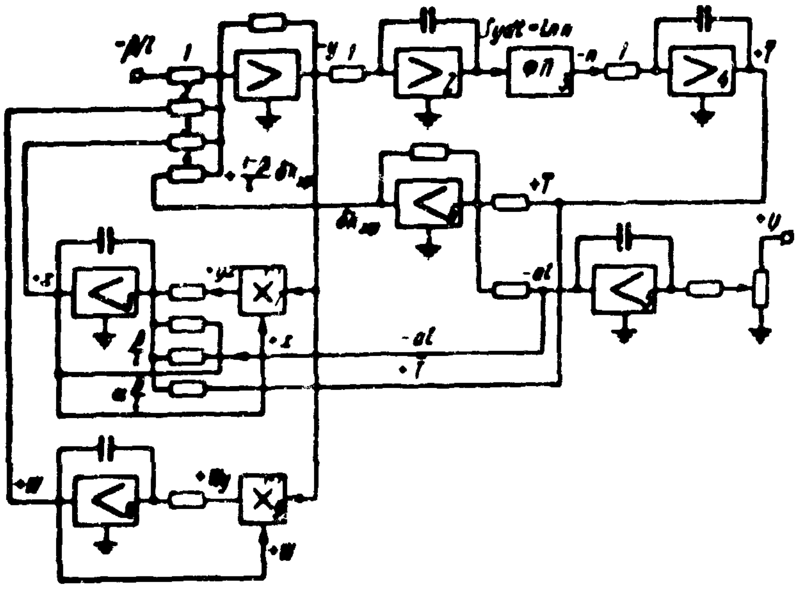

Fig. 256. Setup diagram of equations of reactor, transformed to logarithmic representation of power.

Steepness is

$$S_e = \left(\frac{dU_3}{dU_2}\right) = 10 \text{ when } U_2 = 100 \text{ v}.$$

Scale of representation of power is

$$M_n = \frac{1}{v}\frac{e^{10}}{100} = 220.26.$$

When maximum output voltage $U_3 = 100$ v, maximum power, which can be represented on model, constitutes

$$n_{max} = 22\,026.$$

It is necessary to note that during reproduction of dependence (14.7) by functional generator with piecewise-linear approximation when permissible error $\varepsilon = 0.25$ the least length of interval of decomposition of axis of argument

(See Chapter VI) constitutes

$$h_1 = \sqrt{\frac{8\varepsilon}{(e''_{вх})_{max}}} = \sqrt{\frac{8\cdot 0.25}{1}} = 1.41 \text{ v.}$$

Length of adjacent section will be

$$h_2 = \sqrt{\frac{8\cdot 0.25e^{10}}{e^{9.95}}} = 1.45 \text{ v.}$$

Realization of such a functional generator by diode elements, as calculation shows, requires at least 12 diode elements and runs into large difficulties from comparative proximity of points of switching of diode elements toward the end of the argument scale.

Application of quadratic approximation on diode elements with a carrier (R. A. Bruns [1], A. A. Maslov and Yu. G. Purlov [1]) or by combining thyrite nonlinear resistances (L. D. Kovach and W. Comley [2]) significantly simplifies the circuit of functional generator and increases accuracy and reliability of its work.

Indeed, with quadratic approximation with permissible error $\varepsilon = 0.25$ the length of least interval of decomposition of argument will be

$$h_1 = 5\sqrt[3]{\frac{\varepsilon}{(e'''_{вх})_{max}}} = 5\sqrt[3]{\frac{0.25}{1}} = 6.3 \text{ v.} \qquad (14.8)$$

Rounding to lower whole figure, we take $h_1 = 6$ v. Then remaining sections will be: $h_2 = 8$ v, $h_3 = 10$ v, $h_4 = 15$ v, $h_5 = 25$ v.

Thus, the total number of elements in case of quadratic approximation decreases more than half as compared with case of linear approximation. Diagram of functional generator with quadratic approximation is shown in Fig. 257.* Here for lowering of

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*Diagram was developed by A. A. Maslov and Yu. G. Purlov.

current characteristics of separate diode elements their coupling is taken mixed. First four diode elements (numeration is taken in order of growth of argument) are coupled at input, and two—in a circuit, parallel to amplifier. Ideal current-corroded characteristics of input circuit and circuit, parallel to amplifier, are shown in Fig. 258. From Fig. 258 it follows that with accuracy of 0.16% current characteristic of first diode element can be replaced by a segment of axis of abscissas.

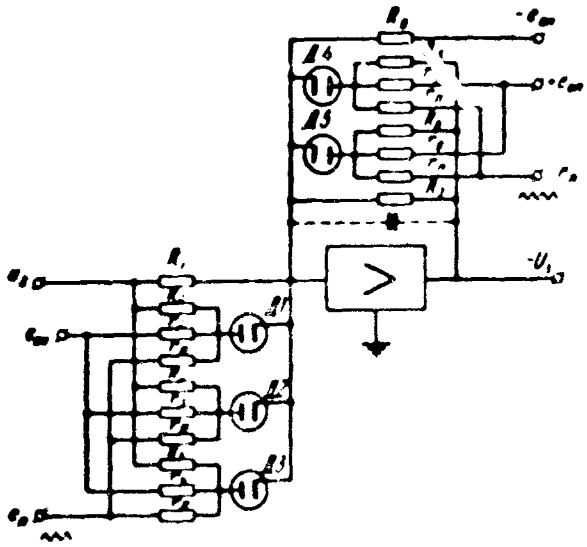

Fig. 257. Diagram of functional generator with diode elements and carrier.

In Fig. 259 is presented oscillogram of process of starting with logarithmic representation of power and use of electromechanical multiplier. Comparison of this solution with solution, received on digital machine, shows that maximum error constitutes 26-30%, including and error of reading of magnitudes from oscillogram (from finite thickness of lin , which themselves can reach 10%. It is natural that application of electronic multiplier based on thyrites in this case leads to increase of error.

In logarithmic representation of power to sources of error, taking place in case of solution of problem by sections, inaccuracy of work of functional generator (antilogarithm). However, inaccuracy of work of functional generator leads to increase of error of resolution only starting from moment of time, when thermal negative feedback becomes effective, i.e., after first maximum attained by power in process of startup.

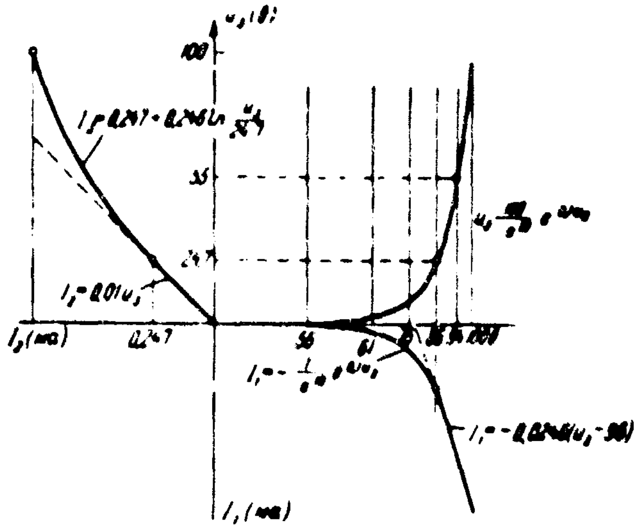

Fig. 258. Current characteristics of functional generator with carrier.

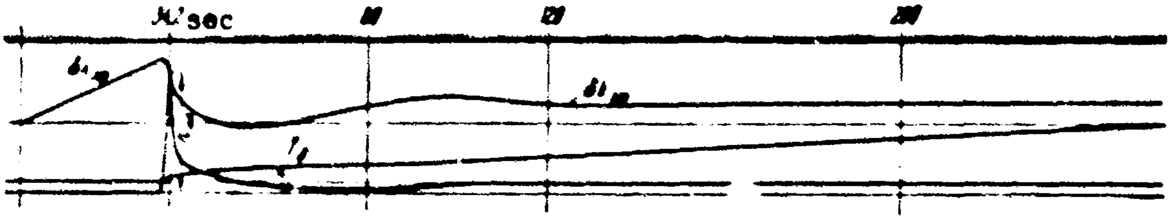

Fig. 259. Oscillogram of process of starting with logarithmic representation of power.

Thus, during solution on electronic models of problems connected with investigation of processes with wide range of change of variables, it is expedient to apply circuit of solution by sections with automatic transition from one section to the next. This circuit gives higher accuracy as compared with solution during logarithmic presentation of variables, although it requires certain increase of number of computing elements and a special control circuit.

When investigation is conducted only on section of build-up of variable or when lowering of variable after reaching a maximum does not exceed two orders, modeling should also be conducted by sections with automatic change of scale of representation of variables with the help of circuit with additional capacitors. This circuit is marked for simplicity of control system and does not require increase of number of computer elements.

With use of logarithmic representation of power the circuit is complicated by necessity to have an additional generator (reproducing antilogarithmic dependence) and gives less accuracy of solution by sections.

From what has been presented, furthermore, it follows that functional generator in circuit with logarithmic representation can be practically realized only with application of piecewise-quadratic approximation and limits magnitude of variable, which can be represented in electronic model to five orders.

APPENDIXES

APPENDIX I

REPRODUCTION OF COMBINED LINEAR OPERATIONS BY ONE OPERATIONAL AMPLIFIER

Linear operations, executed by operational amplifier in combination with simplest passive electric circuits, already were considered in Chapter II. It turns out that by means of complication of these passive electric circuits it is possible by one amplifier to obtain more complicated linear operations. With such use of operational amplifier we can reduce quantity of amplifiers necessary for solution of problem, and in certain cases (for example, during reproduction of oscillatory sections of CAP) lower limitations, caused by imperfectness of frequency responses of computing blocks.

However, such complication of passive circuits of operational amplifier often leads to a state where we can no longer clearly divide input circuit from the circuit, parallel to the amplifier. This in turn demands a more general approach to derivation of transfer function of such an operational amplifier.*

Let us consider the most general circuit diagram of linear circuits and amplifier (Fig. 260).

In this diagram to input of operational amplifier and parallel to it are switched passive multipole networks Y_1 and Y_2. For finding transfer function of such operational amplifier we will write equation of currents at the poles:

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*Such approach is met in a number of works of F. H. Raymond [1] and R. Peretz [1].

$$\left.\begin{aligned}\bar{I}_{11}&=[Y_{11}(P)]_1\bar{e}_{\text{вх}}+[Y_{12}(P)]_1\bar{e}_{\text{вых}}+[Y_{11}(P)]_1\bar{e}_{\Sigma}.\\ \bar{I}_{21}&=[Y_{21}(P)]_1\bar{e}_{\text{вх}}+[Y_{22}(P)]_1\bar{e}_{\text{вых}}+[Y_{23}(P)]_1\bar{e}_{\Sigma}.\\ \bar{I}_{31}&=[Y_{31}(P)]_1\bar{e}_{\text{вх}}+[Y_{32}(P)]_1\bar{e}_{\text{вых}}+[Y_{33}(P)]_1\bar{e}_{\Sigma}\end{aligned}\right\}\quad(1.1)$$

and

$$\left.\begin{aligned}\bar{I}_{12}&=[Y_{11}(P)]_2\bar{e}_{\text{вх}}+[Y_{12}(P)]_2\bar{e}_{\text{вых}}+[Y_{13}(P)]_2\bar{e}_{\Sigma}.\\ \bar{I}_{22}&=[Y_{21}(P)]_2\bar{e}_{\text{вх}}+[Y_{22}(P)]_2\bar{e}_{\text{вых}}+[Y_{23}(P)]_2\bar{e}_{\Sigma}.\\ \bar{I}_{32}&=[Y_{31}(P)]_2\bar{e}_{\text{вх}}+[Y_{32}(P)]_2\bar{e}_{\text{вых}}+[Y_{33}(F)]_2\bar{e}_{\Sigma}.\end{aligned}\right\}\quad(1.2)$$

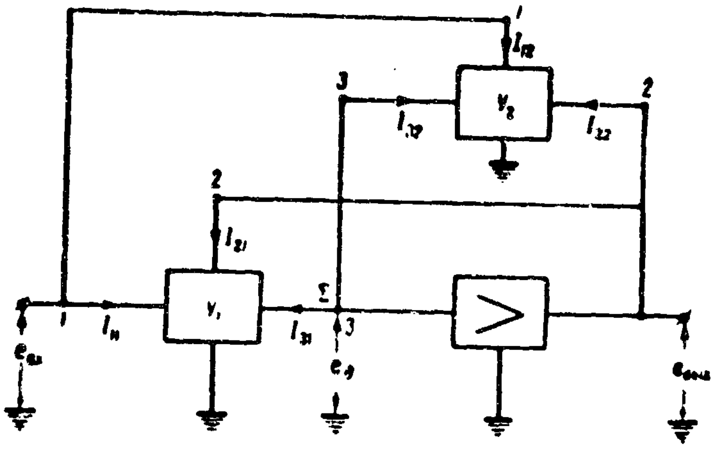


Fig. 260. Skeleton diagram for deriving basic relationships.

In expressions (1.1) and (1.2) the index after brackets indicates number of multiterminal network. Conductances $[Y_{ij}(P)]_1$ and $[Y_{ij}(P)]_2$ are determined as ratio of current I_{i1} or I_{i2} in considered pole to voltage, standing as a factor in sought for conductance on the condition that all remaining voltages, determining current of pole, are equal to zero.

Thus, for example, $[Y_{11}(P)]_1=\frac{\bar{I}_{11}}{\bar{e}_{\text{вх}}}$ when $e_{\text{вых}}=e_{\Sigma}=0$; $[Y_{12}(P)]_1=\frac{\bar{I}_{11}}{\bar{e}_{\text{вых}}}$ when $e_{\text{вх}}=e_{\Sigma}=0$. Disregarding grid current of input cascade of amplifier, we arrive, as before, at equations of operational amplifier in the form

$$\left.\begin{aligned}\bar{I}_{31}+\bar{I}_{32}&=0,\\ e_{\text{вых}}&=-K_y e_{\Sigma}.\end{aligned}\right\}\quad(1.3)$$

where $\bar{I}_{31}$ and $\bar{I}_{32}$ are determined by third lines of expressions (1.1) and (1.2).

On basis of (1.1), (1.2) and (1.3) we obtain finally operational equation of operational amplifier in the most general form:

$$\bar{e}_{\text{вых}} = -\frac{[Y_{31}(P)]_1 + [Y_{31}(P)]_2}{[Y_{32}(P)]_1 + [Y_{32}(P)]_2} \cdot \frac{\bar{e}_{\text{вх}}}{1 - \frac{1}{K_y}\frac{[Y_{33}(P)]_1 + [Y_{33}(P)]_2}{[Y_{32}(P)]_1 + [Y_{32}(P)]_2}}. \tag{1.4}$$

With very great K_y, when in working range of frequencies the following is correct

$$\left|\frac{1}{K_y}\frac{[Y_{33}(j\omega)]_1 + [Y_{33}(j\omega)]_2}{[Y_{32}(j\omega)]_1 + [Y_{32}(j\omega)]_2}\right| \ll 1,$$

equation (1.4) is simplified:

$$\bar{e}_{\text{вых}} \approx -\frac{[Y_{31}(P)]_1 + [Y_{31}(P)]_2}{[Y_{32}(P)]_1 + [Y_{32}(P)]_2}\bar{e}_{\text{вх}}. \tag{1.5}$$

Relative error, caused by such simplification (finite value of K_y), is determined by expression

$$\delta\bar{e}_{\text{вых}} = \frac{1}{K_y}\frac{[Y_{33}(P)]_1 + [Y_{33}(P)]_2}{[Y_{32}(P)]_1 + [Y_{32}(P)]_2}. \tag{1.6}$$

Let us consider several particular cases, flowing from general relationships (1.5) and (1.6).

Let in circuit of operational amplifier there be used only multiterminal network Y_1. Then in expressions (1.5) and (1.6) all conductances with index "2" after square brackets must be set equal to zero.

As a result we arrive at relationships:

$$\bar{e}_{\text{вых}} = -\frac{[Y_{31}(P)]_1}{[Y_{32}(P)]_1}\bar{e}_{\text{вх}}. \tag{1.7}$$

$$\delta\bar{e}_{\text{вых}} = \frac{1}{K_y}\frac{[Y_{33}(P)]_1}{[Y_{32}(P)]_1}. \tag{1.8}$$

where $[Y_{31}(P)]_1 = \frac{\bar{I}_{31}}{\bar{e}_{\text{вх}}}$ when $e_{\text{вых}} = e_{\Delta} = 0$, $[Y_{32}(P)]_1 = \frac{\bar{I}_{31}}{\bar{e}_{\text{вых}}}$ when $e_{\text{вх}} = e_{\Delta} = 0$, $[Y_{33}(P)]_1 = \frac{\bar{I}_{31}}{\bar{e}_{\Delta}}$ when $e_{\text{вых}} = e_{\text{вх}} = 0$.

If we use only multiterminal network Y_2, then, proceeding as in preceding case, we arrive at expression of form

$$\bar{e}_{\text{вых}} = -\frac{|Y_{31}(P)|_2}{|Y_{32}(P)|_2}\,\bar{e}_{\text{вх}}. \tag{1.9}$$

$$\delta\bar{e}_{\text{вых}} = \frac{1}{K_y}\,\frac{|Y_{33}(P)|_2}{|Y_{32}(P)|_2}. \tag{1.10}$$

When in circuit of operational amplifier we use instead of multiterminal networks two quadripoles Y_1 and Y_2, connected so that Y_2 is not directly coupled with input, and Y_1 is not directly coupled with output of operational amplifier, we obtain

$$|Y_{31}(P)|_2 = |Y_{32}(P)|_1 = 0.$$

Therefore

$$\bar{e}_{\text{вых}} = -\frac{|Y_{31}(P)|_1}{|Y_{32}(P)|_2}\,\bar{e}_{\text{вх}}. \tag{1.11}$$

$$\delta\bar{e}_{\text{вых}} = \frac{1}{K_y}\,\frac{|Y_{33}(P)|_1 + |Y_{33}(P)|_2}{|Y_{32}(P)|_2}. \tag{1.12}$$

Some practically important circuits of operational amplifiers, ensuring fulfillment of combined linear operations, calculated by these formulas, are shown in Table XIII below.

Of great interest is circuit, brought in third column of XIII on page 467 reproducing undamped harmonic oscillations. Use of such a circuit as pickup of sinusoidal oscillations has this deficiency, that for change of frequency of oscillations it is necessary simultaneously to change all parameters of the circuit. However, here there is attained economy of computing blocks and bandwidth of generated sinusoidal oscillations is considerably expanded.

Table XIII

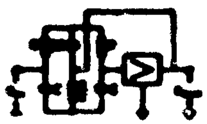
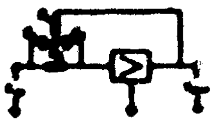
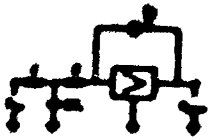
Parameters of random circuits and equations of operational amplifiers / Circuit diagram of random circuits of operational amplifier				
Value of conductance of input circuit	$\lvert Y_{in}(p)\rvert_1$	$\frac{R^2C^2p^2+1}{2R(RCp+1)}$	$\frac{\frac{1}{C_1C_2p^2}+R_1\left(\frac{1}{C_1p}+\frac{1}{C_2p}\right)+R_1R_2}{R_2\left[\frac{1}{C_1C_2p^2}+R_1\left(\frac{1}{C_1p}+\frac{1}{C_2p}\right)\right]}$	$\frac{1}{2R(RC_0p+1)}$
	$\lvert Y_{in}(p)\rvert_2$	$\frac{2Cp}{1+RCp}$	$\frac{C_2p}{1+R_1C_2p\left(\frac{C_1}{C_2}+1\right)}$	—
	$\lvert Y_{in}(p)\rvert_3$	$\frac{(RCp+1)^2+2RCp}{2R(1+RCp)}$	$\frac{R_1C_1p+1}{\left[\frac{1}{C_2p}+R_1\left(\frac{C_1}{C_2}+1\right)\right]}+\frac{1}{R_2}$	$\frac{2RC_0p+1}{2R(RC_0p+1)}$
Value of conductance of circuit parallel to amplifier	$\lvert Y_{os}(p)\rvert_1$	—	—	—
	$\lvert Y_{os}(p)\rvert_2$	—	—	$0.8C_0p$
	$\lvert Y_{os}(p)\rvert_3$	—	—	$0.2C_0p$
Relative error δ		$\delta\bar{e}_{out}=\frac{1}{4A_y}\left[RCp+4+\frac{1}{RCp}\right]$	$\delta\bar{e}_{out}=\frac{1}{A_y}\left[\frac{1}{R_2C_2p}+R_1C_1p+\frac{R_1}{R_2}\left(\frac{C_1}{C_2}+1\right)+1\right]$	$\delta\bar{e}_{out}=\frac{1}{A_y}\left[1+\frac{1-2RC_0p}{RC_0p(RC_0p-1)}\right]$
Equation of operational amplifier		$\bar{e}_{out}=-\frac{\lvert Y_{os}(p)\rvert_1}{\lvert Y_{in}(p)\rvert_1}\bar{e}_{in}$ $\bar{e}_{out}=-\frac{R^2C^2p^2+1}{4RCp}\bar{e}_{in}=$ $=-\frac{1}{4}\left(RCp+\frac{1}{RCp}\right)\bar{e}_{in}$	$\bar{e}_{out}=-\left[\frac{1}{R_2C_2p}+R_1C_1p+\frac{R_1}{R_2}\left(1+\frac{C_1}{C_2}\right)\right]\bar{e}_{in}$	$\bar{e}_{out}=-\frac{1}{RC_0p(RC_0p-1)}\bar{e}_{in}$

Table XIII Continued

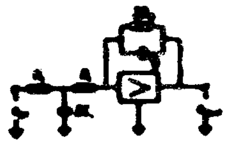
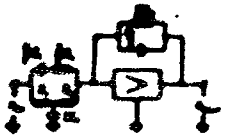
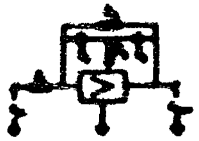
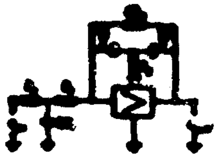
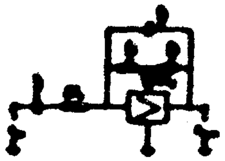
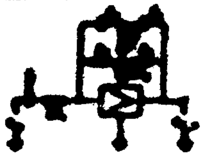
Parameters of passive circuits and equations of operational amplifiers / Circuit diagram of passive circuits of operational amplifier				
Value of conductance of input circuit	$[Y_{11}(p)]_a$	$\frac{1}{2R_1(R_1C_1p+1)}$	$\frac{1+aR_1C_1p}{2R_1(1+R_1C_1p)}$	$\frac{1}{R_1}$
	$[Y_{12}(p)]_a$	—	—	—
	$[Y_{22}(p)]_a$	$\frac{2R_1C_1p+1}{2R_1(R_1C_1p+1)}$	$\frac{1}{R_1}+\frac{2R_1C_1p+1}{2\left[\frac{(R_1C_1p+1)(2R_1C_1p+1)}{aC_1p}+R_1\right]}$	$\frac{1}{R_1}$
Value of conductance of circuit, parallel to amplifier	$[Y_{11}(p)]_b$	—	—	—
	$[Y_{12}(p)]_b$	$\frac{1-R_2C_2p}{2R_2}$	$\frac{1+R_2C_2p}{2R_2}$	$\frac{1+2R_2C_2p+R_2R_3C_2^2p^2}{R_3(2R_2C_2p+1)}$
	$[Y_{22}(p)]_b$	$\frac{1+C_2R_2p}{2R_2}$	$\frac{1+C_2R_2p}{2R_2}$	$\frac{1}{R_3}+C_2p\frac{R_2C_2p+1}{2R_2C_2p+1}$
Relative error δ_{out}		$\bar{\delta}_{out}=\frac{1}{K_y}\left[1+\frac{R_2}{R_1}\frac{(2R_1C_1p+1)}{(R_1C_1p+1)(R_2C_2p+1)}\right]$	$\bar{\delta}_{out}=$ $=\frac{1}{K_y}\left[1+\frac{\frac{R_2}{R_1}+(2-a)R_2C_1p}{(1+R_1C_1p)(1+R_2C_2p)}\right]$	$\bar{\delta}_{out}=$ $\frac{1}{K_y}\left[1+\frac{1+\frac{R_3}{R_1}(2R_2C_2p+1)}{1+2R_2C_2p+R_2R_3C_2^2p^2}\right]$
Equation of operational amplifier		$\bar{e}_{out}=-\frac{R_2}{R_1(R_1C_1p+1)(R_2C_2p+1)}\bar{e}_{in}$	$\bar{e}_{out}=$ $-\frac{R_2}{R_1}\frac{1+aR_1C_1p}{(1+R_1C_1p)(1+R_2C_2p)}\bar{e}_{in}$	$\bar{e}_{out}=$ $-\frac{R_3}{R_1}\frac{(2R_2C_2p+1)\bar{e}_{in}}{1+2R_2C_2p+R_2R_3C_2^2p^2}$

Table XIII Continued

[illegible]				
				
[illegible]	$[Y_m(p)]_1$	$\frac{1}{2R_1(R_1C_1p+1)}$	$\frac{C_1p}{2(1+R_1C_1p)}$	$\frac{C_1p}{2(R_1C_1p+1)}$
	$[Y_m(p)]_2$	—	—	—
	$[Y_m(p)]_3$	$\frac{2R_1C_1p+1}{2R_1(R_1C_1p+1)}$	$\frac{1}{2}\frac{C_1p}{1+R_1C_1p}$	$\frac{C_1p}{2(1+R_1C_1p)}$
[illegible]	$[Y_m(p)]_1$	—	—	—
	$[Y_m(p)]_2$	$\frac{1+R_2C_2p+R_3R_2C_2^2p^2}{4R_3(R_2C_2p+1)}$	$\frac{1+R_2C_2p+R_2^2C_2C_3p^2}{2R_2(R_2C_2p+1)}$	$\frac{(R_2C_2p)^2+1}{2R_2(1+R_2C_2p)}$
	$[Y_m(p)]_3$	$\frac{1}{4R_3}+\frac{C_2}{2}p\frac{\frac{R_3}{2}C_2p+1}{R_2C_2p+1}$	$\frac{C_3p}{2}+\frac{R_2C_2p+1}{\frac{R_2C_2}{2}p+1}$	$\frac{1+4R_2C_2p+R_2^2C_2^2p^2}{2R_2(1+R_2C_2p)}$
[illegible]		$\bar{\varepsilon}_{out} = -\frac{1}{K_y}\left[1+\frac{1+\frac{2R_3}{R_1}\frac{(2R_1C_1p+1)(R_2C_2p+1)}{(R_1C_1p+1)}}{1+R_2C_2p+R_3R_2C_2^2p^2}\right]$	$\bar{\varepsilon}_{out} = \frac{1}{K_y}\left[1+\frac{(R_2C_2p+1)\left[\frac{C_1R_2p}{R_1C_1p+1}+\frac{2R_2(R_2C_2p+1)}{\frac{R_2C_2}{2}p+1}\right]}{1+R_2C_2p+R_2^2C_2C_3p^2}\right]$	$\bar{\varepsilon}_{out} = \frac{1}{K_y}\times\left[1+\frac{2R_2C_2p+\frac{R_2C_1p(1+R_2C_2p)}{1+R_1C_1p}}{(R_2C_2p)^2+1}\right]$
Equation of operational amplifier		$\bar{e}_{out} = -\frac{2R_3}{R_1}\frac{(R_2C_2p+1)e_{in}}{(R_1C_1p+1)(1+R_2C_2p+R_3C_2^2R_2p^2)}$ при $R_2C_2=R_1C_1$ $\bar{e}_{out} = -\frac{2R_3}{R_1}\frac{1}{1+R_2C_2p+R_3R_2C_2^2p^2}e_{in}$	$\bar{e}_{out} = -\frac{R_2C_1p(R_2C_2p+1)e_{in}}{(1+R_1C_1p)(1+R_2C_2p+R_2^2C_2C_3p^2)}$ при $R_2C_2=R_1C_1$ $\bar{e}_{out} = -\frac{R_2C_1p}{(1+R_2C_2p+R_2^2C_2C_3p^2)}\bar{e}_{in}$	$\bar{e}_{out} = -\frac{R_2C_1p(1+R_2C_2p)}{(1+R_1C_1p)(R_2^2C_2^2p^2+1)}\bar{e}_{in}$ при $R_1C_1=R_2C_2$ $\bar{e}_{out} = -\frac{R_2C_1p}{R_2^2C_2^2p^2+1}e_{in}$

APPENDIX II

BRIEF TECHNICAL CHARACTERISTICS OF CERTAIN TYPES OF D-C ELECTRONIC ANALOG COMPUTERS

Table XIV. General Specifications of Certain Electronic Analog Computers, Developed in USSR

No. in order	Type of analog computer	Form of differential equations solved on machine	Maximum order of differential equations solvable or number of first order equations in system	Permissible duration of integration in sec	Dimensions length x width x height in cm	Power consumed from net in kw	
						[illegible]	[illegible]
1	[illegible]	Linear	9	150	200 × 40	0,5	2
2	[illegible]	"	16	200	700 × 80 × 120	1,3	5
3	[illegible]	Linear with typical nonlinearities	9	200—400	822 × 476 × 1320	—	2,1
4	[illegible]	Linear	6	150	120 × 41 × 70	0,23	0,85
5	[illegible]	Nonlinear	6	150	Space occupied 0.1 m²		0,74
6	[illegible]	"	32	1000	Space occupied [illegible] m²		25
7	[illegible]	"	6	200	60 × 30 × 100	0,07	0.76
8	[illegible]	"	6	2000	60 × 50 × 54	0,07	0,35
9	[illegible]	"	6	200	Space occupied 0.1 m²	—	0,130
10	[illegible]	"	Set of standard blocks	2000	35 × 30 × 30	—	0,05
11	[illegible]	Set of nonlinear blocks	—	Not limited	Place occupied on table [illegible] m² Dimensions of base block [illegible] cm	—	0,7
12	[illegible]	Nonlinear	10	120	200 × 30 × 175	—	
13	[illegible]	Linear and nonlinear with [illegible]	30	From 1 sec to 10000 sec	Requires space of about [illegible] m²	—	15
14	[illegible]	Linear and nonlinear with variable coefficients [illegible]	24	1000 sec	Requires space of about [illegible] m²	—	2
15	[illegible]	Linear and nonlinear [illegible]	6—8	Repetition frequency of solution [illegible] solutions/sec	15 m²		10

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*Data for these electronic models are brought in article of E. A. Glusberg [1].

**Figures are given for one base block, designed for resolution of second order equations.

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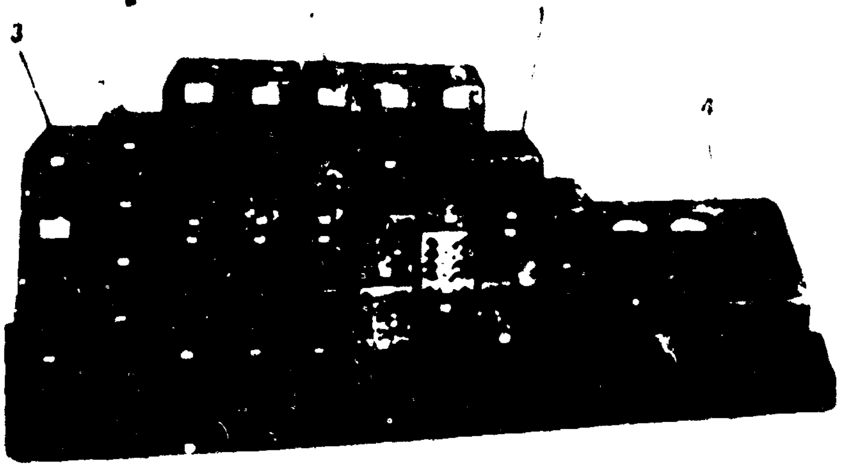

Fig. 261. Electronic analog computer of type IPT-5 (industrial model). 1 - block of operational amplifier, 2 - block of variable coefficient, 3 - block of coefficient, 4 - control panel.

Installation is intended for solution of ordinary linear differential equations up to the ninth order inclusively with constant and variable coefficients. It is constructed in the form of separate blocks (in one block is located one computing element), united by woggle joints in accordance with problem to be solved. Total complex of computing blocks is fed from the stabilized rectifiers.

The installation is composed of:
1. Operational amplifiers — 18 (9 with plate load 20 kilohm, and 9 piece with plate load 10 kilohm).
2. Blocks of constant coefficients (dividers) — 18.
3. Blocks of variable coefficients — 18.
4. Control panel.

Operational amplifiers are three-cascade with triode compensation in first cascade (see diagram of Fig. 37). Installation allows union with automatic control equipment. There is automatic repetition of resolution for possibility of observation of solution of cathode-ray indicator.

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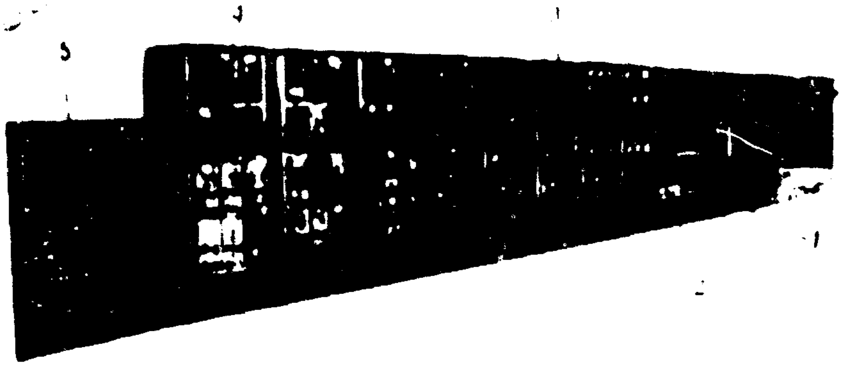

Fig. 262. Electronic analog computer of type MPT-9 (industrial model). 1 - setup field, 2 - control panel, 3 - section of operational amplifiers, 4 - section of blocks of variable coefficients, 5 - section of blocks of constant coefficients.

Installation is intended for solution of ordinary linear differential equations up to 16-th order inclusively with constant and variable coefficients. It is constructed in the form of separate section-stands.

The installation is composed of:

1. Section of operational amplifiers — 4 (total operational amplifiers — 48; 16 integrating and 32 scale — summing).
2. Section of blocks of variable coefficients — 4 (total blocks of variable coefficients -- 48).
3. Sections of blocks of constant coefficients — 4 (total blocks of constant coefficients [dividers] — 48).
4. Control panel.
5. Setting field.
6. Special crystal oscillator 50 cps.
7. Power blocks (electronic stabilized rectifiers with ferroresonant stabilizers) — 4.
8. Power supplies of incandescence — 2.

Operational amplifiers are made with automatic stabilization of zero level (by circuit of Fig. 48). Blocks of variable coefficients are made of stepping selectors.

Installation allows combination with automatic control equipment. There is automatic repetition of solution to ensure possibility of observation of solution on cathode-ray indicator.

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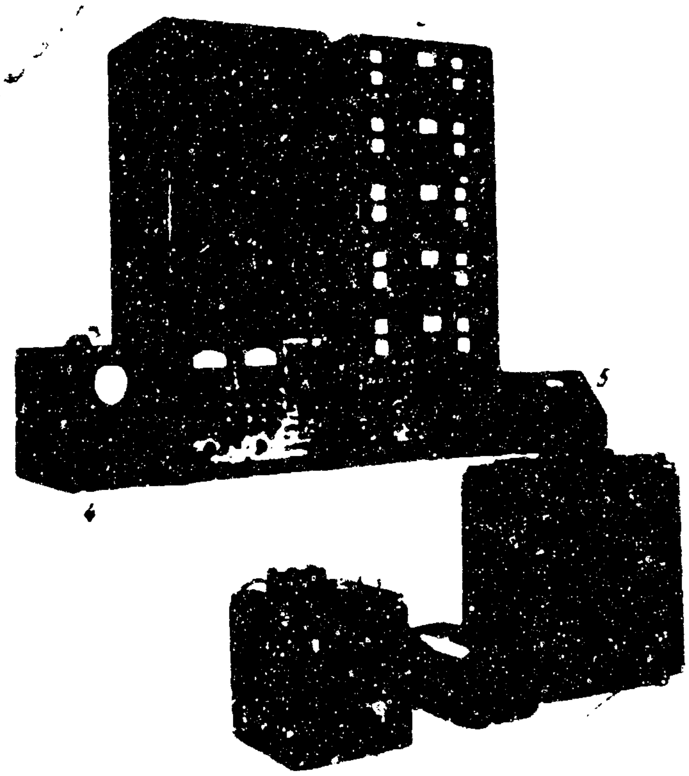

Fig. 263. Electronic analog computer type LMU-1 (industrial model): 1 - section of operational amplifiers — main, 2 - section of variable coefficients, 3 - power units, 4 - cathode-ray indicator, 5 - panel of check of blocks of variable coefficients.

Installation is intended for solution of ordinary linear differential equations with coefficients constant and variable in time up to ninth order, and also reproduction of typical nonlinear characteristics. It is constructed in the form of machine of structural-sectional type with two separate stabilized power supplies.

The installation is composed of: 18 operational amplifiers, 20 variable coefficients, 18 voltage dividers, 68 plug-in input impedances, 1 setting field (not interchangeable), 12 input resistance boxes, 8 diode elements.

Operational amplifiers are three-cascade with triode compensation in first cascade. Gain factor of amplifier at zero frequency is $5 \cdot 10^3$, drift 3 millivolt after 10 minute, bandwith in closed state 500 cps at level 0.99. Amplifiers are located 6 to a plateau.

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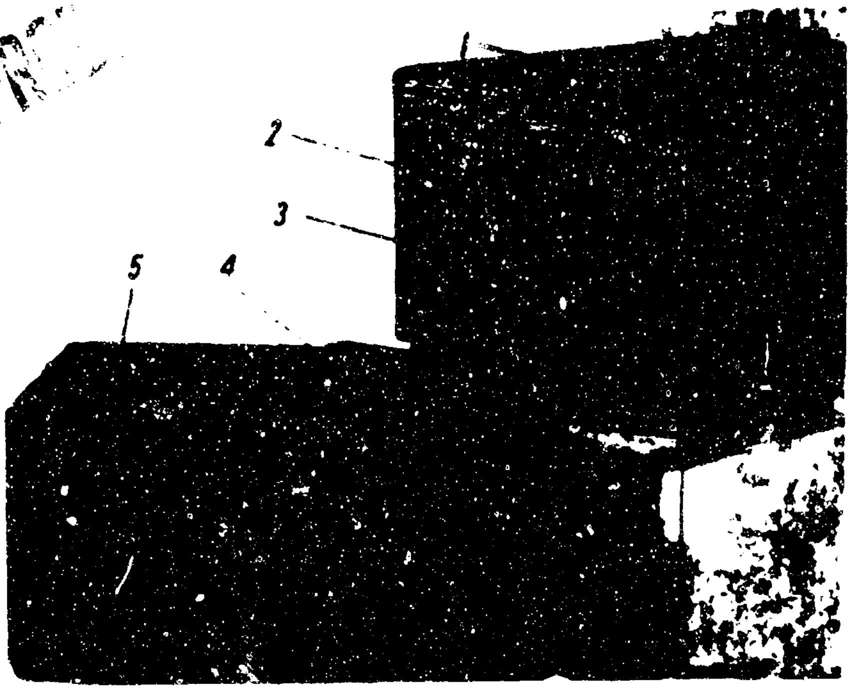

Fig. 264. Electronic analog computer of type EMU-3 (Academy of Sciences of USSR). 1 - block of operational amplifier, 2 - block of constant coefficient, 3 - control panel, 4 - block of adjustment, 5 - power block.

Installation is intended for solution of ordinary linear differential equations up to sixth order inclusively with coefficients constant and variable in time. Structural type in the form of a stand. Setup is carried out by connection of separate blocks by flexible cords.

Feeding is carried out from one stabilised rectifier.

The installation is composed of:

1. Operational amplifiers — 12 (allowing external load of 10 ma).
2. Blocks of constant coefficients — 22.
3. Blocks with capacitors — 2 (0.01 — 1 microfarad).
4. Block of dividers — 1 (0.01 — 1).
5. Electromechanical variable speed drive of coefficients — 1 per 20 variable coefficients (added separately).
6. Control panel.

Operational amplifiers are three-cascade with triode compensation in first cascade (see circuit of Fig. 37). Installation allows combination with automatic control equipment.

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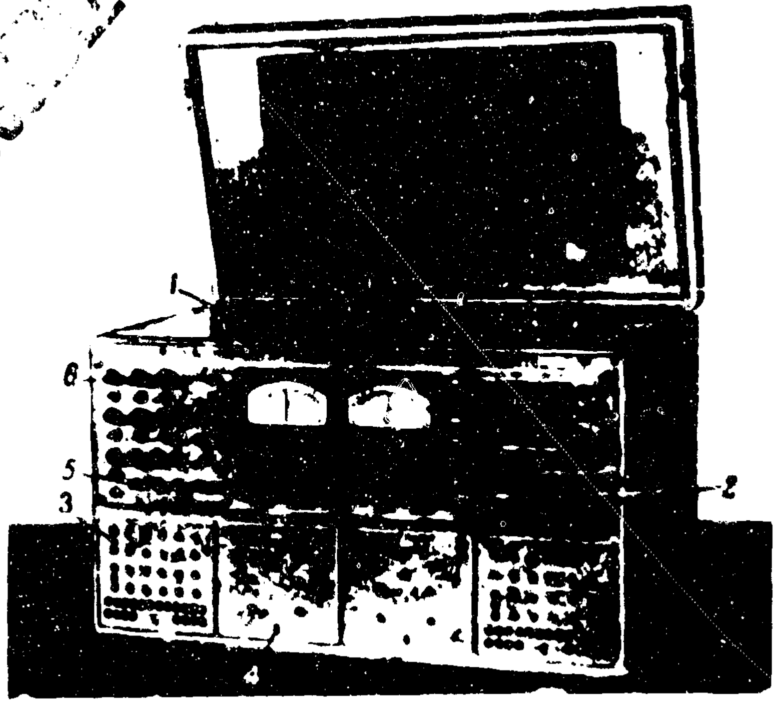

Fig. 265. Electronic analog computer of type MN-7 (industrial model). 1 - setting field, 2 - knobs for adjustment of zero, 3 - functional generator, 4 - multiplier block, 5 - control buttons, 6 - knobs for setting initial conditions.

Installation is intended for solution of ordinary linear and nonlinear differential equations up to sixth order inclusively, with constant coefficients. Installation is in the form of portable desk instrument with separate power block.

In MN-7 there are 16 operational amplifiers with triode compensation (see circuit of Fig. 37), six of which can execute function of integration. Remaining amplifiers are used as adders and sign-inverters.

In lower part of installation are four cells, into which can be inserted multiplier blocks or blocks of universal diode functional generators. These blocks do not have their own operational amplifiers. Multiplier blocks are made from silicon diodes and for work require three operational amplifiers. In functional generator they use tube diodes. Functional generator can be coupled both to input of operational amplifier and in feedback circuit. For work of generator there are required one or two operational amplifiers. Setup is carried out by plugs and short wires on setting field, located on top of the computer.

simultaneous two separate problems.

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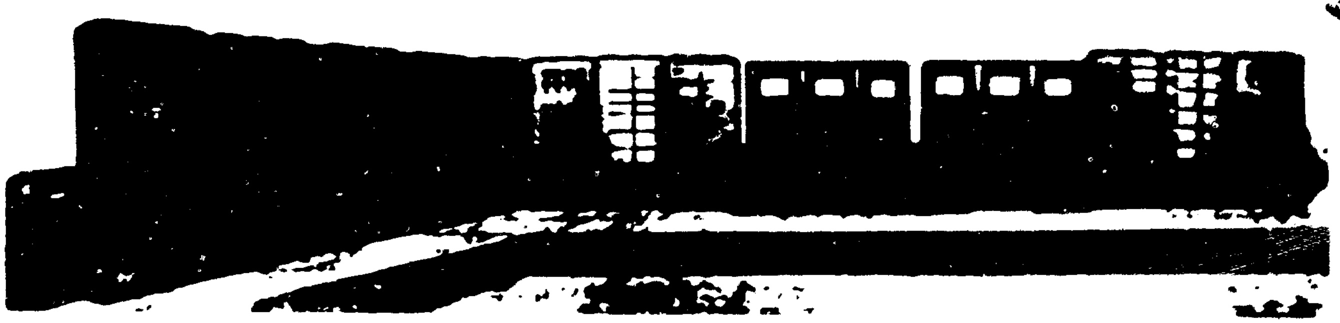

Fig. 266. Electronic analog computer of type MN-8 (industrial model).

Installation is intended for solution of ordinary linear and nonlinear differential equations up to 32-nd order inclusively with constant and variable coefficients. Of structural type, in the form of separate sections. Total sections — 11.

Connection of separate blocks is carried out by flexible cords.

In model is composed of:

1. Blocks of integrating operational amplifiers (4 amplifiers in each) — 8.
2. Blocks of summing amplifiers and amplifiers of change of sign (4 amplifiers in each) — 12.
3. Blocks of constant coefficients —48.
4. Blocks of variable coefficients (in every block 4 variable coefficients) — 12.
5. Functional generators of one argument — 10.
6. Multipliers (with 3 inputs) — 12.
7. Blocks of limitation of coordinates and zone of insensitivity — 6.
8. Differentiating blocks — 4.
9. Delay blocks — 4.
10. Dynamic blocks — 3.
11. Linear load sections — 3.
12. Cathode-ray indicators — 3.
13. Control panels — 2

Operational amplifiers are made in a circuit with automatic stabilization of zero level (see circuit of Fig. 48). Functional generators use principle of piecewise-linear approximation by diodes. Multipliers are made with application of stepping selector with possibility of multiplication of three input magnitudes in one device. Delay block uses of operational amplifiers. Model gives possibility of investigating transient with connection of automatic control equipment.

There is iteration of solution with possibility to examine solution on cathode-ray indicator. Model MN-8 allows us to solve simultaneous two separate problems.

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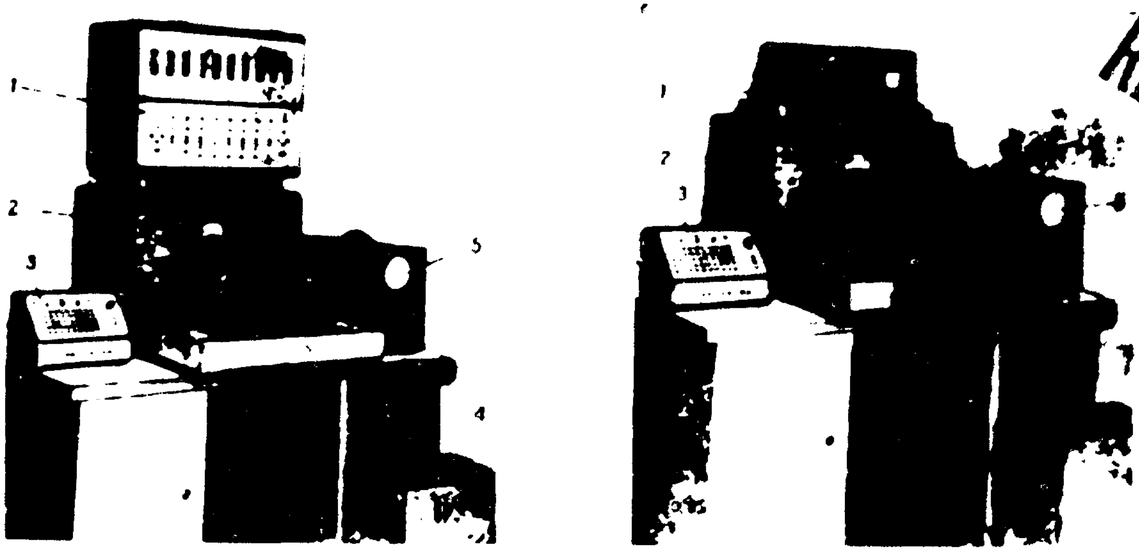

Fig. 267. Electronic analog computer of type EMU-5 (Academy of Sciences of USSR).

Variant I: 1 - nonlinear blocks, 2 - linear part, 3 - additional control block, 4 - power block, 5 - oscillograph EO-7 with tube 13LO36.

Variant II: 1 - nonlinear attachment, 2 - linear part, 3 - additional control block, 4 - power block, 5 - oscillograph EO-7 with tube 13LO36.

Installation is intended for solution of ordinary linear and nonlinear differential equations up to and including sixth order. Of structural type, in the form of portable desk instrument.

Installation is developed in two modifications. In composition Variant I is composed of:

1. Operational amplifiers — 12.
2. Multipliers — 1.
3. Functional generators — 1.
4. Blocks of typical nonlinearities — 4.
5. Additional control block — 1.
6. Power block — 1.
7. Cathode-ray indicator — 1.

In Variant II nonlinear blocks are replaced by nonlinear attachment composed of two multipliers and eight functional generators with 24 operational amplifiers. Operational amplifiers are made with automatic stabilization of zero level and economic output cascade (see circuit of Fig. 52). Installation allows combination with control equipment. There is possibility of automatic iteration of solution.

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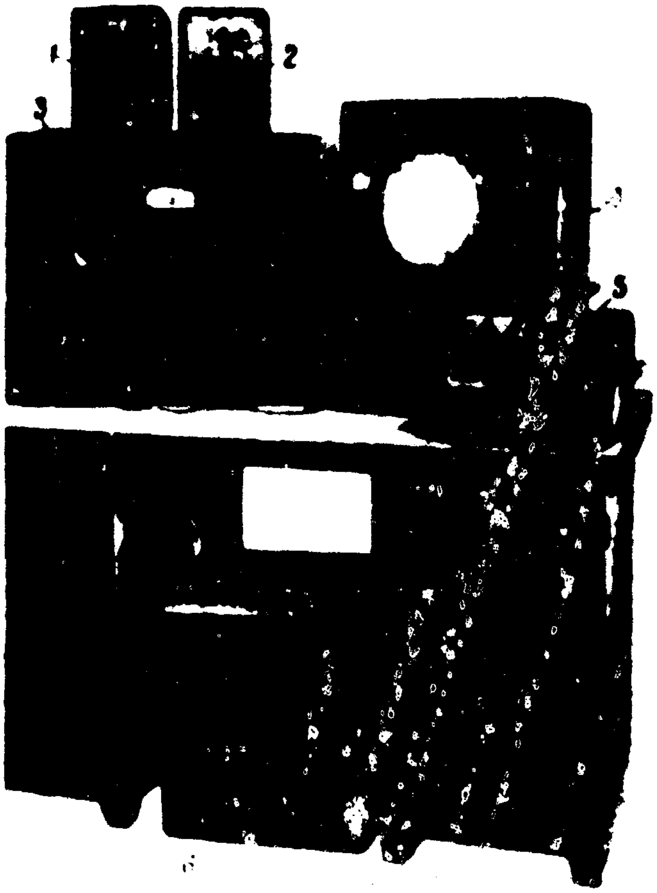

Fig. 268. Electronic analog computer of type EMU-6 (Academy of Sciences of USSR). 1 - multiplier, 2 - functional generator, 3 - linear part, 4 - electronic indicator, 5 - additional control block, 6 - power block.

Installation is intended for solution of ordinary linear and nonlinear differential equations up to and including the sixth order. Of structural type, in the form of portable desk instrument with separate power block.

Model is composed of:
1. Operational amplifiers — 12.
2. Multiplier — 1.
3. Functional generator — 1.
4. Blocks of typical nonlinearities — 4.
5. Additional control block — 1.
6. Cathode-ray indicator — 1.

Operational amplifiers are made with automatic stabilisation of zero level and economic output cascade (see circuit of Fig. 52). In installation and separate computing elements is conducted "ground insulation". Setup is carried out by bipolar plugs and short woggle joints on face of setting field.

Installation allows combination with control equipment. There is possibility of automatic iteration of solution.

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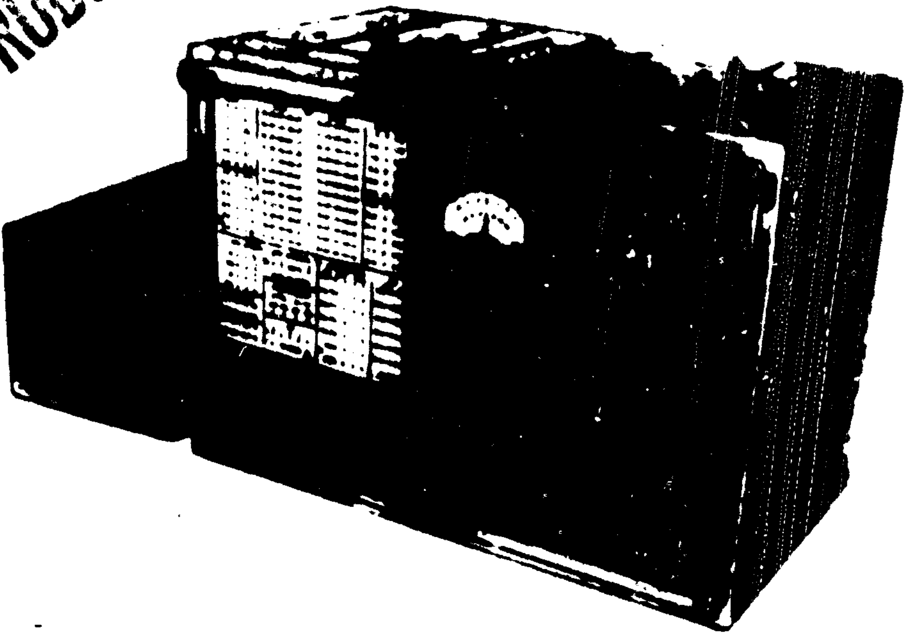

Fig. 269. Electronic analog computer of type MN-10 (industrial model).

Installation is intended for solution of nonlinear differential equations with constant coefficients up to and including sixth order. Number of nonlinear computing blocks (functional generators of one variable and multipliers) does not exceed 6. Setup is carried out on special switching field. All computing elements of machine and sources of stabilised power (block ESV-10) are made of semiconductors.

Installation is composed of: 24 operational amplifiers, 6 diode cells for reproduction of typical nonlinearities, 4 universal functional generators of one variable, 4 multipliers. Operational amplifiers are made of semiconductors by a scheme with automatic stabilization of zero level.

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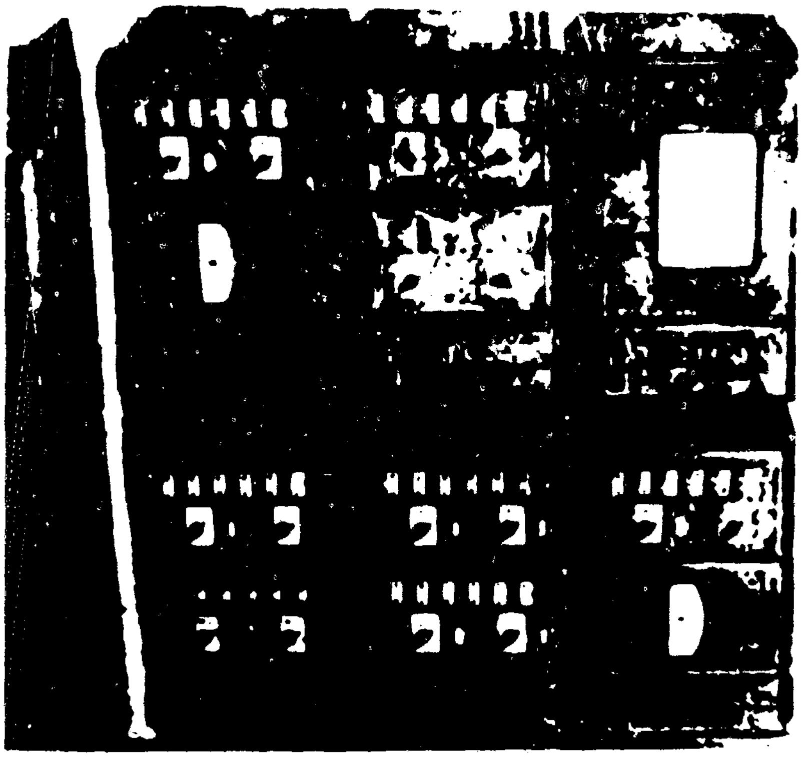

Fig. 270. Electronic analog computer of type EMU-8 (Academy of Sciences of USSR).

Installation is intended for solution of ordinary linear and nonlinear differential equations. Constructed in the form of separate base blocks, designed for solution of differential equations of second order. Every base block is supplied with four operational amplifiers (see circuit of Fig. 58) not requiring stabilized feeding, power supplies and a certain number of linear and nonlinear feedback circuits, in the form of plug-in inserts.

In typical setup of model, intended for solution of linear and nonlinear differential equations up to and including the 10-th order, there are foreseen following varieties of inserts: a) linear (integration, summation, change of sign, setting of scale, reproduction of typical nonlinearities); b) nonlinear, intended to execute multiplier-divider operations; c) nonlinear, intended for reproduction of nonlinear dependences of one argument; d) nonlinear, intended for reproduction of fixed nonlinear dependences of type sine, cosine; e) intended to execute operations of control; f) measuring, intended to execute operations of measurement during adjustment. Cathode-ray indicator is mounted in separate base block.

Installation allows union with real equipment.

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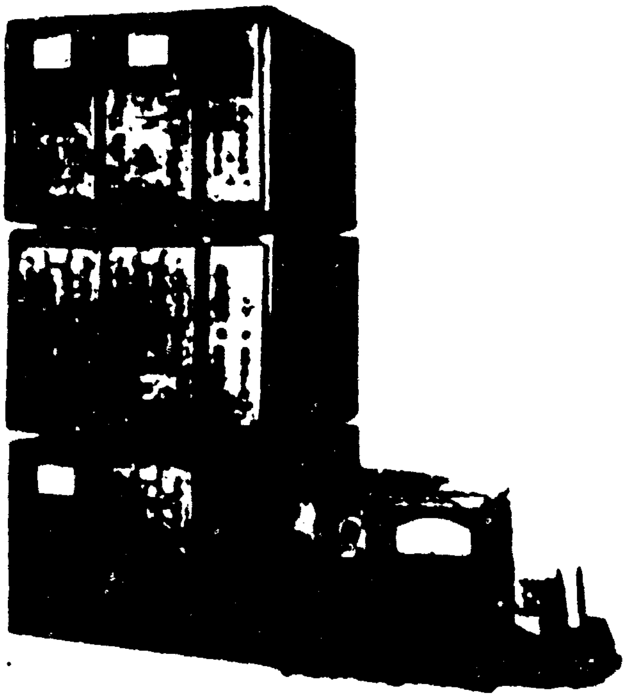

Fig. 271. Set of nonlinear blocks NNB (industrial model).

Setup of nonlinear blocks is intended for expansion of the circle of problems, which can be solved by linear electronic analogs of type IPT-5, MPT-9, LMU-1, and increase of number of nonlinear computing elements of MN-7 and others.

NNB ensures fulfillment of following nonlinear operations: reproduction of three nonlinear dependences of one variable, three operations of multiplication or division. It is constructed in the form of three base blocks. In each base block are two doubled operational amplifiers, made in a circuit with parallel amplification channels (see Fig. 58), two plug-in inserts of nonlinear feedback circuits and a power supply. Development on basis of installation EMU-8.

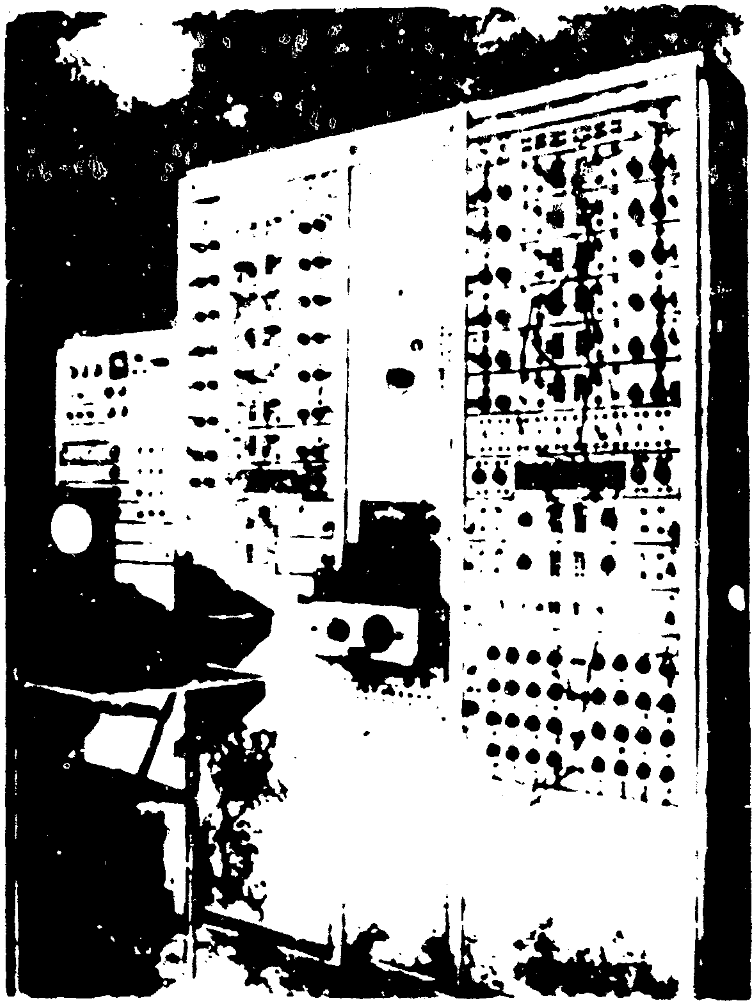

Fig. 272. Electronic analog computer of type EDA (Academy of Sciences of USSR).

Installation is intended for solution of ordinary linear and nonlinear differential equations up to and including 19-th order. Of structural type, constructed in the form of two cabinets.

Installation is composed of:

1. Operational amplifiers — 38.
2. Multipliers — 4.
3. Functional generators of one variable — 4.
4. Generator of harmonics — 1.

Operational amplifiers are supplied with system of periodic setting of zero level (see Fig. 150). Multipliers are based on the pulse-time principle. Functional generator uses approximation of given function by 8 terms of a Fourier series (I. S. Bruk and N. N. Lenov [1]).

Installation allows combination with control equipment.

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Fig. 273. Complex of electronic simulation equipment of type MN-14 (industrial model). Intended for solution of complicated dynamic problems, described by ordinary linear or nonlinear differential equations up to and including 30-th order with large number of nonlinear dependences.

Installation includes: 90 operational amplifiers with automatic stabilization of zero level, of which 30 can work as integrator, 30 — as adder, 20 — in change of sign made and 10 — auxiliary to execute certain linear, nonlinear and logical operations. To execute nonlinear operations there are: 50 blocks, executing operations and 20 blocks of universal functional generators of one variable, 4 blocks of typical nonlinearities, 6 blocks for reproduction of trigonometric functions, 12 pulse-time multipliers. Total number of amplifiers in machine is 360. There is automation of introduction and recovery of data. Setup is carried out on plug-in setting field provided with digital voltmeter and a two-coordinate table.

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Fig. 274. Complex of electronic simulation equipment of type EMU-10 (industrial model); on the right and on the left — universal stands; in center — specialized stand.

Complex of simulation equipment EMU-10 is intended for solution of linear and nonlinear differential equations up to and including 24-th order, and also solution of certain problems of optimization with number of regulated parameters, not exceeding 7. Equipment consists of universal and specialized stands. Universal stand contains: 48 broad-band operational amplifiers, assembled in circuit with parallel amplification channels (see Fig. 59), of which 24 can work as integrator; 4 electronic multipliers; 4 electronic universal functional generators; 4 combined electromechanical functional generators (for 20 coefficients; variable in time; 4 electromechanical multiplier-dividers; 84 ten-turn potentiometers, of which 64 are set automatically by servo system of digital voltmeter; plug-in setting field, standard of tension and time. System of control ensures static and dynamic control of problem, stop at a given moment, iteration of solution for all or part of the computing elements. It includes special block for automatic change of scale of representation of variable. Specialised stand includes semi-channel relay optimiser and 4 blocks of variable delay, for which time lag can continuously change into voltage functions of the model.

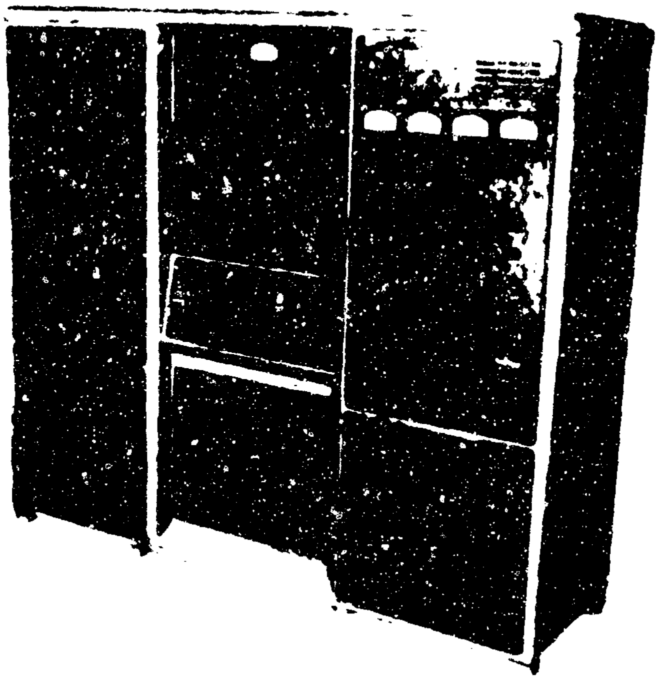

Fig. 275. Electronic analog computer with automated solution scanning of type MN-11 (industrial model).

Installation MN-11 is intended for automatic finding of a solution, satisfying certain prescribed criteria. Installation includes: 9 integrating operational amplifiers, 38 summing operational amplifiers, 6 multipliers, 3 blocks of variable coefficients, 4 three-decade voltage dividers. There are three main regimes: detecting of solution by method "minimization," semiautomatic detecting of solution by method "survey", ordinary solution of problem with selection manually of required solution.

Structually the installation consists of the electronic model itself, working with frequency of repetition of solution 100 times per second, power section and control table for checking and tuning plug-in elements and functional blocks.

Table XV. Main Indices of Electronic Analog Computers Produced Abroad

No. in order	Designation of electronic model and firm	Country	Year of issue (tentatively)	Main characteristics (composition of computing blocks)	Work with real objects	Quantity of production	Note
1	GAP/R of G. A. Philbrick	USA	1944 and following	Set of electronic linear and nonlinear functional blocks. Nonlinear blocks: multipliers and functional generators of one variable. Blocks are connected in computing circuit by cords	Provided	Released in series	
2	Anacom of Westinghouse Electric Corp.	USA	1948	A-c model	Provided	Unit	
3	Flight simulator of M.I.T.	USA	1948	Nonlinear electronic model up to ninth order on a-c electromechanical blocks at 400 cps. More than 200 decoupling amplifiers	Provided	Unique machine for US Navy	In composition of machine is [illegible] block
4	BEAC of Boeing Airplane Co.	USA	1948	Linear model — 1 stand per 12 operational amplifiers, 12 linear potentiometers, diode limiters, voltmeter and power supply	Provided	Released in series	
5	Analog computer "Boeing" of Boeing Airplane Co.	USA	1952 and following	20 operational amplifiers (of them 12 for summation and integration). 12 diode elements for nonlinear circuits, 20 potentiometers, 4 electronic voltage stabilizers; furthermore, functional generator in servo system and device for automatic cutting of zero	Provided	Released in series	
6	Simulator GEDA of (Goodyear Aircraft Corp.)	USA	1950 and following	Nonlinear model up to 12-th order with electromechanical and electronic nonlinear computing elements	Provided	The same	
7	Electronic model of Sperry Gyroscope	USA	1950	Linear (possibly, nonlinear) simulator	Provided	Produced for own needs	
8	Curtiss, of Curtiss Wright Corp.	USA	1950	Linear and nonlinear	Provided	Released in series	
9	IDA of Computer Corporation of America	USA	1951	Linear model up to 8-th order, for which there is produced separately a set of nonlinear blocks for nonlinearities inherent in control equipment	Provided	Released in series	
10	Electronic simulator of Bell	USA	1950-1951	Nonlinear model up to 12-th order with electromechanical nonlinear computing elements	Provided	The same	

Table XV Continued

No. in order	Designation of electronic model and firm	Country	Year of issue (tentatively)	Main characteristics (composition of computing blocks)	Work with real objects	Quantity of production	Note
11	Typhoon of RCA	USA	1950-1951	Nonlinear installation, on which were solved problem of control of rocket missiles. It has accurate blocks of multiplication, built with application of digital elements. In installation are 443 operational d-c amplifiers with automatic stabilization of zero level, of which 80 are integrating. There are also 18 servo systems for nonlinear blocks	Not provided	Unique machine for US Navy	
12	[illegible] of Northrop Aircraft Inc.	USA	1951-1952	Nonlinear model	Provided	Unique machine for US Navy	
13	[illegible] of Northrop Aircraft Inc.	USA	1951-1952	Nonlinear model	Provided	Small lot for own needs	
14	EASE of Beckman with participation of Berkeley	USA	1952 and following	Linear and nonlinear blocks. In 1 stand there are 20 operational amplifiers and a power supply. Separate are control setting field and potentiometers for assignment of constant coefficients. Block of multiplication and functional generators are electronic	Provided	Released in series	
15	Electronic model of [illegible] Aircraft Computers	USA	1952	Nonlinear and linear installations	Provided	In 1952 was development first of all for private needs	
16	Type "16-31" of Electronic Associates Inc.	USA	1953 and following	Precision nonlinear electronic model. Various composition of stands can have from 20 to 96 operational amplifiers, from 4 to 20 multiplication blocks with servo system, a number of electromechanical blocks with servo system for functional generation, a setting field and other blocks	Provided	Released in series	
17	Type "[illegible]" of [illegible] Instruments	USA	1953-1954 and following	Small portable nonlinear electronic model with 12 operational amplifiers, 1 block of nonlinearities with servo system, 4 electronic diode elements	Provided	Released in series	
18	[illegible]-Model 300-4 of [illegible] Laboratories	USA	In 1953 was in development	Small cheap desk nonlinear simulator. Consists of set of blocks connected by cords	Provided	In development	

Table XV Continued

No. in order	Designation of electronic model and firm	Country	Year of issue (tentatively)	Main characteristics (composition of computing blocks)	Work with real object	Quantity of production	Note
19	Analog computer of type REAC-301, C-202 of Reeves Instrument Corp.	USA	1954	Small portable linear simulator with 12 stabilized operational amplifiers	Provided	Released in series	
20	Analog computer of type REAC-400 of Reeves Instrument Corp.	USA	1955	Nonlinear simulator. It contains 17 doubled operational amplifiers nonlinear functional blocks of both electronic and electromechanical type and a plug-in setting field	Provided	Released in series	Firm has produced since 1947 300 installations
21	Analog computer of Donner Scientific Co.	USA	1954	Small portable linear model. It contains 10 operational amplifiers	Provided	Released in series	
22	Analog computer of Raytheon Mfg. Co.	USA	1954	Small portable linear model. It contains 9 unstabilized operational amplifiers	Provided	Released in series	For description see B. V. [illegible] and N. A. [illegible] (1)
23	Analog computer of Central Specialists, Inc.	USA	1955	Small portable linear model of sixth order. It contains 12 operational amplifiers	Provided	Released in series	
24	F-86 R of [illegible] Redifon Ltd. (London)	England	1953	Nonlinear specialized machine	Provided	Unit issue	
25	Analog computer of Short Brothers and Harland	England	1953	Analog computer, intended for solution of three linear second order differential equations. It consists of 18 d-c amplifiers and corresponding number of RC circuits and 20 three-decade resistance boxes	Provided	Released in series	
26	Electronic model GEPT of Plessey Co.	England		Universal model of 16-th order, intended for solution of problems of dynamics. It consists of 90 operational amplifiers, 12 multipliers, 12 functional generators. Multipliers are based on the pulse-time principle. Functional generators with application of cathode-ray tubes. Operational amplifiers are equipped with a system of group setting of zero level	Provided		Total number of tubes is [illegible]

Table XV Continued

No. in order	Designation of electronic model and firm	Country	Year of issue (tentatively)	Main characteristics (composition of computing blocks)	Work with real object	Quantity of production	Note
47	Simulator "Tri-dac" of Elliott Brothers	England	1954	Simulator is intended for solution of three-dimensional problems of dynamics of flight (guided rockets and other objects). Installation contains 650 operational amplifiers with automatic stabilization of zero level, electronic and electro-hydraulic nonlinear computing elements (multipliers, functional generators, generators of coordinates and others). Total power consumption of 650 kilowatts	Provided	Unit sample. Similar installation (AGWAC), but of less power was set up in Australia	For installation a special building was built. Description in [illegible]
48	Simulator OME-L2 of SEA (Société d'Electronique et d'Automatisme)	France	1950 and following	Linear model of 15th order	Provided	Released in series	
49	Simulator of type OME-L 2 of SEA	France	1954 and following	Linear model of [illegible] order with 18 potentiometers for setting of constant coefficients. It is supplemented by a separate nonlinear computing blocks	Provided	Released in series	
50	Model assembly of stands of OME, forming the simulator of SEA	France	1954 and following	Nonlinear model, consisting of two stands. In one there are 12 operational amplifiers with automatic stabilization of zero level and individual power supplies, in the second stand are nonlinear computing elements (multipliers, square-law and sine functional generators). Recording devices and control panel	Provided	Released in series	
51	Simulator OME-P2 of SEA	France	1955	Linear model of 12-th order of increased accuracy. 24 operational amplifiers with automatic stabilization of zero level. 14 blocks of constant coefficients. It can be augmented by nonlinear computing elements	Provided	Released in series	
52	Simulator of type [illegible] of Laboratories R. Derveaux	France	1955	Installation is intended for modeling kinetics of nuclear reactors. It consists of 10 operational amplifiers, 64 potentiometers and is supplied with memory units for control of correctness of work	Provided	Released in series	

Table XV Continued

No. in order	Designation of electronic model	Country	Year of issue (tentatively)	Main characteristics (composition of computing blocks)	Work with real object	Quantity of production	Note
	Electronic simulator of Brussels Free University	Belgium	1953-1954	Large nonlinear simulator, containing 6 high-power operational amplifiers, 60 operational amplifiers 9 multipliers, 6 functional generators, 4 generators of given function of time, control panel, recording and writing equipment	Provided	Unit sample	
14	Simulator of Polish Academy of Sciences	Poland	1954	Nonlinear model, working with iteration of solution. In composition of model enter 8 summation blocks, 6 blocks of multipliers, based on double amplitude modulation, 6 blocks of cathode-ray functional generators, measuring and recording devices	Not provided	Unit sample	
15	Simulator of Research Institute of National Society in Stockholm	Sweden	1952	Linear block with 4 d-c amplifiers (2 integrating), 5 accurate potentiometers. In all, 8 linear blocks. Later machine was augmented by block of multiplication with pulse-time circuit. There is a functional device, built on the basis of magnetic drum	Provided	Unit sample	
16	Analog computer for investigation of servo systems. Prepared in Norway on basis of GEDA	Norway	1953	Model with 24 amplifiers. It contains adders, integrators, phase inverters, multipliers and functional generators, for which they use a circuit with diodes	Provided	In Norway by 1955 there were 6 such installations	
17	Electronic simulator Tosac III of Tokyo-Shibaura Electric Company	Japan	1953	Installation with artificial iteration of equation for designing systems of automatic control. It is assembled from d-c amplifiers, which execute operation of integration, summation and separation of circuits. For modeling of transfer functions of separate sections of CAP there are used passive circuits	Impossible	Released in series	Firm releases also installations without iteration of equation Tosac I & II (i.e. operational amplifiers) and Tosac IV, a basic installation with [illegible] operational amplifiers

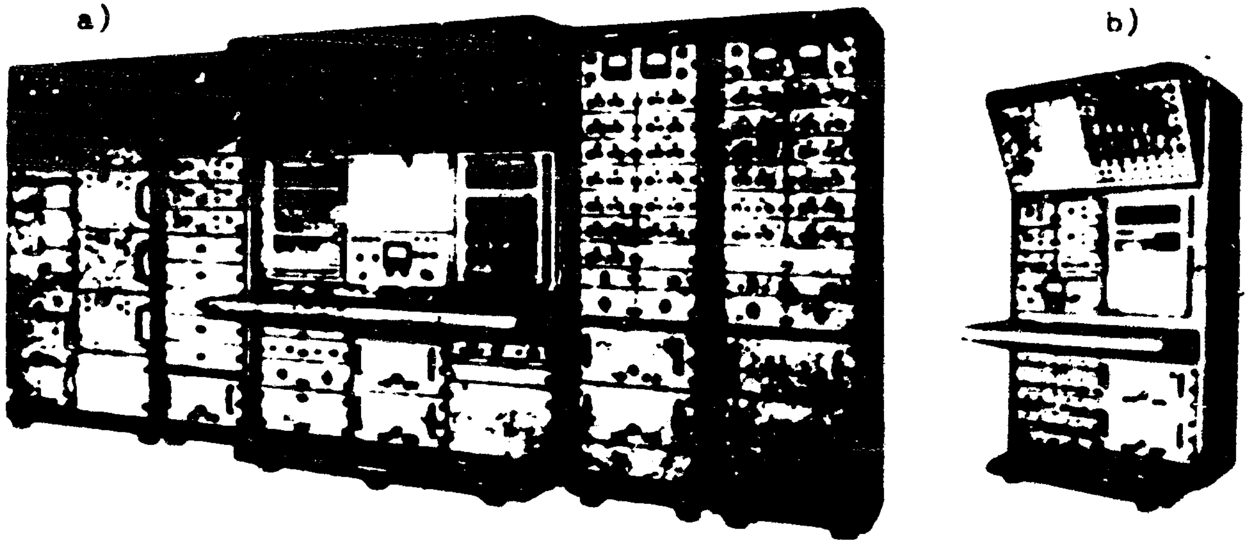

Fig. 276. Electronic analog computer of firm "Electronic Associates" (United States) consists of 50 operational amplifiers, 10 electromechanical multipliers, made with servo systems, control panel with two setting fields, 156 potentiometers for setting initial conditions, two voltmeters and other equipment.

Firm also releases installation of smaller dimensions of type 16-31 R (see Fig. 276b). It contains 20 operational amplifiers, four multipliers with servo systems, 32 voltage dividers and one plug-in setting field.

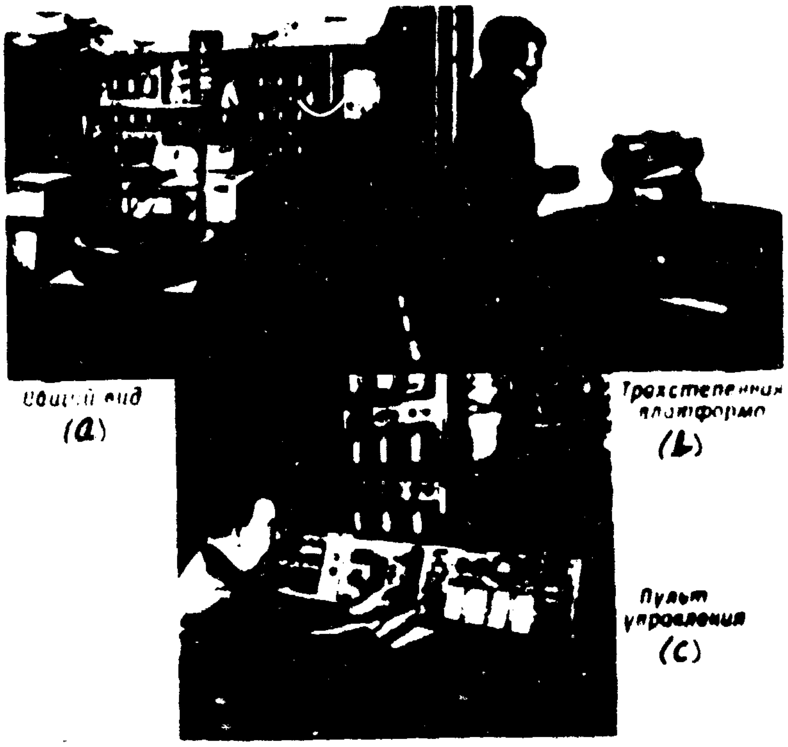

Fig. 277. Electronic analog computer (Radio Corporation of America) of type "Typhoon".
KEY: (a) General form; (b) Three-stage platform; (c) Control panel.

It is intended for investigation of problems of control of spatial motion of missiles. It consists of four main blocks: block of rocket missile, carrying out solution of equation of motion of missile for given initial conditions and current values of forces and moments; aerodynamics unit, executing, according to given data of wind tunnel test and current values of coordinates and speed of object, calculation of forces and moments; target block, giving coordinates and speed of target; guidance unit, producing control signals and stabilization by data on motion of target and missile.

Composition of computing elements of installation can be characterized by the following table:

Designation of blocks of model	Number of computing elements		Note
	Stabilized operational amplifiers	Servo systems	
Block of aerodynamics	135	6	There are 36 multipliers of increased accuracy. From total of 445 operational amplifiers, 80 are intended for work as integrator. All equipment is mounted on 43 panels
Guidance unit	148	11	
Block of rocket missile	135	—	
Target block	9	2	Total number of tubes 4000
Recording devices	18	—	Total power consumption from network 46 kilowatts
In all	445	19	

Results of solution are fixed on two recording tables. For visual observation there is a three-stage platform with model of missile, reproducing motion about center of gravity. Trajectory of motion of center of gravity of rocket missile and the target can be observed on special table with luminescent balls. Possibility of conjunction with real equipment of stabilization and control is not foreseen.

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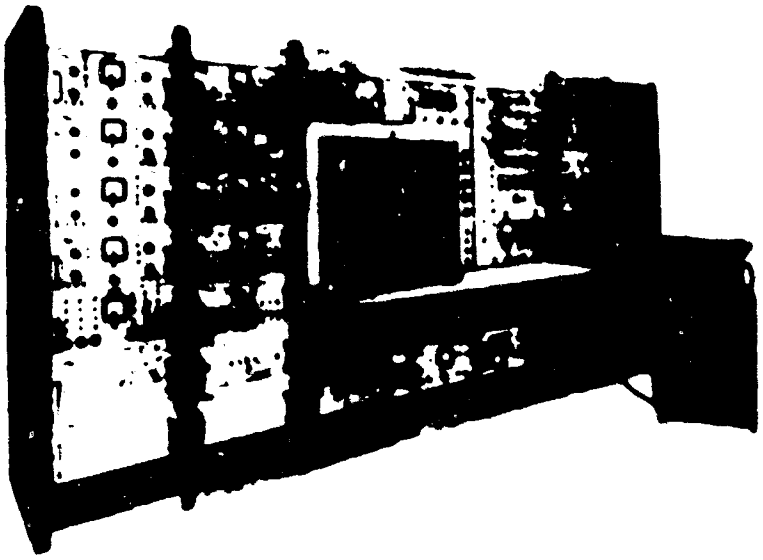

Fig. 278. Electronic analog computer of "Beckman Instruments, Inc." type EASE-1100 (the United States).

It is intended for solution of linear and nonlinear differential equations; it consists of 60 operational amplifiers with automatic stabilization of zero level (made by Goldberg's diagram). To execute nonlinear operations there are 8 electronic pulse-time multipliers with two frequencies of saw-toothed voltage for work with narrow and broad band width, 10 electromechanical multipliers, made with servo systems. Electronic diode functional generators (20) allow us to carry out approximation in 20 sections. With the help of additional block there is foreseen possibility of automatic adjustment of these generators for reproduction of given nonlinear dependence. Furthermore, in installation are mounted 6 devices for transformation of coordinates, 8 sine-cosine generators, 10 pairs of diode elements. Setup is carried out on plug-in setting field with 3600 holes, by shielded connecting wires. With machine there can be used system of automatic setting of potentiometers from punched tape and electric typewriter. By this it is possible to give signal for measurement of output of any computing element of the installation.

Passive circuits of operational amplifiers are located in special housing, inside which a constant temperature is maintained.

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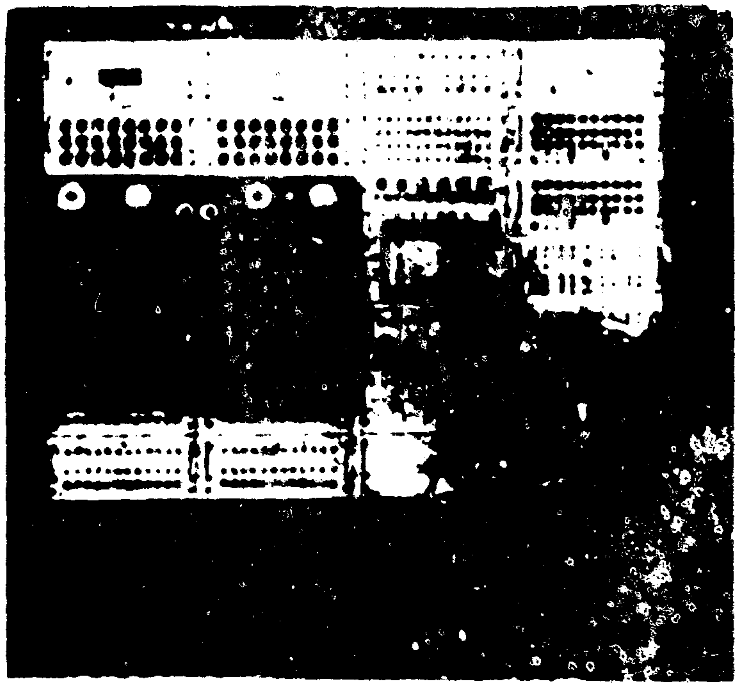

Fig. 279. Electronic analog computer of "Goodyear Avicraft Corporation" type GEDA.

Installation is intended for solution of linear and nonlinear differential equations. Installation includes: 108 operational amplifiers with centralized system of compensation of drift of zero level, 6 electronic diode functional generators, approximating given nonlinear dependence in 10 sections, 6 electronic pulse-time multipliers.

For setup there serve two plug-in setting fields. There is a digital voltmeter, which in combination with accurate bridge is used for setting potentiometers and automatic printing of results. Servo systems are used for automatic setting of potentiometers and functional generators. In installation is a system, allowing us automatically to control and read output voltage from any computing element.

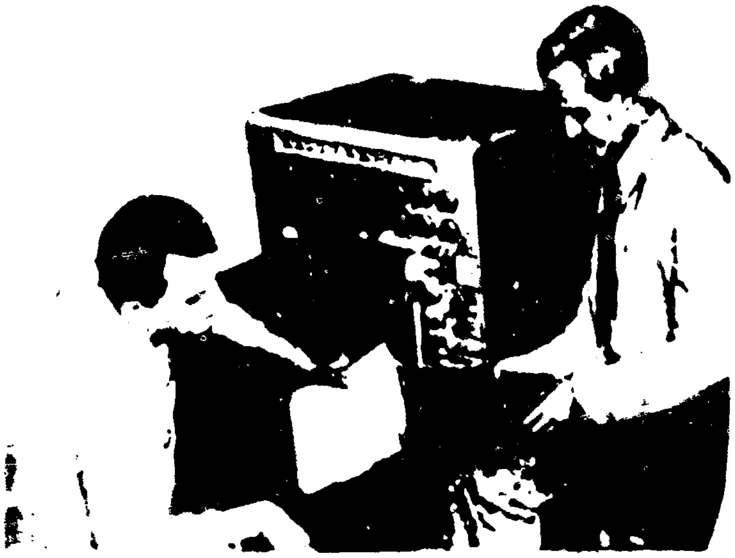

Fig. 280. Electronic analog computer of Reeves Instruments Corporation of type REAC-301.

Installation is intended for solution of ordinary linear differential equations up to and including sixth order. It consists of 12 operational amplifiers with automatic stabilisation of zero level, 18 accurate potentiometers, stabilized power supplies and a 26-volt supply for relays. Setup is carried out on plug-in setting field.

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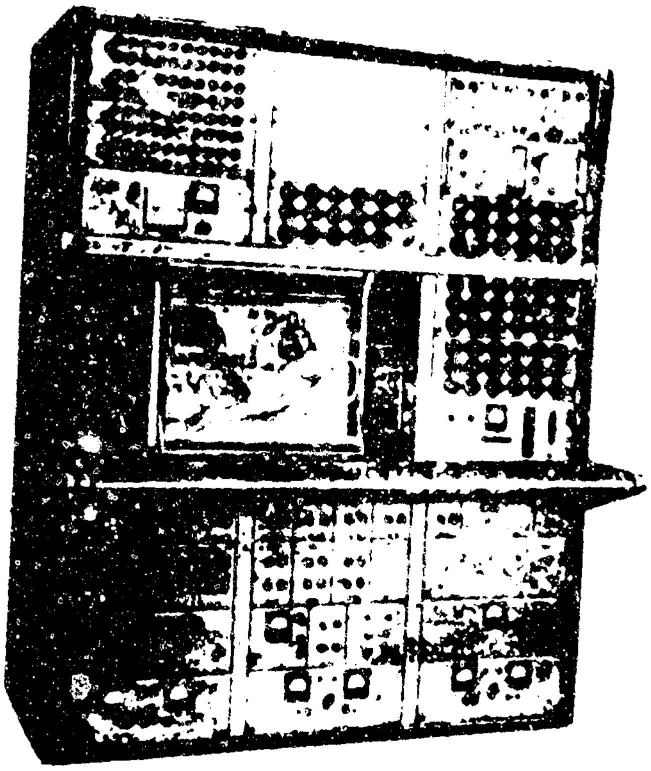

Fig. 281. Electronic analog computer REAC-400 of Reeves Instruments Corporation.

Installation contains 17 doubled operational amplifiers, four high-speed servo systems for multipliers, two transformers of coordinates and plug-in setting field.

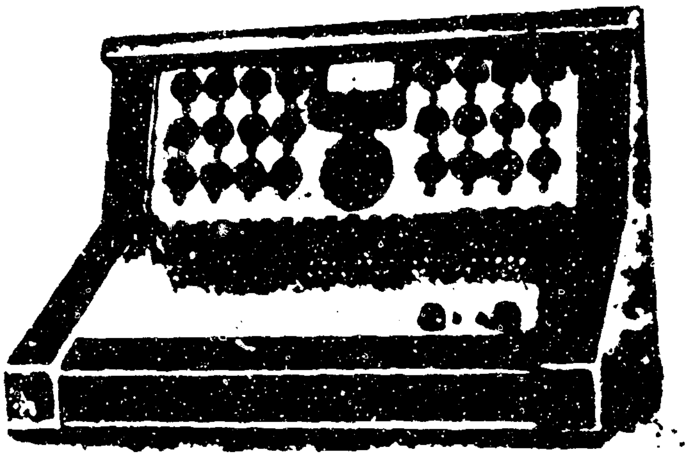

Fig. 282. Electronic analog computer of Control Specialists, Inc.

Installation is intended for solution of ordinary linear differential equations up to and including sixth order. It consists of 12 d-c operational amplifiers with gain factor near $3 \cdot 10^4$ (six of them can work as an integrator).

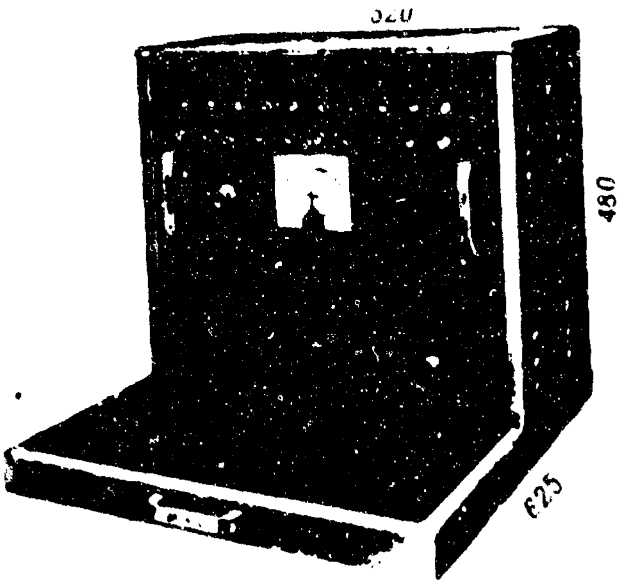

Fig. 283. Electronic analog computer of Donner Scientific Company.

Installation consists of 10 operational amplifiers, each of which is made with one pentode. Setup is carried out on a separate setting field.

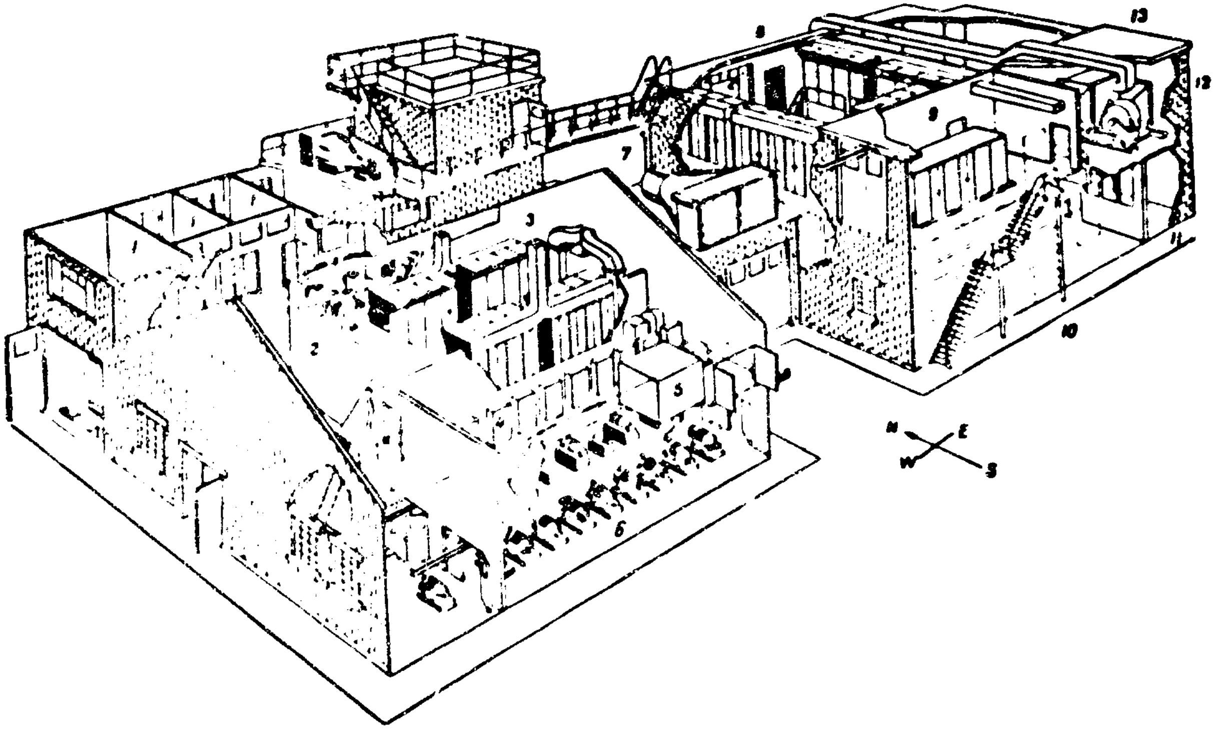

Fig. 284. General view of site of computer of type "TRIDAC" of Elliot Brothers (England). 1 — laboratories, 2 — generating substation, 3 — site for electromechanical servo systems, 4 — hydraulic reductors, 5 — oil tank, 6 — location of hydraulic pumps, 7 — oil cooler, 8 — control station, 9 — location for rectifiers, 10 — location for checking electronic devices, 11 — location for storage of instruments, 12 — fan, 13 — air cooler.

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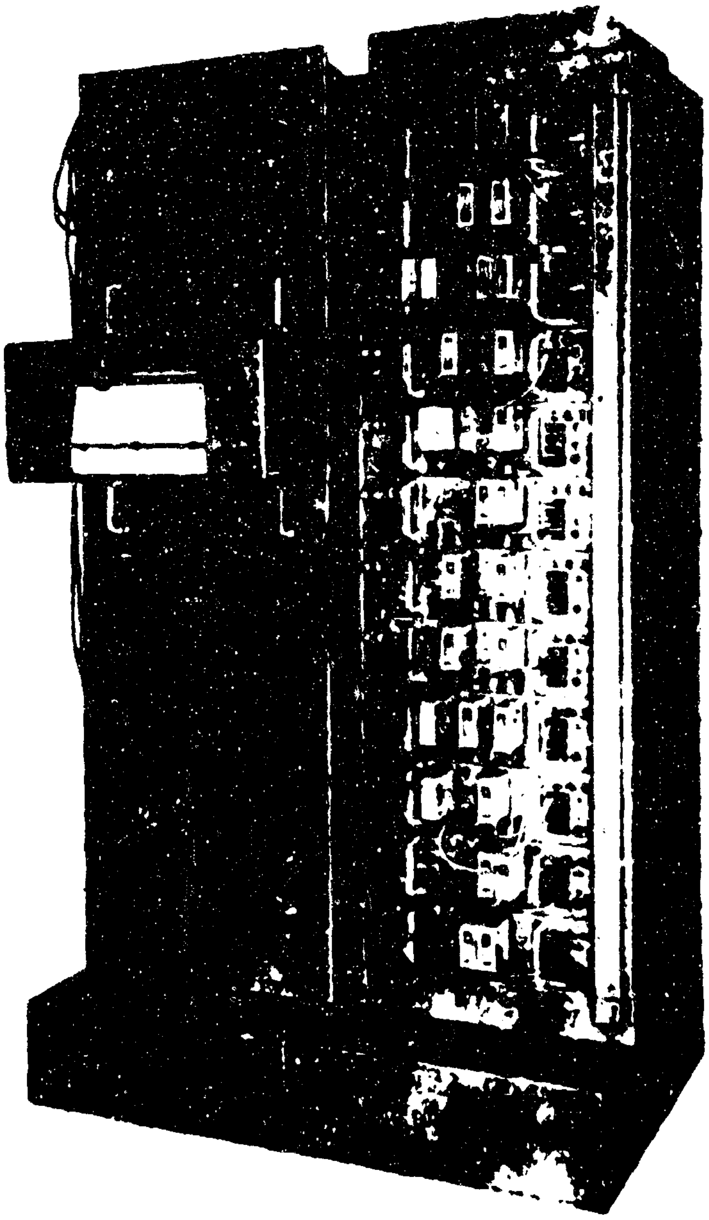

Fig. 285. Model assembly of stands of non-linear analog with 12 operational amplifiers. SEA (France).

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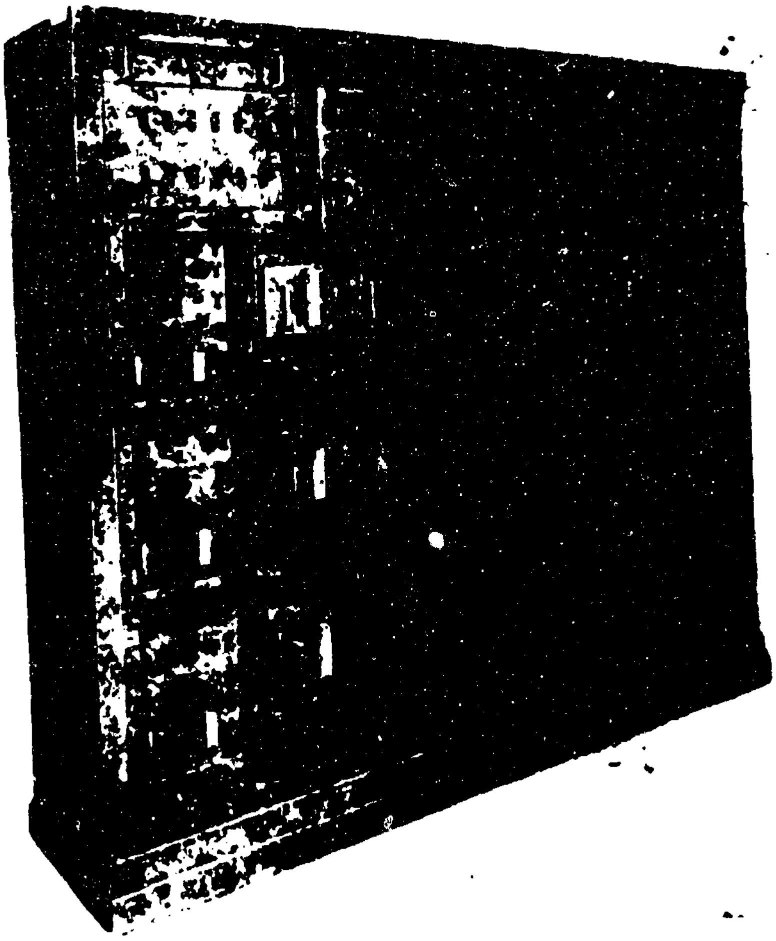

Fig. 286. Simulator of increased accuracy of type OME-P-2 of SEA (France).

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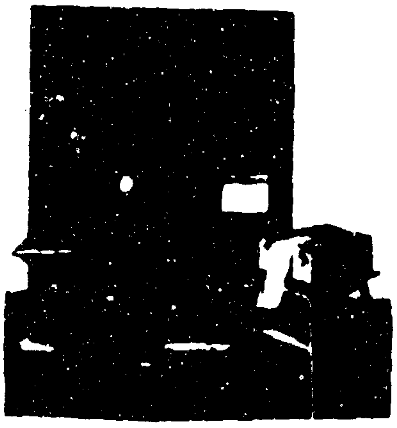

Fig. 287. Electronic analog computer, with iteration of solution, of Tokyo Shibaura Electric Co.

Installation consists of 12 operational amplifiers, intended for work as integrator, adder, and also for fulfillment of other linear operations, and 6 amplifiers, utilized for multiplication by constant coefficient and inversion. To execute nonlinear operations there is foreseen a multiplier, two functional generators and a certain number of elements for reproduction of type nonlinear characteristics. Limits of linearity of output voltage ± 50 v. Control voltage is fed from separate generator of cyclic pulses. For observation of solution there is used a built-in cathode oscillograph.

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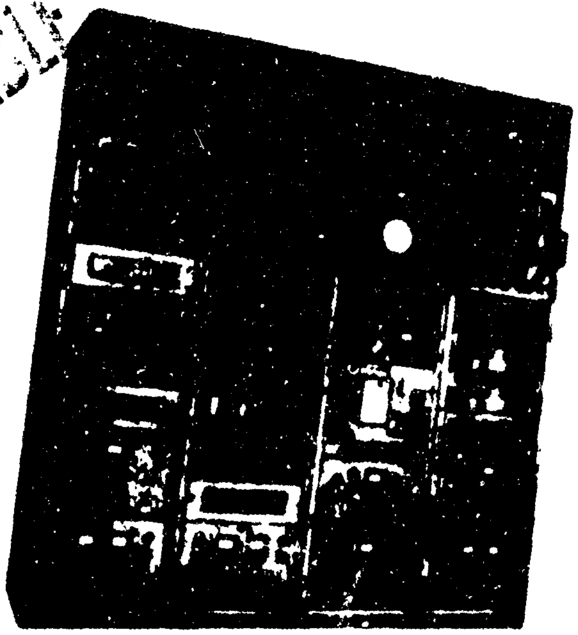

Fig. 288. Electronic analog computer with iteration of solution, Nippon Electric Co.

Installation provides solution of problem in 20, 50 and 100 milliseconds; it consists of 12 integrators, 10 summers, 6 scale blocks and 4 inverters. To execute nonlinear operations there are: one multiplier, one diode functional generator, one cathode-ray functional generator and three blocks for reproduction of typical nonlinearities.

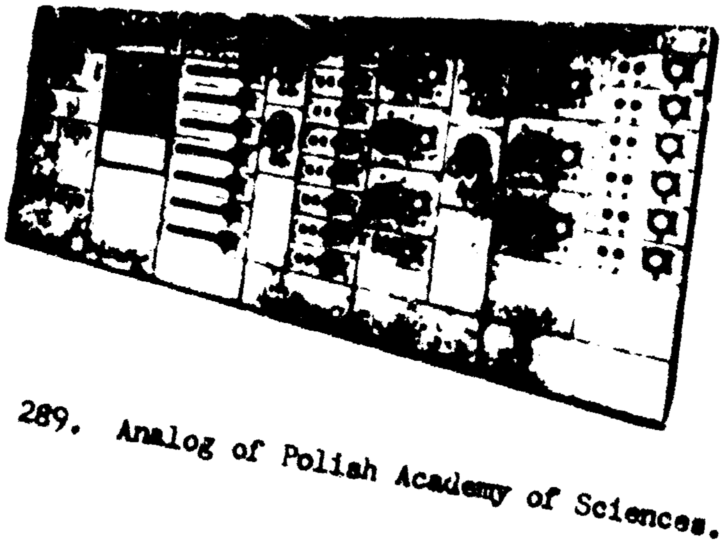

Fig. 289. Analog of Polish Academy of Sciences.

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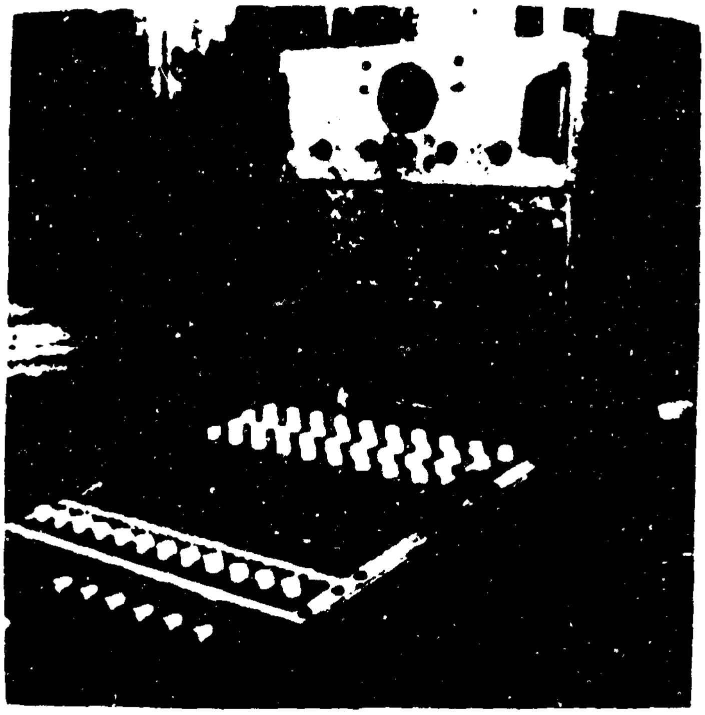

Fig. 290. Small electronic analog computer of type MEDA (Czechoslovakia).

It is intended for solution of linear and nonlinear differential equations. It contains 20 operational amplifiers with automatic stabilization of zero level.

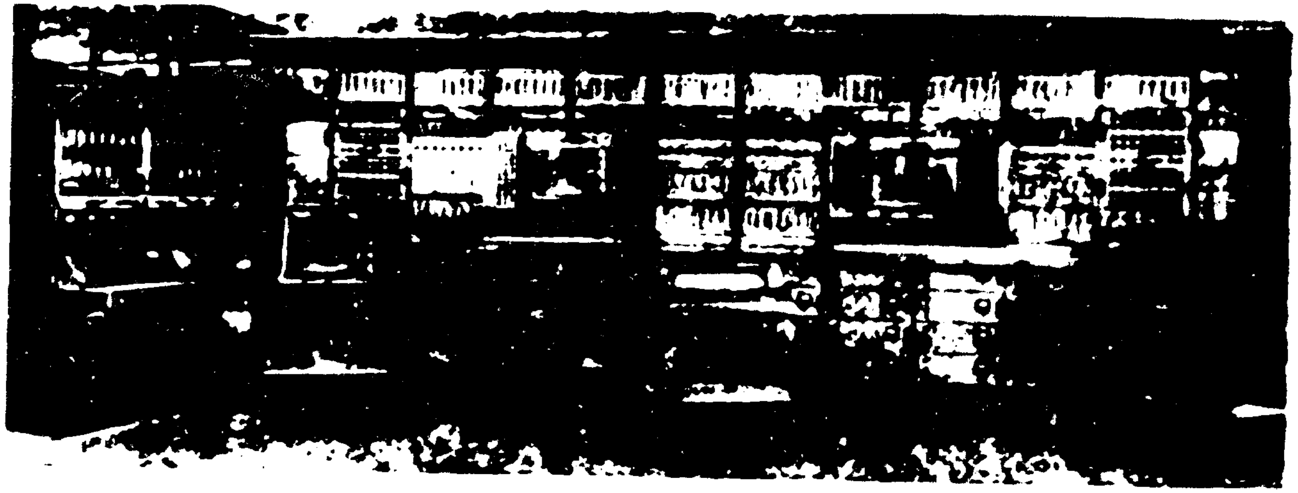

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Fig. 291 Complex of electronic analog equipment (developed VVZ TESLA Pardubice) of type AR3 (left part), AR4 (right side) (Czechoslovakia).

AR3 is electronic analog computer of average divension, consisting of 112 operational amplifiers, of which 64 are used to execute linear, and 48, nonlinear operations. Nonlinear part includes: 16 diode limiters, 4 universal functional generators, 16 specialized functional generators, 4 diode multipliers, 8 electromechanical multipliers, 10 voltage comparators, 6 devices for reproduction of time dependences.

AR4 — small electronic analog computer, having 16 operational amplifiers and the following nonlinear computing elements: 4 diode limiters for inclusion in feedback circuit, 2 diode bridge limiters for inclusion at input of operational amplifier, 8 specialized functional generators (4 for reproduction of function x^2, 2 for reproduction of function x^3 and 2 for reproduction of function sin x).

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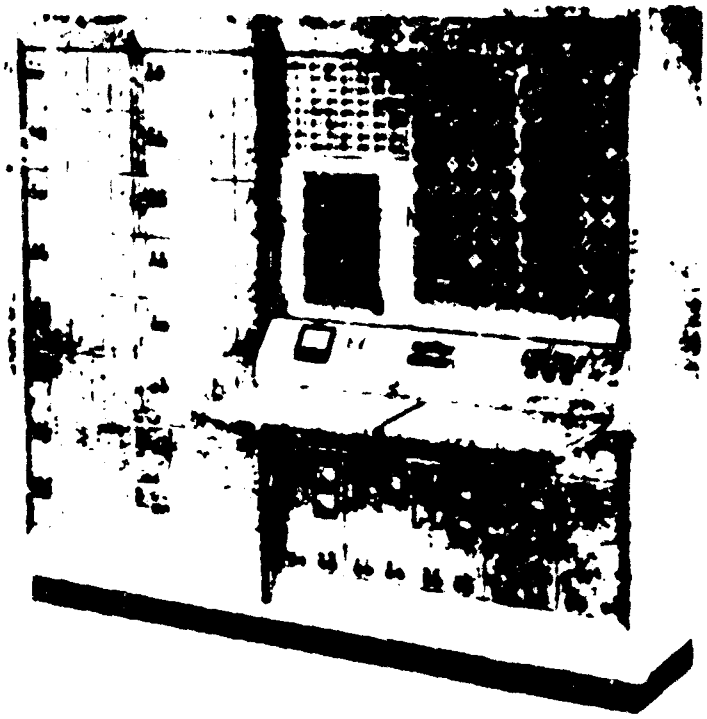

Fig. 292. Electronic analog computer of VEB Archimedes Rechenmaschinenfabrik of type EAR-64 (German Democratic Republic).

Universal installation for solution of linear and nonlinear differential equations. It consists of 64 operational amplifiers with automatic stabilization of zero level, of which 32 can work as integrator and summer. To execute nonlinear operations there can be used 16 nonlinear computing elements (electronic multipliers and universal functional generators). Nonlinear computing elements are constructed on basis of piecewise-linear approximation on diode elements. There is a setting field of 100 accurate potentiometers for setting coefficients, 4 amplifiers with relay output.

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